Defying common sense: Epistemological beliefs in an introductory physics course

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Abstract

Students' beliefs about knowledge and learning in physics may have a significant effect on how they approach the material and what they learn. This dissertation describes a study of "epistemological beliefs." I interviewed seven students enrolled in an introductory physics course, meeting several times with each over the semester. The interviews involved a variety of conversations and tasks closely tied to the context of the course.

Through the use and development of a analytic framework, it was possible to construct characterizations of subjects' beliefs along three dimensions:

1) beliefs about the structure of physics knowledge, whether it is made up of individual "pieces" or constitutes a coherent system;

2) beliefs about the content of physics knowledge, whether it is made up of symbols and rules for manipulating them, or consists of the underlying conceptual content these formulas represent;

3) beliefs about how, as a student, one develops that knowledge, whether by receiving it as information or by constructing and modifying one's understanding.

It was essential to show that the characterizations were consistent across physics content and across interviewing tasks. This was in order to distinguish beliefs about physics and learning from knowledge tied to specific content, as well as from subjects' interpretations of the interviewing task.

This dissertation will discuss the analyses and characterizations, as well as evidence of the involvement of beliefs in reasoning and learning. The results support the value of epistemological beliefs as a theoretical perspective, with implications for instructional practice.

To my parents, Laurie and Avram Hammer.

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The whole of science is nothing more than a refinement of everyday thinking. Albert Einstein

Preface

In an introductory physics lab I once supervised, students were to measure human reaction times using an oscilloscope. One student pressed a key that both started a trace on the oscilloscope and produced an audible tone; another, on hearing the tone, pressed a different key to generate a spike on the oscilloscope trace. The procedure was to count the divisions on the screen between the start of the trace and the spike, and then multiply by the "time/division" setting. Due to various errors in calibration or arithmetic, a number of groups found reaction times of 3 seconds, and one group reported a time of 8 seconds.

It is difficult to imagine these students believed reaction times could be so long, when it seems common sense that reaction times should not be more than about a second.[1] Yet about a third of the class, working in groups of two or more, reported such high values. In other cases, I have had students give negative or very large positive numbers for the time it would take a ball to fall a meter, report outrageous speeds and weights for everyday objects, and so on. Every physics instructor with whom I've discussed these issues has recognized the phenomena as familiar: students often fail to apply useful common sense to their physics course.

They also fail to apply the course to their common sense. There is a large body of research documenting student "misconceptions" (Clement 1982; Halloun and Hestenes 1985ab; McClosky 1983ab; Viennot, 1979). The problems shown are various, but they are all noteworthy mainly because they concern basic concepts. In mechanics, many students cannot

- identify gravitational attraction as the only force acting on a ball thrown upward;
- correctly predict the path of a ball when there are no forces acting on it, such as after it emerges from a curved hollow tube;

[1] I have since taken an informal poll of physics-naive adults. I have asked more than 20 people, and only one guessed a reaction time longer than 1 second. The exception was idiosyncratic in another way as well: he began his response with an extended account of the processes involved in perception.

• draw simple inferences from plots of velocity or acceleration, such as using a plot of velocity to distinguish a change in sign of acceleration from a change in direction of motion.

It is generally accepted that these frequently observed misconceptions originate in common sense notions; several authors have noted strong similarities between common sense mechanics and early theories of motion (Clement, 1983; McCloskey, 1983a; Whitaker, 1983).

What is most disturbing is that these results obtain in students both before and after successful completion of introductory courses, even honors level (Peters, 1981). How can students maintain ideas directly in contradiction with principles they not only have encountered in lectures, demonstrations, and readings but have applied correctly on problem sets and exams?

Some teachers complain of students' intellectual laziness, poor preparation, or lack of discipline. Two aspects of my experience lead me to question such explanations. First, there are students who "get it," and they don't always seem to be the ones who work the hardest or who come to the course with the strongest background in either science or mathematics. Physics education research, unfortunately, has had little to say about the difference between the students who succeed from those who do not.

Second, I have found that, when I have time to work with students in small groups, simple prompts of the flavor of 'You know how to do this' or 'Forget you're taking physics and just think about it' often have tremendous effects. Such experience led me to suppose that perhaps this is what the students who 'get it' are doing: they know to stop and think about it for themselves, to compare it against what they already know, to question what doesn't seem right at first. In a sense, it seems to me, that *is* physics, the thinking that these students know to do, and that the other students can do if they are reminded.

Perhaps, then, for some students the difficulties stem more from what they think learning physics entails. For whatever reasons, a good number of students do not seem to expect physics to make sense; they memorize and reproduce procedures simply because they believe this is what the course involves. If students conceive of physics as a collection of facts and formulas, it may never occur to them to pay attention to the underlying reasoning or to compare what they learn with what they know from common sense.

The purpose of this dissertation is to explore students' beliefs about physics, specifically to determine whether students can be characterized as having such epistemological beliefs, as I will call them, and, if so, how these beliefs are involved in learning.

The first two chapters are introductory. In Chapter 1, I will review the literature on beliefs, recent and not so recent. In Chapter 2, I will give an overview of the theoretical perspective and assumptions of this work, including some definitions of terms as I will use them.

Chapters 3 and 4 discuss the methodology of the thesis. Chapter 3 discusses the interviewing methods and the considerations in their development; Chapter 4 presents the categories of the analytic framework used to describe subjects' beliefs.

Chapters 5, and 6 constitute the core of the dissertation. Chapter 5, by far the longest, reviews the process of characterization for each of the subjects. Chapter 6 then discusses the evidence for the involvement of these beliefs in subjects' learning and problem solving.

Finally, Chapter 7 summarizes the results of this dissertation and discusses their relevance for physics instruction.

Advice to readers:

The thesis contains a number of details and arguments that may not be of interest to all. Readers interested in the essentials would do well to begin with Chapter 4, read in Chapter 5 the reviews of characterizations for the first two subjects (Tony and Daniel) and the summary, followed by Chapters 6 and 7.

Readers interested in the full story may find it useful to begin with a browse through Chapter 5, to have a general idea of the interviews and data before reading the largely abstract preliminary discussions.

Acknowledgements

First and foremost, I am indebted to Andy diSessa. For starters, I appreciated his flexibility in allowing me to pursue this line of research as his advisee, despite his reservations, and I thank him for providing funds for subjects and tapes. Most important, his comments and arguments challenged me throughout, in our conversations and in his extremely thoughtful, detailed critiques of drafts. Andy's suggestions contributed to the substance and form of this dissertation in more ways than I could list.

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Ann Brown made me aware of some related literature, including Judd (1908), and gave encouraging feedback on my thesis proposal. Uri Treisman directed me to Cicourel (1964) and other reading on methodology. I appreciated his support through the proposal process.

John Bowden made me aware of the work at Göteborg. He also invited me for a wonderful visit to ERADU, at the Royal Melbourne Institute of Technology, in November, 1990. Lively discussions with John, Gloria Dall'Alba, Mike Prosser, and Jörgen Sandberg raised many difficult methodological issues, some of which I tried to address in Chapter 7 and Appendix C.

Michael Ranney suggested the method I used for checking inter-rater reliability.

I thank Kate Capps and Carol Rader for guiding me through the maze of Berkeley's administration and for allowing me occasionally to intrude on their office and equipment.

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Chapter 1: Background

Research in physics cognition

Efforts to understand student difficulties in learning physics generally involve observations and analyses at the level of physics cognition, that is, at the level of how students reason about the material: what prior knowledge they have, how that knowledge is organized, and how it is or is not affected by instruction.

Much of this work is very specifically focused, such as on details of student understanding of one-dimensional graphs of velocity (Trowbridge and McDermott, 1980; Goldberg and Anderson, 1989). The purpose of such research, motivated by constructivist ideas regarding the involvement of student conceptions, is to understand the details of how students reason about the material in certain domains. Many of these projects involve training studies to test both the specific models of conceptions and particular instructional interventions.

These studies have been very useful. They have provided specific information about student reasoning to prepare instructors for teaching certain concepts, as well as models for instructional interventions (Arons, 1990). In addition, they have informed more general models of student reasoning, such as those noted below.

There are limitations, however, to the direct applicability of a research program made up of such fine-grained studies. First, it would lay a heavy burden on the research community to produce and to replicate detailed information for the many parts of a physics curriculum. Second, as a practical matter, it is not feasible for instruction to address all possible student conceptions, even if these could be known. Moreover, given the level of our understanding of human cognition, it is not reasonable to assume that the models developed in these studies are sufficient. To the extent that the models are not valid or complete, instructional methods derived from them may be too restrictive. The interventions may be ineffective or, worse, counter-productive.

Other work at the level of physics cognition is more broadly oriented. This research often makes use of results from studies of conceptions of specific content, but the purpose is more toward understanding general properties of student reasoning or approaches. Often this involves comparing novices with experts or with some ideal model of productive reasoning (Reif, 1986).

There have been a number of robust results. Novices tend to solve problems by symbol manipulation (Larkin, McDermott, Simon, and Simon, 1980; Larkin, 1983). The knowledge they acquire in a physics course tends to be poorly organized (Eylon and Reif, 1984); one aspect of the disorganization is indicated by their categorization and representation of problems by surface features rather than by underlying physical principles (Chi, Feltovich, and Glaser, 1981; Larkin, 1983). Novices' prior intuitive knowledge is fragmented (diSessa, 1988, in press; Viennot, 1979) and involves a number of conceptions inconsistent with the physics we hope them to learn (Halloun and Hestenes, 1985a; McCloskey, 1983a). In many cases, these misconceptions remain even after students have successfully completed an introductory course (Peters, 1981).

The explanations given in these analyses are powerful, but there are issues they do not address. First, because much of this research, in comparing experts and novices, treats novices as a single class, it does not offer insight into why there are substantial differences between introductory-level students, in particular into why some develop expertise while most do not. There are certainly exceptions (Chi *et al*, 1989; Reiner, 1990), but the majority of research in physics cognition has not differentiated between novices who succeed from those who do not.

Second, these accounts do not address influences on performance beyond that of individual student reasoning about the content: affect, social and cultural aspects of reasoning, and, as this paper will discuss, beliefs about what reasoning in the domain entails.

To be sure, the framing of the issue in terms of differences between experts and novices implicitly – and many of these studies make this explicit – emphasizes the importance of experience with the material. Framed in this way, the analysis does not account for differences between novices with comparable experience. Nor can it account for why some novices, including two in the present study, show "expert"-like behavior: focusing on the conceptual content of the formulas, building their understanding around physical principles, and resolving their intuitive misconceptions.

Early literature background

Judd

Judd (1908) argued strongly against claims that "the training of mental functions is always specific," in other words that learning does not transfer between different specific contexts. He described a number of experiments in which subjects demonstrated transfer. Students' inability to generalize, Judd argued, has a great deal to do with how they approach a discipline, which he took as directly related to how they are taught:

> [I]f we ask whether arithmetic is helpful as an introduction to algebra, the answer depends on what we mean by arithmetic. One of the most vivid educational lessons I ever learned came to me when I once undertook to help some candidates for teachers' certificates review arithmetic. I gave them examples, and the question they always asked me was which rule in the book the example belonged under. Those girls had a kind of arithmetic which would not carry the weight of any algebraic superstructure. (p. 39)
>
> Our textbooks make boys and girls learn in such a way that there shall be a wall of division between arithmetic and everything else. (p. 41)

Wertheimer

Wertheimer (1945/1982) described differences in students' performance as arising from general characteristics of their approach. He discussed as a prominent example his observations of a class learning how to find the area of a parallelogram. Although they had mastered the procedure, Wertheimer discovered many of the students had learned it "blindly," without a sense for the structure of the situation or for why the procedure worked. When he asked the class to find the area of a parallelogram oriented with the long side vertical, instead of horizontal as they had seen, most were not able to do so correctly. Many continued to apply literally the procedure they had been taught, with nonsensical results.

Later, he asked a number of people to find the area of a parallelogram, but he instructed them only in how to find the area of a rectangle. He found a number of subjects, including children as young as 6, who were able to invent a variety of ways to convert the parallelogram into a rectangle in order to find its area. By Wertheimer's analysis, these people were successful because they were engaged in an entirely different mode of reasoning. Rather than mechanically repeating a procedure, they were reasoning based on the structure of the situation.

Wertheimer and other Gestalt psychologists blamed "blind repetition," or the "mechanization" of thought (Luchins, 1942), on the way students were taught. Children had abilities to reason and to understand that they were trained not to use by drill-and-practice pedagogy:

> No doubt there is some advantage in getting some operations down mechanically in order to free the person to deal with more difficult problems. But though it may be necessary for such a purpose, the procedure is at the same time dangerous. Because instead of opening the mind, instead of giving experience with reasonable dealings with situations, it mechanizes the mind and uses procedures that change the free and sensible manner of children. (Wertheimer, p. 166)

Piaget

Piaget, who conducted extensive studies of cognitive development, stated strongly that differences in students' abilities were not sufficient to account for differences in their performance in school (1976, p. 13):

> Many people accept that there exist obvious differences which increase in importance with age: if some students are plainly more gifted for mathematics or physics, etc., others will never achieve more than mediocre results in these fields. Having studied the development of the logico-mathematic functions in children for many years, I went on to study, with B. Inhelder at first, the education of elementary physical laws, then, in our International Center for Genetic Epistemology — with the constant assistance of several distinguished physicists — the development of notions of physical causality between ages four to five and twelve to fifteen.
>
> More than 120 detailed investigations were made of the manifold aspects of this highly complex matter... With the exception of a few girls who were not unintelligent, but simply lacked interest in these questions, we were unable to obtain any systematic data showing the existence of special aptitudes... Consequently our hypothesis is that the so called aptitudes of "good" students in mathematics or physics, etc., consist above all in their being able to adapt to the type of instruction offered them...
>
> What [unsuccessful students] do not understand are the "lessons" and not the subject.

In other words, Piaget concluded, differences in cognitive structure and resources alone could not account for differences in performance in mathematics or physics courses.

Research in metacognition

Over the last 20 years, some education/cognition research has begun to look into what is often referred to as *metacognition*. There have been a variety of emphases (Brown, Bransford, Ferrara, and Campione, 1983): how children understand memory (Flavell and Wellman, 1977); the development of children's ability to monitor their understanding (Markman, 1977); "hidden curricula" that affect students' conceptions of how they should approach school (Snyder, 1971). Attention to metacognitive aspects of learning has been involved in the development of a number of successful interventions, such as in reading (Palincsar and Brown, 1984) and in mathematics (Lampert, 1987; Schoenfeld, 1985).

This dissertation is specifically concerned with what I will refer to as *epistemological beliefs*, or, simply as *beliefs,* following Schoenfeld's (1983) definition of a "belief system" and diSessa's (1985) of "intuitive epistemology":

> Belief system: "not necessarily conscious" beliefs about the setting, the self, or the discipline that can affect a student's behavior. (Schoenfeld, 1983, p. 331)
>
> Intuitive epistemology: the set of "assumptions that students make about the nature of knowledge and knowing that may affect what they actually pay attention to and do in acquiring knowledge." (diSessa, 1985, p. 98)

Like both of these authors, I distinguish epistemological beliefs from regulatory or control knowledge applied in the course of problem solving, with which the term *metacognition* is more closely associated (Brown *et al*, 1983).

Perry

Perry (1970) studied the development of college students' beliefs about knowledge and learning, interviewing subjects at the end of each of their undergraduate years. Much of the challenge students face, Perry argued, has to do with their understanding of what knowledge and learning involve. He delineated stages that progressed, broadly speaking, from a position at which students expect teachers to be supplying absolute truths, to one at which they think of their teachers as fallible and of truth as relative, to stages involving commitment to some point of view among many.

A number of Perry's subjects complained, for example, that they did not understand the criteria by which they were graded. Perry attributed this largely to the students' understanding that the purpose was for them to be familiar with the presented material, and that instructors were interested in verification of that familiarity. The instructors, on the other hand, were generally looking for creative contributions from the students.

Perry's interviews were generally concerned with students' experiences in liberal arts courses. Conceptions about knowledge as absolute, relative, or involving commitment to a point of view would likely take on forms in physics different from in, for example, political science. Still, some aspects of the results should apply. In particular, the progression from an acceptance of results by authority to the criterion that they make sense from one's own perspective is, I will argue, a relevant dimension for analysis of approaches to physics learning.

INOM group

The INOM Group at the University of Göteborg (Marton, 1986; Pramling, 1983; Säljö, 1982)[1] has investigated the relationship between learning outcomes and the different approaches learners take, particularly in the context of reading text, as well as the relationship between these approaches and the learners' conceptions of knowledge and learning. They have described two qualitatively different approaches to reading, "deep" and "surface," which Säljö has associated with different epistemological beliefs. He distinguished between "Learning in a 'taken-for-granted' perspective" and "as a 'thematic' phenomenon."

In the former, learning is thought of as something that happens automatically. There is no reflection on the process, except for a minimal "reproductive" view of learning, associated with a "'thing-like' conception of knowledge." People with a reproductive view read text literally, as if the knowledge were located in the words themselves rather than in the underlying message the author was trying to convey.

The thematic perspective involves an awareness of learning as "an object of reflection" in itself. People with a thematic perspective have a "reconstructive" view, in which "the learner perceives himself as an agent responsible for making sense of what is presented... perceived as a

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[1] Säljö (1982) provided an excellent review of the work of the INOM Group through 1982, including an extensive bibliography. Marton (1986) mainly discussed "phenomenography," the qualitative methodology developed by the group for education research.

demand over and about the basic task of learning in a restricted sense." Reconstructive learners understand reading as involving active interpretation, with the focus on the intended message rather than on the literal words.

Thus, Säljö argued,

> [W]e are inclined to conceive of the difficulties in understanding which some participants show not as indicative of a lack in their intellectual capacities, but rather as revealing an inadequate familiarity with the premisses according to which this community organizes knowledge. (p. 195)

Pramling's dissertation (1983) studied the development of children's conceptions of learning. She recognized a split between deep and surface approaches and went on to delineate elements of epistemological beliefs and their development at a finer grain. She also noted a similar split in teachers' conceptions of learning, that teachers often seemed to think "learning occurs by repetition." The teachers she observed

> never speak about learning: instead, they say 'now we will speak about...' and then take for granted that children will connect the telling to learning automatically. (p. 80)

That children think of learning as "meaningless remembering," Pramling blamed on the "gap between the teacher's and the child's way of thinking."

Schoenfeld

Schoenfeld (1983, 85, 88) conducted extensive investigations of students' beliefs in mathematics. In his analysis of a New York State Regents mathematics class (1988), he argued that "the students... learned some inappropriate and counterproductive conceptualizations of the nature of mathematics." He listed some "typical beliefs," which he associated with characteristics of how students approach learning:

- "formal mathematics have little or nothing to do with discovery or invention," which Schoenfeld associated with students' failure "to use information from formal mathematics" in solving problems.
- "students who understand the subject matter can solve assigned mathematics problems in five minutes or less," so students "stop working on a problem after just a few minutes."

• "only geniuses are capable of discovering, creating, or really understanding mathematics," having students study mathematics "passively," "accepting what is passed down 'from above'."

• "one succeeds in school by performing the tasks, to the letter, as described by the teacher," thus students learn only as "an incidental by-product to 'getting the work done.'

Schoenfeld showed that cognitive "resources" (1983) are only part of what accounts for student performance, that beliefs about the discipline and the setting are also involved. Moreover, he argued these beliefs are largely shaped by mathematics instruction. The methodology of the present study, described in Chapters 3 and 4, differed somewhat from Schoenfeld's, but the agenda here in physics corresponded closely with Schoenfeld's in mathematics.

Songer and Linn

Songer and Linn (1990) studied how students' beliefs might affect their ability to integrate instructed and natural world knowledge about thermodynamics. This was motivated, in part, by their earlier finding that this integration seemed to be something that occurred "all-or-nothing": students either "had the skills and mindset to find and execute connections and were successful... or they did not have these skills and were unsuccessful in nearly all instances... regardless of training intervention."

They used a survey, asking questions such as "Will studying heat energy and temperature in this class help you figure out any situations in your everyday life?" The results allowed them to distinguish students with "dissociated beliefs" from those with "cohesive beliefs," the former seeing instructed and everyday knowledge as separate, the latter seeing them as related. The results of a subsequent training study showed significant differences in performance between the two groups.

diSessa

diSessa (1985) came across, during a study of intuitive physics, two students who had interesting and articulate beliefs. One appeared to have "a version of the traditional view": "that the learning of physics is a matter of acquiring new knowledge specifically located in the laws, principles, and equations of textbooks"; while the other appeared to hold views more in line with diSessa's: "that the learning of physics intimately involves a substantial reorganization of intuition."

diSessa claimed these assumptions had a dramatic influence on how the students approached learning and on what they learned, giving anecdotal evidence of their behavior and of the latter student's superior conceptual understanding of the material.

The present study supports diSessa's work and extends it: the selection process for subjects did not involve any screening for articulate or unusual views, and the interviews were designed specifically to access epistemological beliefs, providing data for extensive analysis.

Summary

These accounts are similar in many respects. They all describe the presence and involvement of beliefs about knowledge and learning. Moreover, there are a number of common elements between different accounts, such as that student beliefs vary regarding what learning involves and regarding whether knowledge is made up of literal text and equations or underlying messages and theories. I will discuss these matters in detail in Chapter 4.

To close this chapter, I note what I expect this dissertation to add. First, it will extend the analysis of epistemological beliefs in physics, to provide insight into the differences between successful and unsuccessful novices. One category of beliefs to come out of this analysis, which I will present in Chapter 4, has not to my knowledge been described before. Second, I will show that this level of explanation can meet a criterion of consistency: a student's epistemological beliefs can be characterized consistently across interviewing tasks and physics content. I will discuss this further in Chapters 4 and 5. Third, the results of this work have implications for instruction, mostly adding new arguments in support of what others have asserted in various contexts. Finally, I believe the approach to research discussed here will contribute to the development of qualitative methodology, especially in regard to the study of beliefs.

Chapter 2: Theoretical overview

The purpose of this chapter is to define terminology and to provide an overview of the theoretical perspective of the dissertation. In particular, it is to compare, contrast, and explain the relationship between physics knowledge, here delineated as made up of intuitive and formal knowledge, and epistemological beliefs.

Intuitive knowledge.

Intuition is the knowledge one uses to anticipate weight before lifting an object, for example (Clement, 1988), or to estimate the trajectory of a projectile. One uses it in simple "problem solving," such as in deciding how to place a ladder or whether it is safe to cross the street, and in explaining surprises, such as a large box weighing very little or a ball rolling in an unexpected manner. Intuitions develop as one acquires experience or makes discoveries: drivers learn when to shift gears (Dreyfus and Dreyfus, 1986); pilots learn not to pull back on the stick to gain elevation. Physicists develop new associations, for example between stiffness and speed, and modify old associations, such as between motion and force (diSessa, in press).

More precisely, intuition is informal knowledge, of which I discern the following types:

Common sense: the body of cultural and experiential knowledge people acquire about the world in the course of everyday life. For example, it is common sense that heavier objects are harder to move, or that longer falls hurt more.

Intuition or *intuitive knowledge: one's personal sense of truth; the knowledge and reasoning one uses to decide if something seems right.* Following the authors cited below, I take intuition to be largely inarticulate and experiential. Common sense, as used here, is intuitive, but not all intuitive knowledge is common sense: to a physicist, it is intuitive that objects move with constant velocities when no forces act upon them, but this is not common sense.

Conceptual knowledge: a qualitative sense of mechanism or structure. In general, conceptual knowledge overlaps substantially with intuition, but neither category subsumes the other. For many people the fact that one has to spin a top to make it stand is intuitive but not conceptual. It is simply a familiar fact about the world; there is no sense of a mechanism behind it. On the other hand, some physicists have a conceptual understanding of the laws of quantum mechanics, a qualitative sense of how the theory functions, but this is not integrated with their intuition. That is, they understand the structure, but what the theory says does not seem right.

For the purposes of this paper, however, I will only be concerned with conceptual knowledge that is also intuitive. It will be important to distinguish between physics as formulas and physics as (intuitive) concepts, but it will not be important to distinguish between conceptual and non-conceptual understandings of physics as a (non-intuitive) formal system. Hence I will use *conceptual* and *concepts* to refer to a subset of intuitive knowledge.

Intuitive knowledge has been characterized by a number of authors (diSessa, in press; Dreyfus and Dreyfus, 1986; Lakoff and Johnson, 1980; Rosch, 1977) as made up of knowledge elements having the properties that they 1) arise out of experience, 2) are closely tied to contexts, and 3) are largely inarticulate. The emphases in these accounts vary widely, as do the terminologies and details of the models; in fact, of those cited, diSessa is the only one to deal specifically with physics intuition. Nevertheless, on the whole the views are quite consonant. I will assume the same general features here.

The contextual dependence of intuitive knowledge is one explanation for why people often do not apply their knowledge across different situations. For example, a student may expect that a ball will fall behind the runner who dropped it but that an object dropped in a plane will land at the passenger's feet.

The general model of development adopted here is constructivist (Piaget, 1970; Strike and Posner, 1985). People observe and interpret the world through their current knowledge structures, building on those structures as they incorporate new information. Conflict drives accommodation:[1] when new information is perceived to conflict with existing knowledge, the latter may be modified to accommodate. If the student were to notice a disagreement between her knowledge of objects falling in planes and her knowledge of objects dropped by runners, she would need to modify of one or the other.

However, to motivate accommodation it is not sufficient, nor even necessary, that the new information conflict. It is necessary that the information be *perceived* as conflicting. Students may hear an explanation in class, but if they never perceives it as conflicting with their intuitive knowledge there would be no need for accommodation. Beliefs may play a significant role in the perception of conflict.

Finally, I am assuming that one need not be aware of one's intuitive knowledge for it to be involved in one's reasoning. This point is worth discussing briefly with some examples.

There is an old riddle: *A boy and his father were in a car accident. The boy was injured, the father killed. At the hospital, the surgeon assigned to the boy took a look at him and exclaimed, "This is my son!" How can this be?* Many people will guess it is a case of mistaken identity, or that the surgeon is a step-father, and so on. Apparently, these people interpret the story with the tacit assumption that doctors are male. In other words, they have a piece of intuitive knowledge involved in their reasoning, although they are not aware of its involvement. In fact, the story contradicts that knowledge, which is what makes the riddle. If the involvement were conscious and deliberate, the contradiction would be perceived, and the conflict resolved. Moreover, this piece of knowledge would probably be modified such that, in the next similar situation, it would be applied with more caution.

Similarly in physics, there are pieces of intuitive knowledge tacitly involved in students' reasoning. One learns from everyday life that motion is caused by forces, which probably contributes to the common expectation that a ball dropped by a runner will fall behind, the ball no longer being carried forward. In a physics course one is told that forces cause accelerations,

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[1] I should note that my use of the term *accommodation* here is less precise than Piaget's. To be consistent with his definitions, I should really speak of accomodation dominating over assimilation, or vice versa, rather than speaking of one occurring instead of the other.

not motion. However, a student may never perceive the contradiction between this and the everyday knowledge. The student may retain the naive conception, using it tacitly to interpret the meaning, for example, of *acceleration*. If the involvement of the intuitive knowledge became conscious, the contradiction might be perceived, although the resolution would likely be more difficult than for the riddle above.

In Chapter 6, I will argue that beliefs can affect whether students examine their intuitive knowledge in solving problems or in deciding they understand new information.

Formal knowledge

Formal knowledge is explicit, literal knowledge, expressible in rules or simple statements. As distinct from intuition, formal knowledge is, for example, what people use when they do not understand the domain. A novice to computers may have formal knowledge in the form of a sequence of steps to follow, such as to read electronic mail. The students Wertheimer (1945/1982) observed, who applied literally the procedure for finding the area of a parallelogram, displayed formal knowledge. A student driver may begin with rules like 'shift at 30 mph' (Dreyfus and Dreyfus, 1986). Children's understanding of multiplication often involves literal memorization and practice of procedures (Lampert, 1987). Novice physics students often solve problems by purely formal analysis, manipulating formulas without a conceptual understanding of what the formulas mean (Larkin *et al*, 1980; Clement, 1981).

Formal knowledge: explicit, precise rules that can be expressed in symbols or as simple statements.

> "$\boldsymbol{F} = m\boldsymbol{a}$" and "all objects fall with the same acceleration" are examples of formal knowledge. In contrast to intuitive knowledge, the involvement of formal knowledge is always deliberate.

It is essential to note, however, that rules may also have conceptual content. To a physicist, $\boldsymbol{F} = m\boldsymbol{a}$ is both formal and intuitive. Physics knowledge may be intuitive, formal, or both.

This delineation, in terms of intuitive and formal knowledge, is certainly not the only one possible, and some may object that it does not adequately describe human cognition. I will not argue that matter here, except to assert that this is one of a number of useful descriptions, each valid depending on its purpose, but each a limited simplification of an extremely complicated phenomenon. In particular, as I will show, distinguishing between intuitive and formal knowledge is useful for describing student conceptions of physics cognition.

Epistemological beliefs.

Epistemological beliefs are the knowledge one uses, for example, to anticipate the kinds of knowledge and reasoning an issue will require. A novice cook may believe one cannot prepare a meal without precise knowledge of a recipe. An experienced dancer would have ideas about how to learn a new routine. For students, beliefs largely determine how they study a discipline. A student who thinks knowing history means having an understanding for the causal relationships among events would pay attention to more than would a student who believes it means knowing the dates events occurred.

> *Epistemological beliefs*, or, simply, *beliefs: knowledge about cognition; in particular here, knowledge about what physics and learning physics entail.* This includes beliefs about the domain ('Physics involves a lot of math'; 'Physical laws are just models people invent'), and beliefs about learning in the domain ('It helps to visualize things'; 'You need to practice using the formulas').

I exclude "control" knowledge invoked during the course of problem solving, such as planning and monitoring strategies, except insofar as these may reflect on beliefs about the domain. For example, Schoenfeld (1983) described as a "control decision" the choice made by a pair of students to perform a calculation as a step in a proof. This was not knowledge about a domain or learning, but a tactical decision. It is possible, however, that the decision reflected beliefs about mathematics, and in that respect it would be relevant here.

Like intuitive knowledge about physical phenomena, beliefs seem to arise out of experience, such as in school (Pramling, 1983; Schoenfeld, 1988). They also appear to be closely tied to context, so that, for example, the kinds of reasoning people think to apply in performing arithmetic operations in school differ significantly from those they think to use in buying groceries (Lave, 1988).

Moreover, as with physics intuition, beliefs need not be conscious or articulate (diSessa, 1985; Schoenfeld, 1983). diSessa's (1985) subjects, to be sure, were quite explicit in stating their beliefs, but they were chosen largely for that reason. It is unlikely the subjects in Säljö's study (1982) who showed a conception of learning as "reproductive" could have explicitly described this view. Indeed, had Säljö asked these subjects to choose between his descriptions of reproductive and "reconstructive" views of learning, it is not clear which the subjects would have chosen.

For these reasons, beliefs should properly be considered a subset of intuitive knowledge (diSessa, 1985). To avoid confusion, however, I will use *intuition* and *intuitive knowledge* only in reference to physics cognition.

Summary

In a physics course, a student may hear an explanation or see a demonstration showing that a ball dropped by a runner will land at his feet. In trying to understand this result, the student may invoke her knowledge that a ball dropped in a plane would land at the passenger's feet. Her making this comparison might depend on her beliefs about how to approach learning physics. Whether people apply their knowledge across different situations may thus reflect not only the structure of their content-level knowledge but also their beliefs about how to use that knowledge.

The importance of noticing the relevance of prior knowledge has been treated in the literature in discussions of *transfer*. Brown and Campione (1984) cited several studies in which performance on a new problem was greatly enhanced by only very general metacognitive scaffolding, such as by simply telling subjects the new problem resembled one they had already solved. Brown and Kane's (1988) 4 year old subjects showed significant improvement in their ability to transfer when asked to teach Kermit the frog how to solve the examples. Children, the authors argued, have abilities for which they are not usually given credit; what they often lack is the metacognitive knowledge to use what they already know.

These studies point to an alternative way to think of the distinction between beliefs and content-level knowledge. If a general metacognitive intervention ('Do you know anything from your experience that might help'; 'Forget you're taking physics and just think about it'; 'This is a very easy problem') is sufficient to induce a student to invoke other knowledge, then one may attribute the student's not having already done so to epistemological beliefs. This operational definition is consistent with the definitions given above. It was useful in drawing inferences from subjects' protocols, as I will describe in Chapter 5.

Chapter 3: Methodology – The interviews

The purpose of this chapter is to motivate and describe the method of data collection. The first section gives an overview of the study; the second discusses the methodological considerations that went into the design of the interviews; and the third section describes the interviews themselves.

Overview

I met with seven students enrolled in the same introductory physics course, in 3 to 5 one-hour sessions each distributed over the semester. In all, there were 30 hours of interviews, which I audio-recorded and transcribed.[1] The interviews involved a range of conversations and tasks, including open discussions about the course ("How's the course going?"), more directed tasks, such as a account of the previous lecture, and discussions about specific content. Subjects also solved problems, often from an assignment, and reviewed solutions to problems they had previously solved. The attempt throughout was to remain close to the context of the course itself, for reasons discussed below.[2]

In addition to interviewing students, I observed lectures and several laboratory sessions, and I followed the reading, problem sets, and exams. The course, Physics 7A at Berkeley, is a calculus-based, first semester course covering introductory mechanics for engineers and natural scientists. The version I observed followed Ohanian (1985) and was taught by a particularly well-regarded instructor. There were a total of 28 lectures, twice each week for the semester, each 80 minutes. I observed and audio-recorded a third of these, mostly early in the course.

I found five subjects at the beginning of the semester by inviting students from an essentially random list, chosen from the enrollment of about 200.[3] Five agreed out of ten invited:

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[1] There was one exception: I could not transcribe one subject's (Jill's) final interview due to a malfunction of the tape recorder.

[2] It may be useful at this point for the reader to browse through Chapter 5, which contains numerous excerpts from the interviews themselves.

[3] The list was arbitrary except in one respect: I included no more than two subjects from the same laboratory section, mainly to avoid having subjects who might discuss the interviews with each other. Details of the invitation process are included in Appendix A.

Daniel, Evan, Jill, Larry, and Roger.[1] I met with each of these subjects 5 times, with the exception of Larry, with whom I met 4 times.

As it happened, none of these five seemed to be developing a solid understanding of the material. To ensure the involvement in the study of successful students, at least by the measures of the course, I solicited additional subjects based on performance on the first midterm. I invited three students from a list of the top scorers. Two accepted, Ken and Tony. I had 3 meetings with each.

Methodological considerations

I chose to investigate beliefs through a small number of cases studies for the simple reason that it would not otherwise have been feasible for me to obtain the depth of data I needed. A small amount of information from each of many subjects, as from a questionnaire, might have provided information on beliefs, but it would not have allowed me to determine the consistency or depth of those beliefs. It would not have allowed an assessment of the effect of the context of the study, as I discuss below.

Moreover, to design an efficient data collection would have required anticipating relevant issues and limited the possibilities for discovery or exploration of unexpected beliefs. Because there has been so little study of beliefs in physics, and, as diSessa put it, "our measures of epistemology are so feeble," I felt case studies were necessary to discern important issues, to work toward serviceable measures, and, above all, to establish the plausibility of beliefs as a useful theoretical concept.

Of course, there were also challenges in designing case studies. To question subjects directly on their views on the importance of intuition would be to risk influencing the responses, if not the views themselves, because it would create a context different from that of the course, where the beliefs should apply. It would bring attention to knowledge usually left tacit, which could expose conflicts of which the students had not been aware. To put it simply, most students

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[1] These are not the subjects' real names.

probably do not deliberate on their own about the nature of physics, so to have them do so in an interview might yield idiosyncratic results.

An alternative to posing direct questions is to engage subjects in a task in which they might reveal their beliefs by applying them. My first attempt at this involved a "words" task, in which I presented a list of ten attributes, such as *good memory*, *curiosity*, *imagination*, and asked subjects to rank them in order of importance for "doing well in physics." Subjects' justifications for their rankings constituted the primary data. The intent was to engage subjects in a task in which they would make use of their beliefs but without focusing directly on epistemological issues.

The words task failed. Two of the four initial pilot subjects did not find it relevant or interesting. More important, it failed to accomplish what I had mainly intended: the interviews became abstract discussions about the nature of physics. This context was far enough removed from the course that it was not clear how much the beliefs expressed were applicable. Every subject, in fact, commented that the task seemed strange. Moreover, because the initial pilot study only involved this one mode of discussion, I had no means to check the results.

In redesigning the interviews, I decided on three precepts:

1) to continue to engage subjects in baseline activities instead of direct interrogation;
2) to keep the conversations natural and the context as close to the course as possible;
3) to vary the tasks involved.

Baseline activities

The core idea was to involve subjects in conversations and tasks, closely tied to their experiences in the course, in which they might, on their own, reveal something about their beliefs. Beliefs could be indicated either by explicit statements or, more often, implicitly by subjects' comments about the course or by their approach to the material. I took comments made spontaneously and unselfconsciously to be most useful and credible. They revealed matters the subjects' themselves considered relevant, without direct influence by an interview question.

One of the best examples was Tony's response to a question about his background:

> I: I will start out just asking you some background questions. Did you take physics in high school.
>
> T: Yeah, we had a notoriously bad teacher, but yeah.
>
> I: So, did you not learn much physics in high school, or
>
> T: Um, kind of everything we, he gave us, were the kinds of things we already knew but had never actually formalized, if that makes any sense.
>
> I: Huh.
>
> T: You know, if we had sat down and thought about I realize ok a ball falls off [unintelligible] stuff, but we never actually sat down and thought out the equations and everything, but it was common sense type stuff. Just kind of putting together thoughts you already knew.
>
> I: And now is different from that, or
>
> T: Actually it's a lot more of the same. I don't know, everything we do in physics seems like, it, simply, you know, it makes, you think about it and that's what should happen and it's just a matter of putting it, putting common sense into equations.

Because I had never raised the issue – in fact this was the beginning of Tony's first interview – I considered Tony's description of physics as common sense to reflect his attitude independent of the context of the interview.

There is a rich tradition in sociological and anthropological research for the use of open and semi-directed questions in interviews (Bernard, 1988; Cicourel, 1964), as well as for the importance of context (Lave, 1988). Cicourel mentioned in particular the spontaneity of subjects' remarks as one measure of their strength as evidence (Cicourel, p. 48).

The INOM Group at Göteborg developed "phenomenography" (Marton, 1986), a methodology for education research, that emphasizes these points as well. They use

> questions that are as open-ended as possible in order to let the subjects choose the dimensions of the question they want to answer. The dimensions they choose are an important source of data because they reveal an aspect of the individual's relevance structure. (Marton, p. 42)

From their perspective as well, it is essential to study conceptions of knowledge and learning within the context in which they are thought to apply. Phenomenographers do not attribute properties to individuals, but to "relations" between individuals and situations. Rather than "how the human minds develops," they study individuals interacting with situations, "the relation between the individual and some specified aspect of the world" (Marton, p. 33).

Naturalness

The concern for the effect of context applied both to the selection of tasks, which I will describe below, as well as to the use of probes and to my general demeanor as an interviewer. It was important on the one hand to have as little influence as possible on the subjects' responses but, on the other, to keep the conversation unintimidating and natural.

Cicourel (1964) discussed the tension between naturalness and "scientific" impartiality at length. He stressed that one cannot achieve a naturalness by the same procedures for all subjects, because subjects vary considerably in how they approach discourse. Moreover, that questions are asked in precisely the same way does not mean subjects will interpret them in the same way.

In this study, I tried to listen actively without inserting my own views. Probes were mainly requests for clarification and repetitions of subjects' statements. While it was important to have some standardization in interviewing tasks to allow comparison between subjects, I allowed my demeanor and the precise phrasing of most questions to vary as appropriate for natural conversation. For a example, Daniel sometimes seemed leery of my tactic of repeating what he had said, and he would respond with another repetition. This led me to frame more substantive probes. In general, some subjects were quite talkative and did not need to be asked many questions. Others only rarely took control of the flow of conversation and needed frequent prompting.

Variety

The final consideration in designing the interviews was to have a variety of tasks, ranging from open discussions about the course to specific discussions of content and problem solving. Variety was important for three reasons. First, because the method involved providing opportunities for subjects to raise epistemological issues on their own, having a wide variety of conversations increased the likelihood of accessing subjects' beliefs.

Second, to support the existence of epistemological beliefs as general knowledge about the domain, one must be able to assess the consistency of indications across different interviewing situations and content. Consistency was essential: if the indications for beliefs came from only one kind of situation or context, they might have been artefacts of the subject's interpretation of the task or simply statements of knowledge tied to certain content, rather than general beliefs about physics.

Finally, variety was also important for the purpose of evaluating students' understanding. A student might do very well with the quantitative course problems, having memorized the appropriate procedures, but not understand the reasons for those procedures or be able to solve simple qualitative problems on the same material.

In sum, I designed the interviews to include a variety of open and semi-directed discussions to elicit spontaneous remarks relevant to epistemological issues. The exception, as I will describe, was in the final session for each subject, when I asked direct questions about beliefs. In addition, I included some roughly standardized tasks, as well as discussions of specific content and problems by which to evaluate subjects' understanding of the material.

The interviews

In the interest of keeping the conversations natural, I waited until after each interview to write notes. I chose audio rather than videotape for three reasons: 1) I felt videotape recorders would be significantly more intimidating than audio-tapes; 2) visual information was not essential for the analysis; and 3) audio-taping was far more convenient.

The interviews involved a range of tasks: open and semi-directed discussions, semi-directed tasks, discussions of specific content, problem solving, misconceptions probes, and direct questions. I will describe each of these in turn. I should note, though, that these types were not distinct in the interviews, which is to say that open discussions often blended with discussions of specific content, and so on. Further details on the interviews, including a list of questions and examples of probes, can be found in Appendix B.

Open and semi-directed discussions

The open discussions were about the course, physics itself or compared to other disciplines, and on anything else about which the subjects had something to say. These discussions usually started from my general questions, such as "How's the course going?" or "How do you like lectures?" However, subjects sometimes initiated these conversations as well; in fact, some subjects would arrive to an interview with a story, such as of how they lost points unfairly on the previous examination. The purpose of these conversations was to provide the broadest possible context for students to raise relevant issues.

In the interest of keeping the conversations natural, I did not as a rule impose constraints on the topics. Because of this, these discussions were not always useful for providing indications of beliefs. Daniel, for example, complained at length about the sloppy attire of his teaching assistant.

There were also more focused, semi-directed conversations: I asked for accounts of lectures, labs, and discussion sections, and for think-aloud readings of passages from the text. The most generally effective semi-directed task, as a baseline activity, was to go over subjects' graded midterm examinations. This sort of activity was particularly effective with the more reticent subjects, as there was a clear purpose to be achieved. For the more talkative subjects, having such a task was not as important.

Specific content

Discussions of specific content were almost always embedded within another conversation. For example, in going through a lecture, I would frequently ask for an explanation of some specific point. These discussions were also fairly effective for all subjects. They served both as a base for the expression, usually implicit here, of epistemological beliefs, and as an informal means for me to assess understanding of the material.

The most important discussion of specific content arose out of one problem, which I used for all subjects:

$\underline{v = v_0 + at}$

The "two rocks problem" asked which of two rocks thrown with the same speed from the same height, one horizontally and one vertically, would hit the ground first, and which with greater speed. This came from a problem set assigned to students in the pilot study, where there was a strong contrast in subjects' approach (Hammer, 1989). In the present study, there was not so strong a contrast; however, the problem was useful as providing a context in which subjects would make use of some simple relations in one-dimensional kinematics. Specifically, everyone made use at some point of $v = v_0 + at$. This gave me the opportunity to ask subjects where this formula came from, how they knew it was true, and how they might go about teaching it.

I expected this formula to be conceptually accessible to students. There was a strong contrast between subjects who described the formula in terms of literal symbols and those who thought of it as expressing a fairly obvious relationship between acceleration and speed. What subjects had to say in this context was often indicative of their beliefs.

To help distinguish beliefs from abilities, I asked a follow-up question of the subjects whose understanding of $v = v_0 + at$ did not seem conceptual: I asked these subjects to find a formula for the rate of change of the national debt, given an initial value and a rate of change. I hoped to frame a question they would not automatically associate with $v = v_0 + at$, but which had a similar conceptual structure. This was of substantial value in two cases.

Problem solving

Problem solving was useful for evaluating students' understanding and for eliciting information, mostly implicit, about beliefs. I asked students to solve course problems they had previously worked out, to help minimize performance anxiety. This was especially useful with the less confident students. I also asked them to solve problems they had not seen, often from a course assignment they had not finished.

I chose problems sometimes at random, but most often I chose them for their potential at distinguishing a conceptual from a formal approach. There were three problems, which I used for all subjects, that were especially useful. One of these was the two rocks problem, described above; the other two I will refer to as the "airplane" and the "ring":

<u>airplane</u>

The airplane problem, from one of the course assignments, also involved a formula I expected students to be able to interpret conceptually. The problem asked for the angular velocity of an airplane flying in a straight line overhead, given the linear speed of the plane and its location relative to the observer. As a probe, I asked subjects how a plane flying in a straight line could have an angular velocity. Again, there was a clear distinction between subjects, with some applying the formula $v = r\omega$ because it had the right quantities, having no substantive answer to the probe, and others explaining their solutions in solid conceptual terms.

<u>ring</u>

A third problem connected to a semi-directed task. Early in an interview I asked subjects to go through a passage in the text, explaining what they considered important about it and discussing the content. The passage I chose explained the derivation of an expression for rotational kinetic energy, $\frac{1}{2} I\omega^2$, as a sum of linear kinetic energies.

Later in the same interview, I asked subjects to find the kinetic energy of a rotating ring, given its radius, total mass and the linear speed of a point on the rim. When subjects solved this as a rotational energy problem, I asked whether it would be possible in any way to apply the formula for linear kinetic energy. Some subjects made the connection to the passage they had explained earlier, others did not.

This problem resembled Wertheimer's (1982/1945) task of asking students to solve (274 +274+ 274+ 274+ 274) ÷ 5, and Schoenfeld's (1985) straight-edge-and-compass construction connected with a formal geometric proof. In each case, students were known to be familiar with a formal proof or description, and the question was whether they knew to apply it to a certain practical problem.

Misconceptions probes

I also chose a set of three qualitative problems, each of which was directed at a commonly observed misconception. Their purpose was to provide some standardization by which to compare the subjects' conceptual understanding. As these problems tended to be rather different from the problems students encountered in the course, I saved them for the later interviews:

<u>toss</u>

As a probe for a motion-implies-force misconception (Clement, 1982; Viennot, 1979), I asked subjects to identify the forces on an object thrown straight up in the air, after it is released but while it is still on its way up. As a follow-up question, I asked subjects who identified an upward force to explain what happens to the forces as the object rises and falls.

<u>merry-go-round</u>

In order to get at the misconception that an object in free flight might continue to curve if it had been curving before it was released (Halloun and Hestenes, 1985ab; McCloskey, Caramazza, and Green, 1980), I posed a problem about two people playing catch on a merry-go-round: how should they throw the ball, and what will its motion look like, to them and to an observer watching from above on a non-rotating bridge.

<u>juggler</u>

As another probe for misconceptions about force, mainly regarding Newton's Third Law of equal and opposite reaction, I asked the juggler problem: if a juggler who weighs 98 pounds juggles 3 one-pound balls, will he make it across a bridge that can only support 100 pounds? Some subjects did not think a ball would exert a force beyond its weight on the juggler as he throws or catches it.

The coding of *Misconception* or *No misconception* for these problems warrants discussion.

Many assessments of misconceptions (eg Halloun and Hestenes, 1985a) are based on subjects' responses to short answer or multiple choice questions. This practice does not guarantee that the subjects reflected carefully on a question before responding. To assert that a subject has a misconception in such cases may be to assume a model of learning in which old ideas can be eliminated completely from one's knowledge. Such a model is probably insufficient. It is not reasonable to expect knowledge to disappear entirely, so that students do not even consider it for a moment. More likely, the old knowledge remains, so that situations may still bring it to mind, but one becomes able to rule it out. Indeed, even experts, in solving unfamiliar questions, at least consider responses that could be taken to indicate misconceptions (Reif and Allen, to appear).

It is important, then, that subjects give questions adequate consideration. For this reason, I routinely challenged their initial responses, and I based each coding of *Misconception* or *No misconception* on extended discussion. This was to ensure that the coding reflected a subject's best judgement after deliberate consideration, as opposed to a momentary 'slip-of-the-mind.'

Direct questions

At the end of the final interviews, I asked several direct questions regarding the students' views of the role of intuitive knowledge in physics, of the flavor of 'How much do you think common sense is useful for doing well in this course?' As much as possible, I framed these questions in terms of subjects' comments from earlier interviews.

Analysis

The analysis involved, first, finding indications of epistemological beliefs in the form of statements and behavior. Statements could concern physics, learning physics, teaching, the course, and so on. Behavioral indications could be specific problem solving events in which beliefs appeared to play a significant role, or general practices in the course, such as how a subject studied for exams or used the textbook.

I collected significant comments and episodes, and, from them, I tried to assemble a consistent description of each subject's beliefs. This process, which I will describe further in the next chapter, involved two steps for each subject. The first was a coding of indications taken as separate incidents, using the framework of categories also discussed in the next chapter. When a pattern seemed to emerge, I went through the interviews again looking for evidence to support or to contradict a possible classification of the subject's beliefs. The question of consistency then hinged on whether contradictory initial codings could be reconciled with the general pattern.

Chapter 4: Methodology – Analytic framework

Analysis of the protocols involved the use and development of an analytic framework made up of categories along three dimensions. I did not consider the categories of the framework to be natural, either as primitives or as developmental stages; nor were the dimensions independent, as will be clear from their descriptions and in some examples. The framework should be thought of as an analytic tool, a perspective from which to characterize subjects' beliefs.[1]

The first section of this chapter presents the criteria by which to assess the validity of this perspective. The second, which makes up the bulk of the chapter, describes the development of the framework to its final form. The third section explains the process of applying the final framework to the protocols.

Criteria

There were three explicit criteria by which I evaluated the adequacy of the framework and the characterizations of subjects' beliefs.

1) Recognizability

Given the framework, others should be able to recognize the categories in the data. So that readers may judge for themselves, Chapter 5 contains extensive citations from each subject's protocol. As well, the entire transcripts and analyses for two subjects are provided in Appendix E. Finally, as I will describe in the third section of this chapter, I had a second person code a random selection of indications, as a check of reliability.

2) Explanatory power

Of course, the recognizability of the categories would not be useful if the characterizations of beliefs they supported did not provide insight into student reasoning. I will discuss the effects of beliefs on subjects' learning and problem solving in Chapter 6.

3) Consistency

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[1] I defer to Appendix C a more extended discussion on theoretical frameworks in qualitative research.

It was essential for two reasons to show consistency in the characterization of each subject's beliefs. First, it was important to show that a subject's views, as I inferred them, were not distorted by the context of the interview. That a subject, for example, stressed the formalism in recounting a lecture might have simply reflected her understanding of what I was asking her to do, rather than her understanding of what was important in the lecture. That another subject described physics as relevant to everyday life might have represented his views in the context of an abstract discussion, but not his beliefs as related to how he actually approached the course. To address these possibilities, it was important to establish consistency across a variety of tasks and conversations, as described in the previous chapter.

The second reason it was important to show consistency was for distinguishing beliefs from content-level knowledge. For example, a subject's description of $v = v_0 + at$ as an expression of common sense could be interpreted as reflecting either a belief about physics in general or simply content-level knowledge about that particular relation. To distinguish beliefs from knowledge tied to specific content, then, it was not sufficient to identify isolated instances in which beliefs could be recognized or seen as playing a role. It was also necessary to establish that these instances represented a pattern in the subject's protocol across a range of physics content.

The distinction has functional relevance. If student performance can be interpreted as reflecting only content-level knowledge, then it may be appropriate for instructors to consider only content-specific interventions. However, if students can be seen as having more general knowledge about the domain, it may be useful to design instruction for that level as well. I will discuss the implications for instruction further in Chapter 7.

To evaluate consistency, I coded each protocol in two stages. In the first, I considered each indication as an isolated instance; in the second, I considered each in relation to the protocol as a whole. I will discuss the process at the end of this chapter and, for each subject, in Chapter 5.

A framework for describing epistemological beliefs

To give a complete account of the framework, it is important to describe its development. This will provide insight into the choices the final set of categories represent, including how those choices were motivated by data.

There will be three parts to this account: the development of the framework for the pilot study; the "initial" framework for this dissertation; and the modifications of the initial framework that resulted in the final set of categories I used to characterize subjects' beliefs.

Pilot study

As a pilot study for this work, I interviewed four students enrolled in Physics 8A, an introductory course at Berkeley for life-science majors (Hammer, 1989). I began that study with a vaguely defined single dimension of beliefs about the relevance of intuitive knowledge. As with all the dimensions I will describe, I conceptualized this as a range between two extremes. In the course of the pilot study, I differentiated three overlapping dimensions.

The first of these was a range between a complete reliance on formulas, that is the belief that understanding physics consists entirely of knowing facts and formulas ("*formulism*"), and the belief that physics knowledge involves concepts underlying the formulas ("*conceptualism*"). The second was a range between reliance on authority or, more precisely, the belief that the statements of an authority should be accepted without question ("*authoritarianism*"), and the reliance on one's own reasoning and knowledge to determine what to accept ("*autonomy*"). The third was a range between thinking of physics as discovered in nature and made up of absolute truths ("*absolutism*"), and thinking of it as invented by people, with theories having only relative or temporary validity ("*relativism*").

In applying these categories to the data, the first two were useful, but the third was not, at least not in accounting for subjects' performance. While it seemed possible to characterize beliefs about physics knowledge as absolute or relative, this seemed to have nothing to do with how subjects approached learning. I had thought that if students believed physics was at least in part invention, then they might have considered themselves capable of doing some of the inventing; and that if they did not think of the course as providing absolute truths, they would feel more free to question what they were told. However, the results directly contradicted this. Liza, for example, was the least inventive in her approach but the most articulate in describing physics as invention.

This led me to reconsider the involvement of that aspect of students' beliefs. Liza's passiveness could be understood as consistent with her relativism: if the truths are relative, then it could be all the more incumbent on students to find out *which* truths are sanctioned by the course. On the other hand, a student might believe that knowledge is absolute but intuitive, and therefore consider her personal knowledge particularly relevant.[1]

I therefore decided to drop the third dimension of categories for the pilot study. Eventually, this led me to reconsider as well the nature of the second dimension. There was evidence from the protocols that, as described, *authoritarianism* was not appropriate. Students may accept as truth what the professor says without deciding that they understand it. As described, the dimension did not allow what has appeared to be an important distinction between accepting what authority says as true, and believing that is all there is to learning. There was thus a shift in the nature of this dimension as conceptualized for the present study, from a range of beliefs about what is reliable to a range of beliefs about what learning entails.

The first dimension changed as well, although it had been effective for the pilot study. Working with the mathematically more sophisticated subjects in Physics 7A, it became useful to divide this dimension into two, one pertaining to structure and one to content.

[1] That would be consistent with a view commonly attributed to Socrates: all people implicitly know the absolute truths, so learning involves, in a sense, realizing what one already knows.

Initial framework

	Beliefs about structure	
Pieces		*Coherence*
	Beliefs about content	
Formulas		*Concepts*
	Beliefs about learning	
By Authority		*Independent*

Summary of initial framework

There were three dimensions in the initial framework, summarized in this table, each made up of two categories corresponding to the endpoints of a range. The first two dimensions pertained to conceptions about knowledge and reasoning in physics, although I framed them in terms of beliefs only about knowledge. Thus I have assumed that certain conceptions of knowledge necessarily entail certain conceptions of reasoning, as will be clear in the description and use of the categories.

For each dimension, I will first discuss some related observations from the literature on physics cognition. Then I will describe the categories, giving examples of indications from the interviews. Last, I will compare some related results from the literature on beliefs.

In addition to these descriptions, Appendix D contains for each category a list of the kinds of statements and behavior I considered indications. The reader may find it useful to refer to that list to supplement the discussion here.

Pieces ↔ *Coherence*: Beliefs about the structure of physics knowledge.

Related accounts of physics cognition:

There have been a number of accounts of the weak organization of students' physics knowledge. diSessa (1988, in press) and Viennot (1979) have made a strong case that intuitive physics, at least initially, is fragmented and disorganized, made up of many small "pieces" (diSessa, 1988) of knowledge. Reif and his colleagues (Eylon and Reif, 1984; Labudde, Reif, and

Quinn, 1988; Reif, 1987) have argued that the lack of hierarchy to the knowledge students acquire in a physics course leaves them unable to access relevant information.

This fragmentation may not be purely cognitive. In particular, while the initial fragmentation could be inherent in the nature of intuition, that students' understanding *remains* in pieces may derive partly from students' tacit expectations about physics knowledge.

Reiner (1990) described interviews with beginning physics students in which the "good" students, those who went on to do well in the course, showed as many misconceptions as other students but greater coherence. That is, their understanding of physics was no more correct on the whole, as evaluated by number of items correct on an exam, but it was more self-consistent. If students have beliefs about the structure of physics knowledge, then one would expect such results, reflecting different expectations of coherence among students with similar levels of understanding and experience with the material.

Beliefs descriptions, examples:

Pieces: Physics is thought of as a collection of separate pieces. There is no expectation of coherence, and no need to look for it. To know something is to remember it; one either knows a piece or does not.

> Evan's explanation for his inability to solve an exam problem wrong was that he needed a piece he did not have, with no consideration of how it might have related to what he did know – it was 'just different': "...there's just formulas in the book that are different from what I used...I had to have that formula or I couldn't have done this problem." (3/10)[1]
>
> In solving a homework problem, Jill spoke of having "to always try it different ways" (2/6) but could not explain, and did not show interest in, why any particular way would or would not work. This indicated a belief that alternative solution techniques are not related to each other except as candidates.

Coherence: Physics is expected to make up one coherent system. There is a need to perceive that coherence, to resolve conflicts and to find key ideas from which others derive. If some part of one's knowledge is missing, one expects it may be possible to deduce.

> After Ken had solved a problem to find the kinetic energy of a rotating wheel, I asked him what he thought about another possible solution: "[You] could do it

[1] Throughout this paper, I will use double quotes (" ") to denote direct quotation, and single quotes (' ') to denote close paraphrasing.

> that way. Just different ways of thinking about it... because... all rotation is is just... at any time, it's just a bunch of particles, with velocities going off tangentially." (4/18) He was aware, not only that the second would work, but of the underlying relation between the two solutions that accounted for why both were valid.
>
> In working through a qualitative problem, Tony found two ways to think about it that disagreed. Although he was fairly sure one was correct, he was not happy with it because he was not able to account for the other: "I understand the theory... the total momentum of the system has to stay the same, so it's going to have half the speed... but... there's no reason that these two things attached here are going to go slower than these when they are separated..." (4/25)

Related accounts of beliefs:

Pieces corresponds closely with a number of other accounts of students' beliefs. Schoenfeld's (1988) descriptions of the beliefs that formal proofs have little to do with solving problems and that problem solutions should take no more than 5 minutes are related. The former is a kind of fragmentation, with proofs and problems separated as unrelated pieces. The latter could be considered an entailment of a belief that problem solving means applying the appropriate piece of information, as opposed to extended reasoning.

Säljö (1982) described a non-hierarchical understanding as one aspect of a "taken-for-granted" approach to learning. He cited Marton and Wenestam (1978) as having found that many subjects "horizontalize" examples in reading text: they see examples as a "sequential ordering of facts," rather than as connected to or manifestations of an underlying idea. This reflects, Säljö argued, a "thing-like" conception of knowledge, in which knowledge is thought of as made up of facts, located in the literal text, and learning involves taking in the words on the page.

Lakoff and Johnson (1970) argued that abstract, intangible concepts like knowledge are often understood through metaphors to physical experience. One metaphor for knowledge is as a kind of substance, essentially Säljö's "thing-like" conception. As a substance, it could be made up of pieces that would either be 'in one's head' or not.

<u>*Formulas* ↔ *Concepts*: Beliefs about the content of physics knowledge.</u>

Related accounts of physics cognition:

Many studies have described students' solving problems through the "blind" (Wertheimer, 1945/82) application of rules or formulas. An NAEP test question asked how many busses of capacity 36 would be needed to carry 1128 people. Roughly half of the students who successfully performed the division included a fraction of a bus in their response (Carpenter, *et al*, 1983, cited in Schoenfeld, 1988). These students understood how to perform the formal manipulations; certainly they knew one could not actually supply a fraction of a bus. What they failed to do was to attribute real-world, common sense meaning to the result of the calculation.

A number of accounts have discussed the use of formulas in introductory physics (Clement, 1981; Larkin *et al*, 1980; Maloney, 1984). Many students solve problems by pure symbol manipulation: they look to see what quantities are given and what quantities they need to find, then they look for a formula that relates those quantities. Other accounts have described the productive use of intuitive knowledge, mostly in experts, such as through analogies or conceptual models (Clement, 1988; Greeno, 1983).

Usually, formulistic reasoning in students is attributed to the nature of their understanding of the material. It may be, however, that it has as much to do with what they think physics involves.

Beliefs descriptions, examples:

Formulas: Physics knowledge is thought to consist of symbols and rules for manipulating them. An entailment of this position is that one solves problems through purely symbolic methods, by finding the appropriate formulas and manipulating them algebraically.

> A homework problem asked for the angular velocity of an airplane flying overhead. After Evan explained his solution, which was correct, I asked him why a plane flying overhead in a straight line would have an angular velocity. He explained that "looking through the chapter it was like the only formula that would work." (3/31)

> "This formula, distance = $(1/2)at^2$, that I know by heart ... and the one that I'm always not sure is $v = at$, that's I'm kind of shaky... 5 % of the time I'm going to

forget that formula... but if I know this, I'm pretty much going to remember that this is not going to have a square in it..." [$d = (1/2)at^2$ is easy to remember] because "there is this 2 and 2 here, something that just sticks to my mind." (Daniel 3/30)

In both of these examples, which were also indications of *Pieces*, the subjects considered only the literal symbolic make-up of the formulas.

Concepts: Physics knowledge is made up of concepts, often represented by symbols and formulas. One expects problem solving to be guided by conceptualization and measures understanding by one's ability to explain in qualitative terms.[1]

I asked Ken, in his final interview, whether he accepted a formula like $v = v_0 + at$ because it was in the book, or because it was intuitive. In his response, he described the importance of translating from the equation to "what it's saying": "Well, it's probably a combination of the two of those. Basically, I mean, once you see a formula like this you go, well, what does that mean... once you see a formula, if it's just a bunch of... letters... it's not going to make that much sense to you, but if you really sit down and look at what it's saying, then it starts to make sense." (Ken, 4/28)

Tony was talking about angular momentum: "You can see it mathematically, but it doesn't explain common-sensically why it should do that, it just does..." I asked him if that was "pretty much all of physics," and he answered, "No...not at all...in most cases you can change it to map to real life." (Tony, 3/29)

Related accounts of beliefs:

diSessa (1985) described one student who believed "that the learning of physics is a matter of acquiring new knowledge specifically located in the laws, principles, and equations," in contrast to another who believed "that the learning of physics intimately involves a substantial reorganization of intuition." Here I have chosen to consider beliefs about learning as a separate dimension from those about content, but the correspondence with diSessa's account is clear.

The correspondence is also clear with Songer and Linn's (1990) distinction between "dissociated beliefs," a view of instructed and everyday knowledge as separate, and "cohesive beliefs," although these categories probably overlap as well with *Pieces* ↔ *Coherence*.

By Authority ↔ *Independent*: Beliefs about learning physics.

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[1] The label may be misleading: by "concepts" I mean informal knowledge, to include intuition (evolved from experience) and conceptual knowledge (based on a qualitative sense of principles or structure). (Recall the discussion at the beginning of Chapter 2.) These could, perhaps, play different roles in students' beliefs. Several subjects, in fact, showed an articulate sense of different kinds of informal knowledge. For this paper, however, I treat conceptual and intuitive knowledge as a single category.

Related accounts of physics cognition:

Various distinctions between "active" and "passive" learning have been widely discussed in education at least since Dewey (1897/1966). The versions of what constitutes "active" vary, but generally the image of passive learning is of students sitting and listening, taking knowledge as input as if to a 'blank slate.' The image of active learning, from a constructivist viewpoint, is that "in order to know objects, the subject must act upon them" (Piaget, 1970). The issue is often framed in terms of pedagogy: how to teach to support active learning. Certainly students' beliefs about their role in learning should be relevant as well.

Specifically in regard to introductory physics, Chi *et al* (1989) found that successful problem solvers generated more self-explanations and showed better ability to monitor their own understanding while studying examples. They saw implications in these results for the effects on problem solving of "the way knowledge is encoded from the example-exercises." Their results may also show differences in beliefs: the extent to which the subjects' generated self-explanations may have been partly a reflection of their understanding of what learning physics involves.

Beliefs description, examples:

By Authority: One expects to receive physics knowledge from authority. Teaching is telling, and learning means storing what one has been taught. This might be considered a 'blank slate' view of learning, but it is also a kind of abdication of responsibility for learning to the instructor or text.

> "I feel that proving the formula is not really necessary for me, it doesn't matter if I can prove it or not, as long as I know that someone has proven it before... there's a concept, and...here I am paying 15,000 dollars a year... I'm not going to derive this thing for them, they're going to derive it for me and explain to me how it works." (Daniel 4/27)

> In solving a problem, Jill used the formula $x - x_0 = v_0 t + (1/2)at^2$. I asked where that came from, and she said "he derived it in class." When I asked how he derived it, she was not sure how to go about it: "I'm not sure how to derive this, but I remember that's how he told us about it in lecture. So we know about this formula." (2/6)

Independent: One thinks of learning as a process of applying and modifying one's own judgement. It is necessary to make sense of or to recreate the ideas for oneself. This position might be described as a naive constructivism, but it is also an assumption of responsibility for one's own understanding.

> Tony described a kind of debate with the material and the professor: "...like with the gyroscope thing, it doesn't make sense to me right off the bat, so I have to go question it, and, yell and scream and stuff." (4/25)
>
> When I asked Ken directly whether common sense is useful in the course, he described learning as involving a modification of common sense: "...having common sense helps... but it really has to be modified by what you've learned in the course, and added to... so, the common sense is modified, and, I mean, it's not something that you just accept... you have to sit down and think about it." (Ken 4/28)

Related accounts of beliefs:

The beliefs diSessa (1985) described are also relevant here: one student thought of learning as "acquiring new knowledge," the other thought of it as involving "a substantial reorganization" of what he already knew. Lakoff and Johnson's (1970) knowledge as substance metaphor is related to *By Authority*: like substance, knowledge is passed from a source to a recipient. Schoenfeld (1988) described the belief that "only geniuses are capable of discovering, creating, or really understanding mathematics" as resulting in students' passive approach.

The belief that solving a problem should take "five minutes or less" (Schoenfeld, 1988) might be related as well. With a *By Authority* view, solving problems is the application of what one has learned, either as verification or as practice to facilitate memorization. It should not take much time: that one is unable to solve a problem shows one has not received enough information. An *Independent* perspective, in contrast, is that problems are useful for building and modifying one's understanding; routine problems, which do not involve any new ideas, are of little value.

Säljö's (1982) distinction between a "taken-for-granted" view of learning and a "thematic" view also corresponds closely. In the former, students have a "reproductive" conception of learning as something that happens automatically and involves the transfer of information. In the latter, students think of learning as "reconstructive," involving an active, deliberate engagement with the material to interpret the underlying meaning or "theme."

Bereiter and Scardamalia (1989) defined "intentional learning": learning as a deliberate goal of activity, associated with an "implicit theory" of "knowledge as transformative," in which learning "sets off a reflective process that may lead to the overhaul of a substantial portion of one's knowledge." It is in the implicit theory of knowledge as transformative that their account relates to *By Authority* ↔ *Independent*.

Both Säljö (1982) and Bereiter and Scardamalia (1989) focused significant attention on students' awareness and intention to learn. In this respect, the emphasis here is different. At both ends of *By Authority* ↔ *Independent* students can intend to learn; what changes is how they think learning can be achieved. A student characterized by *By Authority* might solve many problems, the better to fix the material in his memory.

Intentionality, then, is not directly a matter of beliefs but of goals: a student made intend either to succeed in the course or to understand physics, which would not necessarily be the same thing. Indeed, several subjects sometimes conveyed a sense that real understanding was not essential for success in the course. Schoenfeld (1988) described the belief that "one *succeeds in school* by performing the tasks, to the letter, as described by the teacher" (emphasis added).

This matter of goals will come up again in the analysis of subjects in Chapter 5. Some subjects' beliefs may have been related to succeeding in school rather than to understanding physics.

Modifications

In its initial form, the framework did not provide consistent characterizations of most subjects' beliefs. The inconsistency was most obvious in Daniel, who was an especially articulate subject.

Many of Daniel's statements indicated *Coherence* and *Concepts*. He explained that he preferred physics to chemistry because, he could see physics "by looking out of the window":

> in physics you can sort of see, ok, if you throw the ball up and it goes down, I can sort of picture that... in my mind... [while in chemistry] you can't picture the atoms going around the molecules. (2/24)

He generally interpreted questions like "Why is $v = v_0 + at$ true" as needing either a qualitative response or a derivation, and, in solving problems, he routinely compared results against what he expected based on his experience.

However, Daniel also routinely arrived at those results through pure symbol manipulation, often trying several methods before deciding an answer was plausible, and showing little inclination to examine the solution process itself to discover why it might or might not work. Although Daniel frequently interpreted 'Why is X true' questions as needing a conceptual response, he was seldom able to give one. He generally ended up appealing to memory or to authority: "That's from the book... I think they proved it too, but I'm not sure" (2/24). Finally, what he thought he could picture in his mind seemed limited to literal replay of familiar situations: a problem involving a pendulum swinging past a peg was not related to his 'looking out the window' knowledge. In these respects, Daniel seemed to fall more toward the *Pieces* and *Formulas* ends of those dimensions.

Thus it was not possible to characterize Daniel consistently along *Formulas* ↔ *Concepts* , nor along *Pieces* ↔ *Coherence*. However, there were patterns to his responses. For one, his references to the coherent or conceptual content of physics were almost exclusively authority based, which is to say he seemed to expect physics to be conceptual or coherent in principle, but he did not expect to understand it in this way himself. Furthermore, when he did make use of informal knowledge it was only in a limited sense, such as to check results involving quantities with which he had experiential familiarity. These observations, in Daniel and in other subjects, led to modifications of the framework.

Certainly, what one believes about the structure, about the content, and about learning in a domain should be related. *Pieces*, for example, fits better with *Formulas* and *By Authority* than with *Concepts* or *Independent*. To be sure, I made no attempt to construct independent dimensions. Moreover, a position along one dimension could affect the nature of possible positions along another. For example, having a *By Authority* conception of learning may allow students to think of physics as involving concepts or coherence in principle, but abdicate responsibility for such content to experts:

Weak Concepts and *Weak Coherence*: Selective abdication to authority.

Description and examples:

> Students with a mainly *By Authority* view of learning may think there is conceptual or coherent content to physics, but that it is the responsibility of experts. It is in principle knowledge that exists, but it is not accessible or essential for a student taking introductory physics, except to the extent it is explicitly 'covered' in the course.[1]
>
> Jill complained that the book spent too much time "talking about the formulas, and why they work this way, and... how scientists found out... but that part... we cannot really see it ourselves." (2/6) She was aware in principle of a level of understanding she did not expect of herself.
>
> During a conversation in his last interview about whether intuition is useful in the course, Evan said, "I mean, I don't really understand it too well really, so I can't really rely on what I think.... I just kind of do it... Well, I kind of understand it, but... if you start asking me questions like that, then I get messed up... [otherwise] I think I understand." He thus distinguished between different kinds of understanding, his own and an intuitive level he did not feel was accessible or useful to him, at least not in regard to the course.
>
> Roger, discussing a theoretical passage in the book, said: "I'm not [sure] that's all too important... I read the words and, I believe the book and say, ok, you are right, and I'm not really going to question it too much. Maybe I should, but then

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[1] For contrast, one subject from my earlier study (Hammer, 1989) was always comfortable responding to 'Why is X true' questions with an appeal to authority, with no acknowledgement of the possibility of deeper understanding even in principle.

> it will take too much time." There was in principle some 'questioning' he could do, but he could get by without it.

Related accounts:

A student who believes that "only geniuses are capable of discovering, creating, or really understanding mathematics" (Schoenfeld, 1988) recognizes that mathematics can be discovered, created, or really understood, but that these are not essential aspects to a student's work in mathematics.

The discussion above on accounts of intentionality and students' goals is especially relevant here. Students may have a *Weak Coherence or Weak Concepts* stand specifically, perhaps cynically, with regard to succeeding in school.

<u>*Apparent Concepts*: Piece-wise connections to conceptual knowledge</u>

The observations of subjects' making use of informal knowledge only in certain ways led me to posit this modification.

Description and examples:

> Physics knowledge is thought to be made up of symbols and formulas loosely associated with conceptual content. One thinks of that content as *Pieces*: a conceptual association is either right or wrong, and one either knows it or does not. There is not a general expectation that physics is conceptual or that such understanding can be developed: one makes these associations when they are apparent, or, as Daniel put it, "convenient." They serve to help one remember facts or to recognize the validity of formulas and calculations.
>
> In particular, some quantities have apparent conceptual content, such as velocity. One can check the results of a calculation to find a velocity against one's informal knowledge of velocities, without thinking of the calculation itself as conceptually accessible.
>
> Note that a classification of *Apparent Concepts* would not exclude or be excluded by one of *Weak Concepts*. In fact, a sense of conceptual knowledge as made up only of

> apparent associations could be consonant with a sense that conceptual understanding is not essential for students.
>
> "It's not that important... if I can't look out the window and see it happening, it would just be a lot more convenient..." (Daniel 4/27)
>
> Jill thought demonstrations were "pretty helpful," because "it's better than just saying, ok, I will turn faster if I pull the masses toward me. He just... shows us, so you can tell, you can kind of memorize it better..." (3/27) Demonstrations are useful because they help one remember, rather than because they help one understand the mechanisms underlying the phenomena.
>
> "I don't know how to explain it... ok, for sure, velocity is equal to distance over time, like when you drive miles per hour, so that's easy to memorize..." (Evan, 3/31)

Apparent Concepts describes a category of beliefs I had not anticipated and have not seen discussed elsewhere. I consider it one of the principle useful results of this study.

Accounts in the literature[1] seem to depict a dichotomy regarding the role of intuition in physics. There are descriptions of students who do not think of physics as related to their everyday experience and for whom intuition plays no deliberate role. There are also descriptions of experts, sometimes of students, who think of physics as related to their experience and for whom intuitive knowledge plays a substantial role.

Apparent Concepts describes a reasonable middle position: physics is related to real-world experience, because one can see and verify the results of calculations and there are apparent, intuitive explanations for certain phenomena. There are a several advantages to this. Students know to check the results of a calculation against their expectations, when possible, and they can use apparent conceptual associations to help them reason, when appropriate.

There are also disadvantages. One is that, from this perspective, calculations are not seen as having conceptual content; formulas are unanalyzable black-boxes. While students may recognize an implausible result, they are limited to formal means by which to check the solution, which may not reveal basic conceptual errors. Often their only option is to try another method.

Another disadvantage is that, from this position, students are not inclined to look for or to construct conceptual understanding, nor to evaluate carefully the validity of the connections

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[1] See pages 37-39.

they perceive. As a result, they are less likely both to find useful but unapparent conceptual knowledge or to rule out misconceived but apparent associations.

I will describe such effects of beliefs on subjects' learning and problem solving further in Chapter 6. As well, I will highlight *Apparent Concepts* in discussing the implications for instruction in Chapter 7.

Summary: Final framework

	Beliefs about structure	
Pieces	*Weak Coherence*	*Coherence*
	Beliefs about content	
Formulas	*Apparent Concepts* and/or *Weak Concepts*	*Concepts*
	Beliefs about learning	
By Authority		*Independent*

Summary of final framework

The final framework was made up of three dimensions. The first pertained to beliefs about the structure of physics knowledge, with three categories in order of increasing expectation of coherence: *Pieces*, *Weak Coherence*, and *Coherence*.

Indications of *Pieces* included subjects' statements implying that knowing physics means remembering facts or that one could not solve a problem without knowing a particular equation.[1] *Weak Coherence* was indicated by remarks that understanding how formulas are derived is not important. Indications of *Coherence* included comments that derivations are important for understanding and self-directed efforts to resolve inconsistencies in an explanation.

There were four categories in the second dimension, varying in regard to the conceptual content ascribed to physics: *Formulas*, *Apparent Concepts*, *Weak Concepts*, and *Concepts*. There were 5 ways to code an indication along this dimension, including the possibility of assigning it to both *Apparent* and *Weak Concepts*.

[1] Appendix D provides a more complete list of indications for codings in each category.

Formulas was indicated by statements that one remembers formulas for their literal content, for example, or by an evident lack of conceptual understanding in cases where it should be accessible. Indications of *Apparent Concepts* included descriptions of intuition as sometimes useful for helping one memorize formulas and subjects' 'shopping' for solutions: trying a number of methods until one yields an intuitively plausible answer, without examining the methods themselves to understand why they might or might not be valid. *Weak Concepts* was indicated by statements to the effect that intuitive knowledge is not necessary or accessible to students. Finally, indications of *Concepts* included remarks linking physics closely to common sense and the use of conceptual argumentation in problem solving.

The third dimension had only two categories, *By Authority* and *Independent*, to describe beliefs about learning physics. Indications of *By Authority* included statements that one could not solve a problem without consulting the text and comments implying that understanding is the direct result of a good explanation. *Independent* was indicated by remarks that students should work through difficulties themselves, for example, or that learning physics involves modifying one's common sense.

Analysis

The framework's application to the transcripts involved two stages. The first was to find indications of beliefs in each subject's statements and behavior, and to code these indications along the three dimensions of analysis. At this step, I considered each indication as if it were an isolated case. Patterns in this preliminary coding suggested a possible characterization of the subject's beliefs.

In the second coding, I considered inconsistent indications in relation to a protocol as a whole, to evaluate whether they could be reconciled. An indication was reconciled if it could be seen as *consistent with*, although not as an indication of, the possible characterization. The question of consistency then hinged on whether the inconsistent indications from the first coding could be reconciled with the general pattern.

Indications

The function of the framework in preliminary analysis was to define the space of beliefs under investigation, in order to guide the discovery of relevant aspects of the protocols. As I described earlier, there was a great deal of iteration between the development of the framework, the discovery of indications, and the trial assignment of indications to categories. This process settled on the final framework, which then guided the final identification of relevant instances in the protocol.

Ultimately, a subject's statements and behavior should be interpreted within the context of his or her interviews as a whole. However, to begin with such an approach would be to risk prejudicing the results, particularly the assessment of consistency: consideration of the entire protocol would naturally support a more consistent set of codings. For this reason, I first coded indications as isolated instances, taking into account only the context immediately surrounding the statement or behavior. This allowed a first assessment of the plausibility of a general characterization.

Each instance was an indication along one or more of the dimensions of the framework, so there were more codings than instances in the analyses. Some of the indications, certainly, could be coded with more confidence than others, as will be clear from the excerpts in Chapter 5.

To measure the reliability of the coding at this stage, I chose 40 indications[1] at random from all of the protocols for a second person to code. I specified only the dimension for each indication, which I presented in random order, without identifying the subjects. In addition, I provided the list of indication types from Appendix D. The second coder agreed precisely with my assignments in 7 out of 7 indications along *Pieces* ↔ *Coherence* (3 choices); 19 out of 26 along *Formulas* ↔ *Concepts* (5 choices), and 4 out of 5 codings along *By Authority* ↔ *Independent* (2 choices).

In almost all cases with subjects in this study, the indications of *Pieces* or *Formulas* were consistent as well with *Weak Coherence* or *Apparent / Weak Concepts*. Moreover, the

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[1] Two of these had to be excluded, due to errors in their presentation. In one case, I inadvertently included my own coding; in the other, I included too much information from the protocol.

differences between the intermediate positions and *Pieces* or *Formulas* did not have as much functional relevance for these subjects as differences between these positions and *Coherence* or *Concepts*. I therefore treated indications of *Pieces* as supporting a characterization of *Weak Coherence*, and indications of *Formulas* as supporting *Apparent* / *Weak Concepts.* For the purposes of generating characterizations for these subjects, then, each dimension was effectively reduced to two categories.[1]

Reconciliations

It would not be a fair test of the consistency of the indications of subjects' beliefs to consider them only as isolated instances. Many could be consistent with an alternative interpretation, particularly when considered in light of the protocol as a whole. Thus inconsistent indications could be reconciled as not necessarily contradicting the overall characterization.

A subject's statement that learning physics is a process of "learning the formulas," would indicate *Formulas* . However, if elsewhere in the protocol it was clear that the subject used the work "formulas" to connote not only the literal symbols but the underlying conceptual content, then the indication could be reconciled as consistent with *Concepts*. It still could not be seen as an indication of *Concepts*, but it would not have to be seen as contradicting that characterization.

Most reconciliations involved the intermediate positions added as modifications to the initial framework. A statement or behavior indicated *Apparent Concepts* only when there was something to distinguish it from *Concepts*. Because of this convention, there were many instances in which subjects employed conceptual argumentation that, considered alone, indicated *Concepts*, but would also be consistent with *Apparent Concepts*.

For example, Evan explained why the separation would not change between two rocks dropped at the same time, one 1 meter above the other (3/10):

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[1] Dividing the dimensions in this way, the second coder and I agreed in 7/7, 21/26, and 4/5 indications along the three dimensions.

> Yeah, actually it doesn't change, cause now I've thought about it, cause, if you just drop two rocks, they're going to go thump-thump [gestures, two hands falling in succession]... I just thought, if you drop two things, [pause] yeah, cause I just thought about dropping things, and it would just stay the same.

One of Daniel's arguments for why the earth does not accelerate away from someone standing on it was that

> the force you exert on the earth is so insignificant compared to the mass of the earth that the earth does not accelerate away from him, even though he exerts a force on it.

Each of these, as isolated instances in the subject's protocol, indicated *Concepts*: Evan's statement described the use of intuitive knowledge; Daniel's argument was conceptual. However, both cases could also be seen as consistent with *Apparent Concepts*, because they both involved apparent intuitive notions: Evan's description focused on a literal image of the two rocks falling in tandem; Daniel's argument was essentially that 'more stuff is harder to push,' a common and apparent intuitive notion.[1]

A list of reconciliation types is provided in Appendix D. I will describe the process and give examples for each subject in the following chapter.

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[1] Evan was correct, but Daniel was not: the earth does not accelerate away because it is also attracted to the person standing on it, just as much as it is pushed.

In general, this reconciliation did not depend on whether the argument was correct or not, but on whether it involved a common intuition. However, in some cases, an incorrect use of intuition supported a coding of *Apparent Concepts*, if the use of the intuition displayed a superficial association.

Chapter 5: Characterization of subjects' beliefs using framework

This chapter discusses the application of the framework to the analysis of the protocols. It was possible to find consistent characterizations of beliefs in all cases, except for Larry, for whom the interviews were not successful at eliciting useful information, and for Ken, whose beliefs could not be characterized along *Pieces* ↔ *Coherence*.

The chapter is organized as follows. First, I will provide some general information about the data presentation and the subjects. Then I will describe each subject in turn, giving examples of indications and explaining the reconciliation of counter-examples. Finally, I will summarize the results and draw conclusions.

I consider the arguments presented in each case to be central to the purpose of supporting the validity of epistemological beliefs as a theoretical construct. The point is to demonstrate that beliefs can be characterized consistently in a number of cases. Readers not interested in this demonstration should read only the descriptions for the first two subjects, Tony and Daniel, and then proceed to the summary on page 117.

The description of each subject consists of a brief introductory overview, followed by a number of examples of indications drawn from "Open and semi-directed discussions," "Specific content," "Problem solving," and "Direct questions," to refer to the various interviewing contexts described in Chapter 3 and in Appendix B. The standard tasks, "$v = v_0 + at$," the "airplane" problem, and the "ring" problem, which are also described in Chapter 3 and Appendix B, were especially useful for comparing and contrasting beliefs. I will discuss each of these tasks for every subject.

Discussion of evidence to support the characterization is followed by an account of the "Counter-indications," indications from the first coding not consistent with the characterization, with a discussion of their reconciliation in the second stage of analysis. Finally, I will provide a numerical summary of the indications and reconciliations.[1]

For each excerpt, I will provide justification for the coding, both in the text of the description of the subject and with a reference to the list of justifications in Appendix D. The latter will be noted simply by abbreviations (eg "CON-3").

The subjects

There were seven subjects, all freshmen, six men and one woman.[2] All had studied physics previously in high school, and they all had disparaging things to say about what they had learned.

Because the selection process involved no screening for beliefs, I am assuming these subjects can be considered typical in regard to beliefs. In regard to their performance in the course, however, it is clear they were not typical: two subjects were chosen for their high scores on the first midterm; in addition, of the remaining five subjects, all had above average scores for problem sets, and only one had a cumulative rank below the 50th percentile. The above average performance of the five original subjects might have been an effect of the selection process, since half of the students invited declined to participate, or it might have been an effect of the interviews. That subjects were asked to discuss the course in interviews could easily have led them to pay it more attention.

For each subject, the overview will include a description of aspects of her or his background that might be considered relevant to work in Physics 7A, specifically prior courses and examination results in physics, mathematics, and chemistry. In addition, I will report the percentage of the problem set points for which each received credit, as an indication of the effort each put into the course.[3] I will use these details to address possible explanations for the

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[1] There were, of course, ambiguities involved in coding. Identifying a statement as a separate indication, for example, rather than as repetition, was often difficult. These numbers should be considered accordingly.

[2] The selection process was blind to gender. This ratio reflects the ratio in the enrollment.

[3] In this course, students could receive help on problem sets, whether from teaching assistants or other students, by going to the "course center" at almost any time of the week. Problems sets were also usually the focus of the discussion sections, run by teaching assistants, and the

differences between subjects' understanding and performance as reflecting only content knowledge, aptitude, or effort. Finally, I will also report each subject's overall performance, by the measures of the course, including grade and percentile ranking in the class.

Tony

Tony was one of the two subjects selected for his high score on the first midterm. He was a strong, motivated student, of all the subjects the most interested in the material. At the end of one interview, although he was late for crew practice, he insisted on staying to figure out a problem that was bothering him, saying "Coach can wait." In fact, Tony showed the most thorough understanding of all the subjects, and at the end of the course he was the top scorer (100^{th} percentile; A+).

Tony felt he did not learn much from his "notoriously bad" high school physics teacher. He did have a strong background in mathematics, including high school courses in calculus and differential equations. The latter, Tony said, had "emphasized physics," and was of some help to him in 7A. Tony did well on standardized tests (Math SAT 740; Math Level 2 Achievement 780). He did not take an advanced placement exam, however, and so he was concurrently in a self-paced course in calculus. Tony was also doing well in Chemistry 1B.

Other subjects had equally impressive records: it would be difficult to attribute the differences in their performance in introductory physics to differences in their backgrounds. It would also be difficult to attribute the differences simply to overall effort: of all the subjects I interviewed, Tony had the lowest final score on problem sets (82%), and he described paying little attention to reading the text.

Tony's beliefs about physics and learning, and his approach to the material, contrasted sharply with those of other subjects who did not do as well. He described physics as essentially "a matter of... putting common sense into equations," although sometimes this meant he had to "modify [his] common sense." In practice, he routinely worked with both the formalism and his

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professor was also available during his office hours. Because help was so readily available, and the "help" was often quite direct, any student who wanted to solve any problem would eventually be able to find out how. For this reason, I took students' scores on problem sets to indicate effort.

intuitions to try to form a coherent understanding, whether in solving problems, in trying to make sense of the result of a calculation, or in trying to understand a presentation from lecture.

The indications in Tony's protocol supported a characterization of his beliefs: *Coherence, Concepts,* and *Independent*. The three interviews took place on 3/29, 4/11, and 4/25.

Open and semi-directed discussions

Most open and semi-directed discussion with Tony took place during the first interview. He was extremely articulate about his epistemological beliefs. In fact, in the first moments of the first interview, he answered my question about his high school course with an explicit, spontaneous statement indicating *Concepts*:

> I: I will start out just asking you some background questions. Did you take physics in high school.
>
> T: Yeah, we had a notoriously bad teacher, but yeah.
>
> I: So, did you not learn much physics in high school, or
>
> T: Um, kind of everything we, he gave us, were the kinds of things we already knew but had never actually formalized, if that makes any sense.
>
> I: Huh.
>
> T: You know, if we had sat down and thought about I realize ok a ball falls off [unintelligible] stuff, but we never actually sat down and thought out the equations and everything, but it was common sense type stuff. Just kind of putting together thoughts you already knew.
>
> I: And now is different from that, or
>
> T: Actually it's a lot more of the same. I don't know, everything we do in physics seems like, it, simply, you know, it makes, you think about it and that's what should happen and it's just a matter of putting it, putting common sense into equations. (CON-1)

Specific content

This attitude was apparent as well when Tony discussed specific content. During the same interview, while recounting a lecture, Tony talked about angular momentum. When I asked him to explain, he identified angular momentum as an exception to his view of physics as common sense:

> T: ...you can see it mathematically, but it doesn't explain common-sensically why it should do that, it just does. You can take that as a given, angular momentum is conserved, it's just a given.
>
> [Tony outlines how to explain it mathematically.]
>
> T: It kind of makes sense, it's hard to talk about it in a non-mathematical way. It's hard to say, well the amount angular momentum changes is directly related to the amount of force on the point, and if that force is parallel to the actual position then it's going to be 0, and that kind of thing, it's easier to prove mathematically, it's like really high level differential equation or something, you can't
>
> I: Is that pretty much all of physics?
>
> T: Hmm?
>
> I: Is that pretty much the way physics is, or
>
> T: No, not really, not at all. A lot of times you can very easily relate the equations to, angular momentum you can too, you can sit there and you can see that the skater gets faster as she pulls, and you can sit there and you can see that, and I can find equations for that and stuff, it's just the proof that is one exception... In most cases you can usually, like in any kind of translational motion you can sit there and say, oh yeah, it's that because it's slowing down, or whatever. And in most cases you can change it to map to real life.
>
> I: But this is a case where it's tough.
>
> T: Well, here you can, it's just hard to explain unless the person you talk to knows a lot about the math and stuff, involved, otherwise you have to go through and reprove all the math to begin with. Cause like in a total mathematical sense, it really does make sense... (COH-1; CON-1)

Tony's statements early in this excerpt and at the end indicated *Coherence*, in that it was implicit Tony felt 'seeing' or being able to 'prove it mathematically' was part of understanding angular momentum.[1] That he described the concept as an exception to a general expectation of a connection with common sense indicated *Concepts*. It is worth noting that angular momentum and rotational dynamics are particularly difficult to understand in terms of familiar intuitions.

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[1] Reminder: single quotes denote close paraphrase, changed for clarity or convenience. Double quotes denote direct quotation.

Tony's notion that physics is mostly common sense was more than rhetoric: this was a meaningful exception to the rule.

$v = v_0 + at$

As with all subjects, I interrupted Tony's solution of the two rocks problem for an explanation of the formula $v = v_0 + at$. This arose for Tony in his first interview, after he had commented (see below) that physics is mostly common sense. I used that comment as a basis for challenging his discussion of the formula:

> I: Wait, can I ask you, where are you, where did you get, say this, v x equals v 0 plus a t.
>
> T: Oh, that's just like a basic equation. Um, you know, like one of the first, few weeks ago that we derived equations for actual position of an element, and speed, and its acceleration, well, this is assuming acceleration is constant, and since acceleration is due to gravity, it is. But that's just an equation we learned right off the bat, we derived. Sometimes they require a little thought to recover though.
>
> I: Does this one require thought to recover?
>
> T: Hmm? Yeah, because we haven't used these in a while. Or not in this way, anyway. But, doesn't, it requires thought, but it's there....
>
> I: But for example, v x equals v 0 plus at, you know that from an equation.
>
> T: Yeah, that's a basic, that's a given equation. What that's saying is the velocity at any given point here is going to be the velocity it started with, plus it's acceleration times how long it's been accelerating.
>
> I: Ok. So that's not common sense, that's an equation.
>
> T: Hmm? Well, it's common sense too, if you think about it. I mean, all it's saying is that if something starts off at a certain speed, and it gets faster for a certain amount of time, the speed it ends up with is going to be what it started with plus how much it accelerated, and, how, what it's accelerating at, for how long. (F-2; CON-1)

Tony's first response indicated *Formulas*, although it could be reconciled as consistent with *Concepts* (see below). The coding of *Concepts* was based on his final response, taken at face value, that he thought of the formula as an expression of common sense. His explanation, to be sure, was in essence no more than a restatement of the formula. Still, insofar as this formula is an expression of common sense, there is little other than such a restatement Tony might have said.

Problem solving

Tony struggled for a while with the two rocks problem. He got stuck

> because the second rock B started with more velocity, but fell for less time, whereas A started with no velocity, but it fell for a greater amount of time, so the trick is was the amount of time enough to overcome that initial velocity.

This realization prompted him to decide that "this is actually a harder problem than it looks." He did not express any expectation about the result, but when he eventually found that the two hit with the same speed, he was surprised:

> T: Looking at the, now I think they're going to be equal, interestingly enough. 2gh, ok, that's its total speed. Well, now that I think about it it kind of doesn't make sense. Here, that's going to be v sub 0 plus, yeah, ha, total speeds are going to be equal.
>
> I: And that doesn't make sense.
>
> T: Well, it might, now that I think about it. Yeah it does, because they both started at the same height, ok, so you can bring energy into it,... they started at the same height, and they both started with the same kinetic energy,... as they fall, their potential energy is decreasing,... since they both fell the same distance... they have to have the same kinetic energy, which means the same speed. And so yeah it does make sense, now that I think about it. If this had started off, not being thrown horizontally, just being dropped like it had before, it would be different, because, its kinetic energy started off as being zero, and the only kinetic energy it has when it hits is it, the amount it got by falling, whereas, being since it was thrown horizontally, it had an initial amount, and they both had the same initial amount, and they both gained the same amount by falling. Interesting. (COH-2; CON-2; I-2)

This indicated *Concepts* and *Coherence*: Tony checked his result against other conceptual knowledge from the course to see that it was consistent. He looked at what he knew about another example and found a difference that explained its apparent inconsistency with what he had found. It also indicated *Independent*, implicitly, as Tony thought of the inconsistency and the difficulty he had discovered earlier as matters he could sort out for himself.

airplane

Tony's response to the airplane problem (4/11) showed *Concepts*, in his use of experiential knowledge to explain angular velocity, and *Coherence*, in the evident coherence in his understanding of straight line motion as "instantaneously" circular:

> I: Ok, but, well, here's the reason that I pull this question out, is that it seems strange, omega, the way you were describing it, with your arm and the record is about things moving in circles, and this is an airplane moving in a straight line overhead.

> T: Here they're talking about instantaneously, as if, if right here, if it were going in circles around you, how fast, how many an, what's the angular velocity it would have, and, yeah, since it's, it's instantaneous angular velocity. Basically what it comes down to is if it were going in circles, what would the angular velocity be, and it's still angular velocity with respect to you...
>
> I: Ok, so it still has an angular velocity with respect to you.
>
> T: Right, it's just that it's not going in a circle, it's, the angular velocity will change, as soon as it, it's different at each point, because it's going in a straight line, that's like when you sit there and you watch a train come, you'll see it come, and it's, kind of sits there, and as it goes by, it's, zoom by, so it's angular velocity, the faster you turn your head that's what angular velocity is, shoooo [Tony turns his head to demonstrate]...
>
> (COH-5; CON-2)

ring

In response to the ring problem, Tony immediately referred to the derivation of the expression for rotational kinetic energy, indicating *Coherence*. His response also indicated *Concepts*, in that his explanation of the derivation consisted mainly of a conceptual summary of the idea of adding the kinetic energies of each "little mass dm" (4/11):

> I: Ok. So, let me see. Somebody did this on a test, from another course, and got it wrong.[1] The way that person tried to do it was write, was use one half mv^2.
>
> T: You actually could do that, if you really wanted to, but you'd have to do the integral around the whole circle.
>
> I: How would you do that? Now, the person didn't take an integral, so, how would you do that?
>
> T: Ok, that's the reason that this moment of inertia thing was invented to begin with, was to get rid of having to do this. Each piece is going to have a kinetic energy of one half of its little mass, dm, times its velocity squared...
>
> (COH-5; CON-2)

The juggler problem (4/25) provided a context for epistemological discussion. Tony decided he had found one way to think about the problem, as "a system," which led him to think that the juggler would not be able to make it across. He was not satisfied with this, however,

[1] I set up the question differently for Tony than for other subjects, and in a misleading way, by telling him that the student who had used linear kinetic energy got the problem wrong. This did not prevent him from thinking of the derivation, but it apparently did confuse him. He took it as a fact that the student was wrong and struggled for some time to understand why. Tony never questioned the use of the linear kinetic energy in principle; he thought the difficulty had to do with the integration. He remained confused, because he could not find any reason the integration should be wrong.

because, as he put it, he could not "justify it physically in [his] own mind." Evidently he was looking for a mechanistic explanation for a force on the bridge larger than 100 pounds:

> T: ...you know, if I get a problem like right or wrong, if I get it right anyway, I can like justify it physically, this one I can't seem to do.
>
> I: I'm not sure what you mean.
>
> T: Well, normally if I'm not sure of a problem, I go and I think about theory, and I go ok, ok, now I see how the theory applies and I can apply that theory to the problem and say ok that makes sense, and the whole bit. This I, here, the theory is, since, ok, you can consider the man and the 3 balls are exerting forces on each other, every time the man throws a ball... they're each exerting a force on the man... So, I'm going to have to say no, I'm going to have to go with theory, but now I still can't really, I'm trying to figure out, if the ball comes, I'm trying to think of it in terms of momentum....
>
> I: Ok. In describing that you said I'm going to have to go with theory.
>
> T: Yeah, I can't seem to, I can't justify it physically in my own mind, without, there's only, use equations on how high he's throwing the balls, on something that shouldn't matter, what it is it shouldn't matter at all. I can see, I can definitely see that as each ball comes down, it imparts momentum to him, which gives him an extra weight, but then, I don't want to go through and figure out, it shouldn't make a difference, how high or anything, and the way I'm thinking of that that does make a difference, so... (CON-3; I-2)

This episode indicated *Concepts* and *Independent*: Tony explicitly expressed the belief that generally in physics he should be able to "justify it physically in [his] own mind." Moreover, his effort to generate such a justification was unsolicited and self-directed.

Direct questions

To challenge Tony's position that physics is mostly common sense, I directed the conversation to a qualitative problem Tony had solved earlier. In that context, he had explained that gravitational potential energy only depends on the height of the object, not on how it got there:

> I: But now there's an example where, I would think someone's sort of common sense would say it should be, you know, if you take a longer way to get up there then it must take more energy. Wouldn't that be sort of a common sense kind of way to look at it?
>
> T: Um, yeah, but that, I usually tend to modify my common sense during the year, cause it also makes sense to me why, I mean it makes the, why I see that and I go oh yeah, then it also makes sense to me why it takes the same amount.
>
> I: So when you say you modify your common sense, what do you mean by that?
>
> T: Um, I mean, common, it's not, it doesn't make much sense to, bring in one thing after you've seen examples of the other and stuff. I just, you know, if I think one thing and it's proved me wrong, and the other thing does make sense, which in this case it does, it makes sense that it would go up all the way, then I have no problem

I: If the other thing makes sense.

T: Yeah, like with the gyroscope thing, it doesn't make sense to me right off the bat, so I have to go question it, and, yell and scream and stuff.

I: So now, how do you tell when it makes sense to you?

T: I don't know, just based on like past experiences with stuff I've seen or read or whatever, it just fits in, that's all I can describe it is it just fits in with everything else. (CON-1; I-4, I-5)

Tony's response to the challenge indicated he thought of the connection with common sense as part of his understanding. There were two indications of *Independent*: the first was Tony's statement that he "tends to modify [his] common sense"; the second was his statement in regard to the gyroscope that he had "to go question it." The latter was a separate indication, because it elaborated 'modification' as involving not simply substitution of new knowledge but a process of questioning, 'yelling and screaming.'

Counter-indications

Out of 107 initial codings, 21 were inconsistent with the characterization of *Coherence*, *Concepts*, and *Independent*. 13 of these could be reconciled.

For example, a number of indications involved Tony's speaking of the formalism in a literal manner. The formalism, however, is extremely useful, in part because one can use it and remember it in a literal manner. That Tony did so should not necessarily require a belief that the formalism is the physics content, although when taken as isolated these instances would indicate *Formulas* or *Apparent Concepts*. Thus the coding of *Formulas* (F-2) assigned to Tony's first response to my question about $v = v_0 + at$, quoted above, could be reconciled (REC-8).

Another counter-indication reconciled in this way was Tony's discussion of the importance of doing "everything abstractly at first" in the course of solving a problem (3/29):

T: Um, the one thing they emphasize in the course is always to go through and do everything abstractly at first and plug in all the numbers at the end, and that's kind of a very good thing to emphasize.

I: And why is that?

T: Because if you go through and plug, well, I'll show you in the problem here. [mumble-work] Like if I went through here and started plugging stuff in, and then I went into some other equations, it's a lot easier to make mistakes with numbers than with just moving a letter around someplace. All right, so the total force. (F-2 —> REC-8)

Throughout the interviews Tony seemed to think of formulas as closely tied to conceptual content. It would not be surprising, then, for him to use words like "formulas" and "equations" to denote both the literal expression and its underlying conceptual content. To be sure, this is a common practice among physicists. The coding of *Formulas* of Tony's description of lectures could be reconciled in this way (3/29):

> The general pattern is he'll give us the equations and what-not....
>
> (F-5 —> REC-9)

There were two cases in which it was clear Tony thought of certain equations or laws as having no conceptual content, as mentioned above for the definition of angular momentum. These initially indicated *Formulas* (F-4) but could be reconciled because, for one, Tony described the lack of conceptual content as exceptions, and, for another, these were cases in which conceptual content would be especially difficult to discover (REC-10).

There were 8 unreconcilable codings. One was a statement Tony made in the course of solving a projectile problem:

> T: Cause there's other, there's 4 basic equations, like, there's 2 of them that are easy to know and the other two are used much less often [$x = x_0 + \frac{1}{2}(v_0 + v)\,t$, and $v^2 - v_0^2 = 2a(x - x_0)$]... if it were a test I would memorize them... Those are equations you can't just recall off the top of your head... [$x - x_0 = 2a(v^2 - v_0^2)$] is a little hard to grasp.... If you modify it some more, it turns into energy...it ends to equate um, kinetic energies together.
>
> I: But these two, if you haven't memorized them you can't get
>
> T: This one kind of [$x = x_0 + \frac{1}{2}(v_0 + v)\,t$]... The trick is you can derive either one of them, from these, well if I remember, this one was pretty hard to derive, I can't remember...

That Tony felt it was difficult to remember $x = x_0 + \frac{1}{2}(v_0 + v)\,t$, simply the statement for constant acceleration that the change in position is the average velocity times time, indicated *Formulas* (F-4). With the assumption that the conceptual content of this equation should have been accessible, this coding could not be reconciled.

Another episode concerned a qualitative problem from lecture with a train car rolling on level, frictionless tracks. The car was filling with water from rain falling straight down. We discussed it during the final interview, as part of a conversation on the role of common sense:

T: Mass goes up, velocity goes down.

I: ...Now one way to look at that is I'm just using... $p = mv$... would you agree with that... you're not using common sense you're just using an equation.

T: Yeah, definitely... Here I would look at the equation, you see, it's like I usually say, I go, common, not common sense, but I'm, just past experience would say ok use the momentum equation, mv, mass is gong up... so velocity has to be going down, and then as a check I would say, well, it's getting heavier, so it's also getting slower... I use the theory first and I just check it with the common sense. (4/25)

There were two indications of *Apparent Concepts* (AC-2, 5) here. For one, he agreed he was reasoning based on a formula that was "not common sense." Second, he said he used common sense in general to check results, rather than to reason about the mechanism of the calculation.

Numerical summary

<u>TONY</u>	Total instances: Total codings:	81 107
Pieces 2*	*Weak Coherence* 2**	*Coherence* 23
Formulas 7* 3**	*Apparent Concepts* 2* 2** *Weak Concepts* 0	*Concepts* 43
By Authority 2* 1**		*Independent* 20
		* = Reconciled ** = Unreconciled

Daniel

Daniel was one of the original five subjects. His protocol first highlighted the issue that led to the framework modifications described in the previous chapter. While he described physics as something he could find out about by "looking out the window," he also described plugging into equations that bore no relation to his experience. It became clear that Daniel saw a limited role for "looking out the window" and for common sense, in the form of apparent connections with the physics of the course. He described connections with his experience as "convenient," but he said he did not expect them in general.

Daniel seemed to enjoy learning about physics. He was interested, he said, because of the connections and applicability to his experience, and because of the relevance for his intended major, architecture/civil engineering. Often, though, he seemed to think about the material in terms of what understanding the course required of him. He was not especially diligent, however, as indicated by his comparatively low score on problem sets (86%) and by his own description. His grade at the end of the course was a B– (59^{th} percentile).

Like other subjects, Daniel often distinguished between different levels of understanding. He said that he got an A in high school physics without learning much. He was currently "getting a borderline A" in the same self-paced calculus course Tony was in. In general, Daniel's background was not as strong as most of the other subjects. He "did not like" Chemistry 1A "at all," and he had not studied calculus in high school. In fact, Daniel never graduated from high school: he began at Berkeley after his junior year. Still, he was by no means a weak student, judging for one from the fact that he was admitted to Berkeley as a high school junior. His score on the Math SAT, 710, was comparable to those of the other subjects.

Like Tony, Daniel was extremely articulate about the course and his approach to it. What he had to say, however, was quite different from Tony. Daniel's beliefs could be characterized consistently by *Weak Coherence*, *Apparent Concepts*, and *By Authority*, although, as I will discuss, the characterization of *By Authority* may not have captured an important aspect of Daniel's beliefs about learning. The interviews took place on 2/24, 3/3, 3/13, 3/30, and 4/27.

<u>Open and semi-directed discussions</u>

Daniel was one of the most articulate subjects. Especially in the early interviews, he had a great deal to say in open discussions about the course. During the conversation about his background, in explaining why he did not like chemistry, Daniel made some explicit remarks about the content of physics (2/24):

> D: And, chemistry was just so much harder to grasp for me because, you can't see, you can't picture the atoms going around the molecules, while in physics you can sort of see, ok, if you throw the ball up and it goes down, I can sort of picture that, yeah, this one's going to go faster as the other one goes down, you know that one where he says, throws two balls up, I can see that, I can picture that in my mind, while in chemistry I can picture nothing in my mind, because you mix acids together, all you see is um, mixtures, fluids mixing together

> while in physics you can actually see things happening, and you can, I can picture things happening, that's why physics is just so much easier for me. (AC-2)

Later in the semester he reiterated this position (3/13):

> D: Chemistry I'm dealing with acid base titrations, and all those things ...you're dealing with more minute things, instead of physical things that I can see and imagine. For instance, um, I would calculate the pH in chemistry, I can't see that, I can't see what's reacting, except with an indicator I can see colors changing, but, I don't see angles, I don't see anything but the changing of colors, while, collisions, ok, so, a car hits something and it's elastic, yeah, of course, I can find examples that I've seen in life, where something to hit is going to scatter, it's going to be elastic, and like that, and it's just more easy to deal with in way, because, you can actually think, or, recreate the process in your mind, while in chemistry, you can't recreate what happens in your mind in acid base titration. (AC-2)

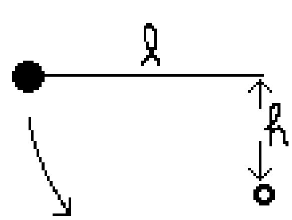
Both of these excerpts indicated *Apparent Concepts*. Daniel described the role of experiential knowledge as limited to apparent, literal connections. What he could "picture in [his] mind" or even "imagine" was limited to what he could perceive directly.

I challenged Daniel on this the second time it came up in regard to his understanding of energy and momentum. We had discussed these concepts earlier, in the second interview, at which time he had decided he did not understand the difference between them. I asked Daniel how being able to "see it and recreate it" helps him with understanding kinetic energy and momentum. He asked me to give him a specific example:

> D: I'm not really sure. Try giving me a specific problem, and how I can portray that in my mind...
>
> I: Ok. This is a ball on a string, and here's the pivot. I'm going to let the ball on the string drop, so that if I would just let it drop, it would swing back and forth like this, except what I'm going to do is over here, I'm going to put in a post, that's some distance below this pivot. The length of the sting is l, and this post is some distance h below it. And the question is, how, how big does h have to be, in order that the ball will swing completely around the post.[1]
>
> D: Ok. Um, so you want me to find h, in this case. [The professor] did the same problem in lecture. So what the string's going to do, is gonna be here, and it's going to be caught, and it's just going to be a smaller loop. [pause]

---

[1] I chose a problem that had recently been discussed in lecture. In presenting it, I drew a picture of a pendulum bob on a string attached to a pivot. I drew in a post to catch the string roughly at its middle:



> That's, and you want to find, h, such that this goes around like that. Can I look at my notes?
>
> I: Sure.
>
> D: With this, you can't really recreate it in your mind, I can't, except that, ok, I see what's going to happen, and I see that, oh yeah it's going to go like that, except that, I haven't seen much examples to know, to remember, at the point it hits this, will the ball actually go faster or slower at the same rate or anything.
>
> I: Say that again?
>
> D: Once it gets caught in the post, and starts making that loop, I can't remember whether the ball, the speed of the ball will actually go faster, decrease, or stay the same after that. (P-1; AC-2)

This episode indicated *Apparent Concepts*, as well as *Pieces*: Daniel said he did not "remember" whether the ball would go faster or slower and so could not picture the movement. Daniel thought of "picturing" only the literal motion in his head. He saw conceptual content, but only as piecewise, familiar associations with what he had observed.

This matter came up again at the end of the semester in his final interview, when Daniel mentioned he still had not understood the difference between kinetic energy and momentum. I asked if this was something he would have expected to be able to learn in the course . His response was a clear statement of a *By Authority* position that a clear explanation is sufficient for learning (4/27):

> D: I don't think you need a lot of physics to get to it, it's just a matter of sitting down with somebody who knows physics and talking about it. I'm sure if I can talk with some grad student for an hour and talk about these things, and have them explain to me, it wouldn't be much problem at all.
> (BA-4)

He had expressed similar ideas in complaining about assigned problems, which he often could not solve, compared to the lectures, which he felt he understood (3/13):

> D: ...I don't think he's giving us a fair chance to be able to work out the problems.
>
> I: Really?
>
> D: He's just not giving, I don't think he's giving enough infos or inputs on how to, um, go about the problem, because, none of his, well, I've looked through my notes, and there's not much correlation between what he says in lecture, and how to approach this problem. It's kind of like there's a missing gap between them, and we're supposed to fill that in by ourselves? I don't know if it's fair or not, but (BA-4)

Daniel thought that his difficulty with the problems was a result of not having been given enough information.

Specific content

$v = v_0 + at$

Daniel's explanation of $v = v_0 + at$ indicated *By Authority*, *Weak Coherence*, and *Formulas* (3/30):

> I: Ok. Well let me ask you a couple more questions, and then I'm going to, ah, be annoying. Um, so, you have, ah, the velocity for ball 2 is v_0 + gt. Where'd you get that, how do you know this formula?
>
> D: Um, book.
>
> I: From the book?
>
> D: Mm-hm. I think so [laugh].
>
> I: Ok. Um, if you had to teach this to somebody, how would you do it?
>
> D: If I had to teach that to somebody?
>
> I: Yeah.
>
> D: The only thing I would say is that that's a given formula, I mean I'm not a deriving person, so, if, that's why I'm not going to be a teacher [laugh] or a professor, but that's what I'd say, like, that's the given formula, you got to memorize that, otherwise you're going to be screwed.
>
> I: Ok. You're not a deriving person.
>
> D: I'm not a deriving person.
>
> I: Ok, but you're sure you got it right?
>
> D: No. [laugh] Something like that. I'm not exactly sure. [looks in book] Yeah, that's right. (WCOH-1; F-1, F-3; BA-3)

The coding of *Formulas* was based partly on an assumption $v = v_0 + at$ should be accessible to conceptual interpretation, if one thinks to look for it. Daniel's only justification for it was that it is in the book. Independent of this assumption, while Daniel acknowledged the existence of a derivation, he did not consider it essential to his own understanding. He believed his only option was to memorize.

Daniel's reliance on memorization and his conception of formulas as literal symbols were even more blatant in his discussion of $d = \frac{1}{2}\,at^2$ (3/30):

> D: ...this formula, distance is one half a t^2, that I know by heart, I've never forgotten that formula, and, the one that I'm always not sure is v = at, that's I'm kind of shaky.
>
> I: This is the one you're shaky on
>
> D: Um, yeah, 5 percent of the time I'm going to forget that formula or be unsure of that formula, but, if I know this, I'm pretty much going to remember that this is not going to have a square in it, this is, the t is only going to be to the power of 1, so if I start thinking about it I can probably come out with this.

I: Using d = one half a t^2.

D: As a reference, not

I: Now why is this one so easy to remember?

D: Oh, just form, I guess, first of all there is this 2 and 2 here, something that just sticks to my mind, I can't forget about it.

I: Um, could you figure, if somehow you had never taken physics, could you figure

D: Derive that, no. I could not have derived that. (P-1, P-4; F-1)

Problem solving

Daniel thought of problem solving as a matter of choosing among solution techniques, based either on intuitive expectations about outcome or on an apparent conceptualization of the procedure. He saw no need to evaluate why one method or expectation might have failed once he decided on an answer.

Most often, in fact, he made the choice of technique simply by means-ends analysis. Thus his accounts of his solutions, and the problem solving behavior I observed, indicated *Apparent Concepts*.

For example, Daniel related how, on a midterm problem, he had ruled out a solution because it had involved only (3/3)

> very simple, very basic thinking, so basic that I thought it was wrong, because it took me like 4 minutes to get 13.5 Newtons...

He then described ruling out another technique because the result seemed high:

> I've never seen that high of a number before. And thinking about it, that's a lot, [laugh] for two blocks together to have a .9 static friction is like glued together almost, so I think there must be something wrong with my answer there. Um, getting .9 was easy, I think it's just a matter of dividing 27 N by 6 or something like that, and it's just not thinking of any, not taking any of this into account.(AC-3)

Thus Daniel described his choice of solution technique as informed by his expectations about the complexity of the problem and by his notion of a plausible result, but not by either the physical mechanism or the conceptual content of the solution process.

On another problem, he described having adjusted his solution in order to use all of the information given, indicating *Formulas*, because it described a solution as purely symbolic manipulation:

> D: First I was trying to solve this equation, F = G mM over r squared, except that, um, at this part, the radius of earth's orbit, that could be r squared, but I wasn't really sure at the time. But then, I read this part, then about π times 10 to the 7 seconds in the year, and when I first read it, I know there's something wrong about putting this in here because, I felt that um, they wouldn't just put this in here for nothing [laugh], because so far, all the midterms I've taken in the university, they've never put information down for nothing, just to throw me off, I've never had that. So, I was thinking about it for a long time, and I couldn't get to it, and suddenly it just came to me that it must imply something about velocity here, so since we're studying centripetal acceleration, I put this equation in, and, it just came to me. (F-2)

Whether Daniel actually solved any of these problems in the manner he described is irrelevant for coding these indications, because the descriptions themselves reflect what he believed was involved in problem solving.

In any case, his accounts were consistent with what I observed directly. In solving the two rocks problem, for example, he tried a variety of techniques that did not lead him to an answer. Eventually, with some assistance, he arrived at the solution that the two rocks would hit at the same speed. I asked whether he was happy with this answer, since it did not agree with what he had expected (3/30):

> D: Of course I have to trust my answers that I calculated better than intuition, so, I'm more than willing to accept this as an answer, as a better answer. [laugh] (AC-3)

In this case, Daniel's intuitions were not strong enough to get him to try another technique, or perhaps he did not expect to be able to find another technique to try. That he did not try to reconcile this result with his intuition was an indication of *Apparent Concepts*.

airplane

Our discussion of the airplane problem was consistent with this characterization (4/27):

> I: Ok, so one of the reasons I picked [problem] three is just cause I wondered if it seemed weird, um, because this is, an angular velocity, which is things moving in circles, but the plane is moving in a straight line.
>
> D: You're asking me whether it seems weird to me, is that your question?
>
> I: Yeah. Well, how did you know to use this, because the plane isn't going in a straight line, I mean the plane isn't going in, you're standing and watching the plane fly overhead, you're not watching it go in a circle around you, you're just watching it go
>
> D: Well, first of all that's the only reasonable formula that is in the book, but
>
> I: I'm sorry, say that again?
>
> D: This is the only reasonable formula in the book to solve the problem, but, if this is the earth, and this is the plane, that I've been on so many times, [laugh] if it's going to go straight, well, I don't want it to go straight [laugh] I want it to go in a circle, [Daniel draws a circle around the earth] so, even though it seems like it's going straight, I know it's not.
>
> I: Ok. So it's really going in a circle around the earth. Well then
>
> D: It's flying, its motion is the same as the circle, as the curvature of the earth.
>
> I: Why aren't you using the radius from the center of the earth then?
>
> D: Um, that's a good question, I should have thought of that. [pause] I don't know, it's just, um, yeah, that's a good point. I don't know what to tell you. I wonder if the solution set would tell. [looks for it, flips pages] Which homework is this, eight? Um, they use the same equation.

(AC-5, AC-6; BA-3)

Daniel thus chose the formula purely by its symbolic content. Asked to explain why it made sense, he invoked convenient but inappropriate conceptual knowledge that the straight line motion of the plane could be extended in a circle around the earth. In the end, he justified the use of the equation through an appeal to authority, which indicated *By Authority.*

ring

His solution to the ring problem indicated similar beliefs (4/27):

> I: Ok, so it's going to be 9 meters, 9 mass times meters per second. Ok, so you used, kinetic energy is $I\omega^2$. What if somebody wanted to solve this using one half m v^2, could they do that?
>
> D: [pause] Yeah, sure.
>
> I: Ok, how would that go?
>
> D: Well, I would just substitute in, solve for v, v is ωr, um [pause] I think this is just a matter of manipulating the formula, can I have a piece of paper? Kinetic energy is supposed to be I ω^2, and v is ωr, um, so ω^2 is v^2 over r^2, v^2 is ω^2 r^2, [pause] it's just going to be I, v^2, over r^2. Is this what you're asking me? This is going to be, oh, you wanted to solve it in terms of m v^2?
>
> I: Right, one half mv^2.
>
> D: Isn't, [pause] are you talking about a general case, or is this in this situation?
>
> I: In this particular case, you started with $I\omega^2$, and I'm just wondering would it be possible, because, the first thing you learned about kinetic energy was to use one half mv^2, and you started with $I\omega^2$, and so you got an answer, and I'm just wondering if it would have been possible to use one half m v ^{2} in this case, to get an answer.
>
> D: Mm, well, the answer for one half mv^2 has to match the answer for $I\omega^2$. It's, am I using the formula correctly, is it one half $I\omega^2$ or $I\omega^2$? [flips pages] It's one half $I\omega^2$, so, um, in this particular case, the case with the ring, one half mv^2 will also work. Just because, the moment of inertia of the ring happens to cancel out the r ^{2} in the bottom of ω.
>
> I: Ok. but if it weren't, if the moment of inertia
>
> D: If the moment of inertia was, like one half mr^3, it wouldn't work. Or, something. If the power of r, in the inertia, has to be r^2 basically. (P-5; F-2)

Daniel thought that the linear equation should apply, but he described this as a coincidence. His decision was based on the fact that $\frac{1}{2}mv^2$ gave the same answer, but not on the formal or conceptual coherence of the two forms of kinetic energy. Although earlier in the interview he had explained the passage from the text deriving the expression for rotational kinetic energy as "the sum of the individual kinetic energies of these little particles," he did not invoke that argument here.

Direct questions

With Daniel, some of the most convincing evidence of his beliefs came from explicit discussion (4/27):

> I: ...you said in the first time that we met actually, that you liked physics because a lot of the stuff you can see it, you can do things with it, and you can know things just by looking out the window, is the way you put it. Um, and so I'm wondering, how much of that kind of thinking have you found you've been doing in this course, is there a lot of, is that helpful?
>
> D: It can be helpful, and it can be really confusing, just like that juggling problem.... because then I can picture, just by experience if I walk across a dinky bridge... and if you start jousting on it, of course it will break, so, just by remembering that I can answer your question, your specific question.... sometimes it's confusing, because I believe in one theory, and then I look out the window, and what I see outside, sort of disproves my theory... if it's, vague enough such that I really don't know which one to choose, which one to believe in, it can get really confusing, you know....
>
> I: Ok, and would you say, is it most of the time that it's like this, or is it most of the time that it's like this, in this course?
>
> D: Once it gets to things that are not really apparent, like, if you look up in the sky it's not really apparent to you, ...you know, there's going to be an angle opening up, that's not really as obvious as driving, or juggling, or something like that. Once it gets to really abstract things, I can't really picture, or use the same method as [with] linear motion....
>
> I: Ok. So, would you say it's important to you, or it's an extra, if you get a new formula, so I'll use this example still, just because we've been using it, you read something like this in the book, um how important is it to you to have it fit with looking out the window stuff?
>
> D: It would be nice.
>
> I: It would be nice. Is it important to this course?
>
> D: It's, [pause] it's not that important such that I'll have a fit over it if I can't look out the window and see it happening, it would just be a lot more convenient if I can look out and see it happening.
>
> I: It would be more convenient.
>
> D: Yeah, it would make life a lot easier, just because I can remember it.
>
> (AC-1, AC-2)

Thus Daniel described intuitive knowledge, not as *essential* to understanding physics but as *convenient*, because it helps him remember. In some cases, one can find an apparent connection to one's experience, but often one cannot.

Shortly later, I asked Daniel how much physics he felt he had learned in the course. This led him to complain again about understanding lectures but not being able to do the homework. In particular, he said he did not feel that the professor should assign proofs and derivations:

> D: Because the way I look at it... there's a concept, and there's a bunch of physicists out there that's getting paid 40,000 dollars a year, or whatever, and here I am paying 15,000 dollars a year, or however I'm paying... I'm not going to derive this thing for them [laugh], they're going to derive it for me and explain to me how it works.... Sure, I understand that maybe doing the homework problem will help me understand the concept, but if it becomes so trivial, such that I can have someone explain it to me, without me having to spend 3 hours... wasting 3 hours just to get extremely frustrated, it's not worth it....
>
> I: Ok, so I'll still play the professor, I don't know whether he'd say these things or not. You want to know more about kinetic energy and momentum than just this is the formula for this one and this is the formula for that one, and so what I'm going to say to you is the way to understand what the difference is, really, is to, is, you can find that in deriving the two, or in doing these derivations, because the derivations, just as you said, involve understanding, and so by doing the derivations you are coming to understand these two formulas.
>
> D: I will probably come back to you and say why should I spend 3 hours and not be able to derive this and get extremely frustrated, while here I am paying 400 dollars that goes indirectly to you somehow, where you can spend 5 minutes and explain this whole thing to me, and make me understand it in less than 10 minutes, the whole thing, how the this whole thing works....
>
> (WCOH-1; BA-4)

I continued to try to challenge his views, recalling his own comments that did not seem consistent (see below), but he remained firm: one learns physics by having it explained clearly, and derivations are not important for students' understanding. These were indications of *By Authority* and *Weak Coherence*.

Counter-indications

Out of 131 initial codings, only 22 were inconsistent with the characterization of *Weak Coherence*, *Apparent Concepts*, and *By Authority*. Of these, 6 could not be reconciled.

There were 12 instances I coded initially as *Concepts* and later reconciled as consistent with *Apparent Concepts*. Most of these involved conceptual arguments based on familiar intuitions (REC-5), such as in Daniel's solution to a problem involving two blocks attached by a cord. He solved the problem correctly by considering the two blocks as a single system (2/24):

> D: Because, it's pulling, let's isolate this part. If you want to look at it this, this is the only force that is going to be acting on the whole system, is just gravity pulling this part, gravity pulling this part will generate a force of 40 Newtons, but it's not only pulling this part, it's also pulling this part. So, you have to have 2 kg of mass, that's why it's pulling 6 kg (CON-2; REC-5)

As an isolated instance, this indicated *Concepts* because it involved the use of conceptual argumentation to think of the pair as a single system. This point, however, was discussed in the course and, moreover, could be seen as an apparent connection. Thus the indication could be reconciled as consistent with *Apparent Concepts*.

In another case, Daniel explained that labs were useful because "you see things more, you're actually doing the procedure...," an indication of *Concepts* (CON-1) if one interprets the comment to mean labs help one build a mental model of the formalism. However, in light of what Daniel had said elsewhere about pictures in his mind, it is likely that by "see things" Daniel meant a literal visualization: "actually doing the procedure" provides a literal image of it. In this way, the statement could be reconciled as consistent with *Apparent Concepts* (REC-7).

Two indications of *Concepts* involved Daniel's discussion of kinetic energy and momentum. Daniel said he did not understand the difference between the two concepts, although he knew the difference between the formulas. These indications could be reconciled with *Apparent Concepts* (REC-7) for two reasons. For one, Daniel was not concerned by his inability to explain the difference. More important, it could be that Daniel’s lack of conceptual understanding reflected his expectations of literal real-life associations with the quantities. Tony, in contrast, noted that kinetic energy “isn't so much a physical” quantity, presumably to say that it cannot be perceived as directly as other quantities like velocity. In fact, kinetic energy is often recognized as a construct without directly perceivable manifestation (Feynman, 1965). From his comments elsewhere, Daniel expected direct associations with what he could perceive.

Three codings of *Independent* could not be reconciled. Daniel thought it a problem that "most students" have things explained to them, but that the TA or the professor is "doing that thinking," that students need to think for themselves (3/3):

> D: It's like, the problem with most students I see is that, there's a tutor sitting by them, explaining to them how to do the problem, most of the time they're going to catch what the tutor's going through, what the tutor's doing, most of the time, except that their problem is, once the tutor works it out they go, oh yeah, that's really easy and they go on to the next problem and go, ok, where do I start.... because he's doing that thinking and we're just following... So, yeah, all his lectures basically made sense, because it's just, he's just deriving equations and saying blah blah, and if you read the book, that's what the book says anyways, so if you read the, especially if you read the book ahead of time, everything will make sense to you, most of the things. So, following his lecture is not a problem for me, at all. Getting, doing the homework problem [laugh] is another thing, it's just two different things. (I-5)

Similarly, Daniel complained about a teaching assistant (3/30):

> D: [pause] No, I understand how he got that far, but if I have to start from scratch, I don't know if I'd be able to do it or not. One of the problems he even set up the whole thing, and all I have to do is plug in some numbers, [laugh] It's pretty bad. I mean, it helps to get homework points, but, it just doesn't get me thinking. (I-5)

In the final interview, I asked Daniel about the consistency between these statements and his "economics" view that it would be more efficient for someone to explain concepts to him than to have him figure them out himself (see above). He did not see the two views as inconsistent (4/27):

> D: How does the economics mind work there? Because I'm not getting as much as I can, because that way I'm just, it's like somebody's just giving me money, and I'm not working for it, no, it's not like that, it's like, they're just stuffing food in my mouth and I'm actually not working, it's actually deteriorating me as a person, because my brain is not being stimulated, because all I have to do is integrals, I mean, if all I am doing is that, I'm going to take a math class. (I-5)

In these statements, there seemed to be aspects of Daniel's beliefs not captured by the analytic framework. Daniel spoke of students as having to think for themselves, but it is not clear that he conceived of "working for it" in any way other than as needing to expend effort to take in information. He appeared to have a strong sense that one needs to *do* something to learn, but his conception of what that might be was sparse. This is a distinction not described by the dimension *By Authority* ↔ *Independent* as presented.

There were also 2 unreconcilable indications of *Coherence*. Daniel discussed a solution to a midterm problem (3/3):

> D: [pause] The thing that makes me decide about how, why I want to do it this way, it's because if I want to apply a top, um, if the maximum force that I am applying, I'm only applying a force to the top block, then there's no reason for me to calculate 9 kg from this, because why take into account the bottom one, because I'm only exerting a force on the top one, and I only want to know, if those two equals, um, something like 9 N, I'm just saying that, well, I'm pushing the top one, and the maximum I can, um, accelerate the system is Newtons, because that's a frictional force, so the acceleration has to be [one?] meters per second squared, over here, so the first time I did it, it was 1.5 meters per second squared, and then I changed this part too, because, I basically modelled this problem after this problem.
>
> I: Ok, so you were happy enough with this one that you decided to
>
> D: To do it the same way, to model it this way. Since it's not right, then I don't know, how, how that is right, or how that is true.
>
> I: Ok, so you got 13.5 this way, when this was .4 up here, so since you redid this you redid this, and got this, and now you know that this is the right answer, are you, do you think that this is why this is the right answer?
>
> D: I'm not sure, to tell you the truth, because it just doesn't make sense for me to do it that way, and so I'm not so happy with this answer. (COH-3)

This was an unreconciled coding of *Coherence*, because it was clear Daniel thought he should understand how it was that his explanation for the first part, which had given him the right answer, did not apply to the second part. The second unreconciled coding involved a similar exchange concerning a pair of lecture examples, where Daniel claimed that since his explanation did not work for the one, he could not accept it for the other, although it had given the right answer. In neither case was he able to sort the matter out for himself. Nevertheless, he seemed to believe it important, at least for the purpose of understanding.

It may be, however, that understanding was not always Daniel's goal. Often, it seemed Daniel was trying only to succeed in the course, and that he did not think of understanding as important for this success:

> (3/30) you got to memorize that, otherwise you're going to be screwed...
>
> (4/27) I didn't think about it for, I mean, I know that, if I want to really go into details and be real trivial about it, but again, that's not what they want, so.

Inconsistencies in the codings along *Pieces* ↔ *Coherence* may relate to Daniel's goals, which in turn may not have been consistent. Of course, his goals could certainly have reflected his epistemological beliefs. He may not have thought of real understanding as an option, largely as a result of his ideas of what such understanding would entail.

In these two respects, then, the inconsistencies in Daniel's protocol may reflect incompleteness of the framework. The codings of *Independent* could not be reconciled, but they may point to an intermediate position along *By Authority* ↔ *Independent*. The codings of *Coherence* may to some extent reflect Daniel's goals, which were not part of this analysis. I will discuss inconsistencies and limitations of the analysis at the end of the chapter.

Numerical summary

DANIEL	Total instances: Total codings:	106 131
Pieces 15	*Weak Coherence* 10	*Coherence* 3* 2**
	Apparent Concepts 35	
Formulas 27	*Weak Concepts* 2	*Concepts* 11* 1**
By Authority 20		*Independent* 2* 3**
		* = Reconciled ** = Unreconciled

Roger

Roger was one of the 5 original subjects. Like others, he said he did not learn much in his high school physics course. He had one of the stronger records in mathematics, having scored a 5 on the BC Calculus Advanced Placement Exam, and was enrolled, concurrently with Physics 7A, in a course in differential equations and linear algebra. Roger also had advanced placement in chemistry, with a score of 3 on that exam.

Roger was a good counter-example to the idea that what distinguishes students in physics is simply aptitude. His record in mathematics, including 760 on the Math SAT in addition to his advanced placement scores, was at least as impressive as Tony's or Ken's, but Roger's performance in 7A was considerably inferior, with a grade of B– (64th percentile). He did give the impression from time to time that he was not working very hard in the course, but so did Tony, and Roger's score on the problem sets (98%), a measure of effort more than of understanding, was significantly higher than either Tony's or Ken's.

It was often clear Roger's goal was more to succeed than to understand, and he did not always consider real understanding to be important for success. He was quite pleased to have found an intuitive understanding of one formula, but he felt such an understanding was not particularly useful, because the formula would be 'just as easy to memorize.' Roger's beliefs, whether about learning for the course or learning for understanding, could be characterized by *Weak Coherence*, *Weak* and *Apparent Concepts,* and *By Authority*. His interviews took place on 2/6, 2/16, 3/8, 4/5, and 4/19.

Open and semi-directed discussions

Roger generally had little to say in open conversations. He sometimes touched on epistemological issues, however, in the semi-directed tasks. In going through a lecture (2/6), Roger said he had "finally learned about why there's acceleration when there's constant velocity" for circular motion, but he was not able to explain. He struggled with it for a few moments, and finally gave up:

> R: Cause, ah, [laugh] it does. First you have the circle [sketching], point, that way [laugh] for r. Huh. I forget his exact words. Can't think of 'em. [laugh] I don't know. (P-1; BA-1)

This was an early indication of *Pieces* and *By Authority*, in the evident importance to Roger of the professor's literal explanation, and in the idea that knowing is remembering.

Specific content

$v = v_0 + at$

The discussion with Roger of $v = v_0 + at$ (4/5), along with the follow-up national debt problem, was especially valuable:

> I: Um, yeah, let me just ask you though, how do you know this, and how do you know this [$v = v_0 + at$ and $d = vt$]?
>
> R: Um, they're just basic equations, that I remember [laugh] [unintelligible]. I worked through it once, well, not once, but, for a long while, at that time, and I believe it, and it's true, and I can use it for this application.
>
> I: Ok. And you believe it, why do you believe it?
>
> R: Because of all the homework problems we did, and some demonstrations, and, and it's in the book [laugh]. And I believe the book.
>
> I: Ok. Just, if you had to teach say, this to someone, how would you do it?
>
> R: If I had to teach that? [pause] Um, [pause] just, look at the book, I don't know. I mean I'd have to go back and then restudy it and the, do it but, I don't see any point in me doing that right now. (AC-4, AC-5; BA-1, BA-2)

The excerpt was an indication of *Apparent Concepts*, for two reasons. First, Roger did not have a conceptual understanding of the formula. This coding is based on my assumption that the conceptual content of the formula should be accessible, if one knows to look for it. Second, he described accepting the formula because he knew it to produce correct results, showing a lack of concern for understanding why it produces those results. The excerpt also indicated *By Authority*, in Roger's use of an appeal to the text to justify his belief of the equation as well as in his belief that in order to understand the formula further he would have to consult the text.

In Roger's final interview, after I had explicitly asked about the role of common sense, we discussed this formula again. As it happened, Roger had used the formula earlier in the same interview, but he had remembered it as $v = v_0 + \frac{1}{2}at$. At that time, I asked him how he knew that, and he answered that it came "from the very first few chapters." Toward the end of the interview I brought the matter up again, specifically to challenge his statement that he used common sense "whenever possible":

> I: So it, suppose that you forgot this formula [$v = v_0 + \frac{1}{2}at$] somehow, um, would you be able to get it, other than by looking in the book?
>
> R: Well, because of math [laugh], I can't forget it, I mean I just don't see how I would forget it.
>
> I: Hm. Ok. Ok. [pause] Could you show me how you'd figure that out, from math, or from anything. Or, better, maybe I already asked you this. If you had to teach this to someone, how would you teach it?
>
> R: How would I teach it? Teach them math first [laugh].
>
> I: Did I ask you that?
>
> R: Um, I think you did.

I: If you had to teach this formula [$v = v_0 + \frac{1}{2}at$] to someone, how would you teach it to them.

R: Using math I think, well, um, [long pause] Ok, as a function of time, this is a function of, functions of time, well, first you got to explain what function is, if they don't know by then, you can't teach it to them [laugh], and, as time varies, the position of the ball, or, the position of whatever is going to vary, and so is the velocity going to, and, the fact is involved in

[I provides scratch paper.]

R: First there's position, velocity, and acceleration. Well, that's what you want to teach them. Well first I would have to use an example, or, a ball falling, I guess that would be the best. And, [pause] then you want to teach them that you can find out where, the ball would be, at a certain time, I guess that's your objective, first of all, and so you would go that, the person learning would say, well, how would you know that?

[tape side ends]

R: I would say that, well it all depends on what the initial position of the ball was, and if it had any initial velocity, and if it had any, depending on its acceleration. [pause] So [pause], what's the best way to do this. [pause] How would I teach this, I don't know.

I: So this really isn't, very much like common sense.

R: Mm-mm. [No.] (F-1, F-4)

Roger's attempts to respond referred only to various apparently relevant symbols, "position, velocity, and acceleration," and to a context, "a ball falling," in which the formula could be used. This excerpt indicated *Formulas*: Roger did not provide any conceptual explanation, and he agreed with my assertion that the expression "isn't very much like common sense."

He went on to derive the equation, by integrating from $a = dv/dt$, and discovered the extra factor of $\frac{1}{2}$, which he thought was the reason I was asking him questions. He said that he would have discovered the mistake if it had mattered in the calculation.[1] I asked him again about common sense (see below), and then introduced the national debt problem.

The question was to find the national debt D_f after T years, if it started at D_i and changed by R every year. After a little initial confusion, Roger arrived at $D_f = D_i + RT$:

I: Ok. How'd you come up with this?

R: First, well I did it by plugging in numbers, but, then um, [pause] it's going to change, yeah, I guess I would have to just show you by plugging in numbers. [pause] Ok, I know that in two years, ok, if T was 2, and the rate was

---

[1] He was correct that it had not mattered in the calculation: the problem earlier had involved a ratio, and the extra factor of $\frac{1}{2}$ was irrelevant.

> 10 dollars per year, it would change by 20, and so, it'll be 20 plus D.... D_f is going to be equal to D plus some change, and the change was 20 and so, so D_f equals D_i plus some change, and the change is going to equal times, times TR... Not too hard... I suppose I could figure that out in 9th grade or 7th grade. Maybe earlier.
>
> I: So, on this. v equals v zero plus a t.
>
> R: Ok. Well, that's pretty good [laugh]. So the velocity is going to change, well, it's gonna, first, have some, it's going to change by what the initial velocity was, and then it's going to change by some function, I mean some rate... the change is going to be dependent on time, it's going to be some constant variable, or some constant times time. [pause] That's interesting. [laugh]
>
> I: Would you call that, what you just did, common sense?
>
> R: Yeah, I guess. Yeah. Going from this, yeah, that's common sense, now that I think about it that way.
>
> I: Do you do that kind of thing much, in the physics course?
>
> R: No, well not like this, because, I don't know, that just seems so elementary. It's probably just as easy memorizing. With, certain things.
>
> I: Ok. All right, we're done.
>
> R: That's pretty, pretty smart [laugh]. (AC-1, 2; WCON-1)

With my minimal intervention, Roger evidently was able to discover a conceptual understanding of the formula. This supports my assumption, noted earlier, that a conceptual understanding of this expression should be accessible. Roger was quite excited to have found it. The explanation he generated was correct, if somewhat inarticulate.

Most important, having found this conceptualization and having decided it was common sense "now that I think about it that way," Roger went on to say explicitly that this would not be especially useful in the course, because "it's probably just as easy memorizing."

Problem solving

Roger clearly thought he should apply intuitive knowledge in solving problems, but only to check results, not to understand or to guide the process. In his first interview, I asked him to solve one of the assigned problems for that week, a problem in projectile motion.

Roger first described his attempts from the previous evening. Then he had focused on two tables in the textbook, each a summary of equations. He had tried using one of the formulas, that for the maximum height ("z max"), but he decided that was not helpful. He then went on to try finding the time, to substitute into another formula for height as a function of time, but that gave "an answer different than the answer in the back," so he "gave up" (2/6):

> R: The height.... I got a, an answer different than the answer in the back, which was lower, which means the answer was wrong. [laugh] That's when I really gave up I guess. But then, I, between that, I was looking, I thought for a second about, maybe finding t, using this and finding t and substituting for z, that's what they did, so I'm gonna try that now.
>
> I: Ok, that's what they did in example 6.
>
> R: Mm-hm [Yes.] [works silently]
>
> I: So if you can just kind of explain what you're doing as you go along, that helps.
>
> R: Oh, just got this equation,... x equals v nought x t [$v_{0x}t$], and, from this equation v nought x equals v nought cosine theta. And, so x would equal v nought cosine theta t. And, x equals, [pause] cosine theta, [pause], t. [works] Ok, I found the t, which also, has an unknown angle in it, and I'm going to put that into the z equation... I guess I should stick in t, for this. [works] That should simplify somehow [laugh]. Where's this going to get me? (P-3, F-2)

Starting the problem again during the interview, Roger's first approach was to look at an example in the text to see if it was similar. He saw "what they did," "using this and finding t and substituting for z," so he went on to try the same approach. This gave him a complicated expression, which he was not able to use.

Roger's behavior here indicated *Pieces* and *Formulas*. First, he selected formulas for their symbolic content. Second, he did not try at all to account for why his answer was lower than the book's; he simply rejected the method and looked for another, again based on the formulas' symbolic content.

airplane

In discussing the airplane problem, Roger again described choosing the formula by the variables it contained. His response to the follow-up question indicated *Weak Coherence* and *By Authority* (4/5):

> R: Well, from my understanding of this problem, at least, um, ok, it's says what is the angular velocity of the aircraft, relative to me when directly overhead. And they give an altitude, and so I figured that altitude would be like the radius, and so I just assumed that it was going like this [gesturing to show plane moving overhead], at 10 thousand feet, and at that, going straight ahead, and so, so the angular velocity was has to do with v and r, and so, ω equals, what is it [flips page] ω equals v divided by r, so ok, so I said velocity divided by, by [flips page] ok, velocity divided by the radius. So I found that part. I think that's how I did it. I don't know, I still haven't looked at the solution set....

> I: So this is a question, it says angular velocity, but the plane's moving in a straight line.
>
> R: Yeah, I was con, I was, I had trouble with that, I asked my friend... he thought that it should be just straight, so I said, ok, I'll do it that way then. Cause, I mean, I don't like questions like this, cause if I have a question, I can't really ask well, are you asking for this, or what am I supposed to assume... I don't fully understand what they're asking. And so, I just assume that that's what they wanted me to say....
>
> I: Is it something that you're happy with, or hesitant about, or
>
> R: Well, I mean, what else can I do? [laugh] [We check the solution set.] Yeah, that's what he did too, so I did do it right. Ok. I'm happy now. [laugh]
>
> (WCOH-2; F-2; BA-3)

Although he "had trouble" with the issue I raised, he felt it was a matter of "what they wanted" and not something he should be concerned with for himself. In the end, that a teaching assistant had used the same equation was sufficient to resolve Roger's discomfort.

ring

Roger had read and explained the passage on rotational energy earlier in the interview, but he did not invoke its content in discussing the ring problem (4/5):

> R: Ah. I'm going to cheat.
>
> I: What is cheating.
>
> R: Well I'm just looking up the equations.
>
> I: Ok.
>
> R: I remember this, but, well, I don't actually remember it, but I remember using it, so I'm gonna use it.
>
> I: One half I omega squared. So this is in the category you once told me that there were some equations that you didn't like using, we were going over the midterm and you had x max and z max, and you said something like, I don't like using these cause they're not the basic equations.[1] So this is cheating.
>
> R: No, I say, I say, it's not like the other.
>
> I: It's not like the other.
>
> R: No. I'm saying I'm cheating because, I don't remember this. I would have thought of this, just, if it was on a test, if I was taking the test right now, I wouldn't have thought of this right now, I, well I don't know, [laugh] I'd have to think for a long long time and I probably wouldn't have ever figured it out, but there's a book handy, and so, that's right.
>
> I: So the cheating is not using this but it's looking it up in the book.
>
> R: Uh-huh.
>
> I: Whereas with x max and z max you said something kind of funny about

[1] See below for the excerpt to which this refers.

> R: Well it's more comfortable going through, you know, using the basic equations, cause I always did that before in high school, and stuff like that, and in calculus, but, [pause] and it's a little harder to memorize than the basic equations.
>
> I: But this is one that you're comfortable using.
>
> R: Yeah, I would use that.... Cause it reminds me of one half m v squared. I mean I thought that at first, and then I said wait a minute, I don't know and then, I'm just going to use the book to make it easier, and so, ok there it is, I'll use that. [works] I keep getting confused which is which. [checks book] Ok, it's v over r. I don't know if it's v equals ω r, or ω equals vr.
>
> (P-4, P-5; F-1, F-3; BA-3)

Roger referred to looking up the equations in the book as cheating, because he felt that they were equations he should have memorized. He spoke of the value of the "basic equations" as being easy to memorize; and he described being comfortable with $\frac{1}{2}I\omega^2$ because it resembles $\frac{1}{2}mv^2$. This excerpt indicated *Pieces* and *Formulas*.

There was a second coding of *Formulas* at the end, in Roger's confusion over $v = r\omega$. Here, again, I have assumed that the conceptual difference between $v = r\omega$ and $\omega = vr$ would have been accessible to Roger if he had thought to look for it.[1] Moreover, that he looked for the answer to his confusion in the book, without trying to make sense of why that would be the right choice, indicated *By Authority*.

Direct questions

In working through the qualitative problems during the final interview, Roger had hefted a few objects to help him decide whether throwing or catching them would exert a force on his arm, and he had spoken about his experience on a merry-go-round.[2] I used his behavior as a way to frame a direct question about conceptual knowledge (4/19):

> I: All right, let's stop with this, and I want to ask you a more general question. In, in working through this stuff, at least with these last couple of problems, you know, you're picking up the backpack, and you're picking up the book, and you're thinking about, you're thinking in a certain kind of way, about, and, and on the mgr, you're talking about your experience on the mgr, and seeing things. How much of that kind of thinking do you use in the course?

---

[1] One way to see the difference between the two formulas is to consider whether, on an object rotating at a given rate (ω), the inside (small r) or the outside (big r) moves with greater speed. For $v = r\omega$ the outside moves with greater speed; for $\omega = vr$, the inside.

[2] These were indications of *Concepts* (CON-2), reconciled as consistent with *Apparent Concepts* (REC-3).

R: [pause] Not as much. Especially with energies and moment of inertias, I can't, you can't really experience it [inaudible word], but um, like, with the first few chapters, I remember one problem was will you be able to throw it into the window, and what velocity, and that, yeah, I kind of think about it that way, baseball, how fast can I throw it, if I throw it that fast, how far would it go, and, what was the last thing, oh, simple harmonic motion, yeah, you I just think of, yeah, I guess I do use it, if possible, whenever possible.

I: So when is it possible?

R: If I've experienced it [laugh], or seen it before.

I: Ok, so, um, something like this, on, on here, [pointing to Roger's calculation from earlier in the interview] v equals v zero plus one half at, is that something that you use, your experience.

R: Well, I could call that an experience, because I've seen it so many times.(AC-2)

Roger's first reaction to the question was that he did not use his experiential knowledge as much in the course as in response to my qualitative questions. With a little reflection, though, he decided it was useful "if possible," 'if he'd experienced it before,' giving an example of using his experience to estimate a plausible velocity. He did not seem to consider such reasoning an essential part of physics. This was in contrast with Tony, who described physics in general as "putting common sense into equations," and the need to "modify" his common sense as part of learning.

We went on to discuss[1] Roger's understanding of $v = v_0 + at$, and he agreed that $v = v_0 + at$ was not common sense:

I: Ok, well that's, ok. So going back to the more general question. Do you, does common sense apply to this course, or not?

R: Yeah, it applies.

I: It does. Does it apply to formulas in the course?

R: In the, most simplest way. (AC-2)

Again, common sense applies, but only in a limited way.

Counter-indications

Many of the initially inconsistent codings of *Concepts* involved instances of the application of common intuitions about effects, such as that a hanging block would pull a lighter block up an incline (3/8) or that friction acts to slow things down (4/5). Several as well involved concepts explicitly discussed in lecture in response to qualitative problems, such as his argument

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[1] See page 78.

that two rocks dropped one second apart from the same height will hit the ground one second apart (3/8). All of these could be reconciled as consistent with *Apparent Concepts* (REC-5, REC-9).

In responding, during the final interview, to the question about tossing a ball on a merry-go-round, Roger invoked his experience on merry-go-rounds. However, this use of experiential knowledge involved no more than Roger's remembering what would actually happen, and so was consistent as well with *Apparent Concepts* (REC-7).

One coding of *Coherence* involved a conversation, referred to earlier, in which Roger expressed a reluctance to use formulas for the maximum height and range of a projectile (3/8):

> R: Well, at first I didn't use it, I tried to derive these formulas, I guess.
>
> I: Say that again, at first you tried to derive this?
>
> R: Uh-huh, from the basic equations, let's see [flips pages] from these equations.

At first he had tried to use calculus to maximize the height and range, but he "kept getting something that wasn't working for some reason." At some point he noticed a formula on his crib sheet "that'll work, it has the right things":

> R: I, I didn't want to use these, because, just because I didn't want to use them. I was hesitant to use them. Because I just don't like formulas, I just like knowing what I [pause] what I know exactly how it came about, and then doing it.
>
> I: So, you didn't want to use these, which is why you started from the other ones.
>
> R: Mm-hm. [Yes.]
>
> I: But those are formulas too, aren't they?
>
> R: Yeah, but, they're just so, basic, and I have memorized them, know that for a fact that it works so many time, cause I've tried them, so many times, and, had many experiences with them, so
>
> I: You're much more comfortable
>
> R: Much more comfortable. But these, these I've never memorized before, and the first time I've seen them that way, but I haven't really used them too much, so, I don't know, even though it's proven that it's right, it's in the book, I'm still not too comfortable with using things like that. I like to, go from what I know is, what's basic, and what I know is positively true, and then work with that, if I can. But if I can't, then of course I use that [laugh].
>
> (COH-1)

There were two codings of *Coherence* in this episode. One was based on his reluctance to use formulas, his statement that he likes to "know exactly how it came about." This could be reconciled (REC-2) based on subsequent conversation in which Roger's reluctance to use some formulas was not based on a concern for coherence, but on concern for accuracy. There he preferred to start from the basic equations, because he had had more faith in them, or in his ability to use them: he had "memorized them" and knew "for a fact that it works... cause I've tried them so many times."

The second coding was based on Roger's reference to his calculus manipulations as an attempt "to derive these formulas." That Roger considered these two outwardly different approaches as essentially the same thing indicated *Coherence*. This coding could not be reconciled.

In fact, there were 7 unreconcilable codings: 3 of *Coherence*, 1 of *Concepts*, and 3 of *Independent*. All of the instances involved Roger's commenting on what he *should* or *could* be doing in the course, and not on what he actually did. As with Daniel, these may have reflected aspects of Roger's reasoning, in particular his goals, that were not captured by the framework.

For example, the coding of *Concepts* involved Roger's comments about a mistake he had made on an exam question (3/8):

> R: It's interesting. [long pause] I guess you can't say there's work being done. Well, I don't know, that's confusing to me. That's still confusing to me, something like, am I doing work or not, for this, I think if I remember, [the professor] said, there's no work being done. There's work being done here, but there's no work being done, on something like that. That troubles me. (CON-3)

He described being troubled by the notion that supporting a stationary object did not involve work. In this case, he seemed to believe this was a matter for him to understand, rather than a case in which he would just dismiss his intuition as "wrong."

When Roger was going over the passage from the text on rotational kinetic energy, I interrupted him to ask for an explanation of the statement in the book that all particles in the rigid, rotating object move with the same angular velocity (4/5):

> R: Um, [reads] Hm. I would think that it's because [pause], cause their regular velocity would vary depending on how far it is from the axis it's turning, and it's [pause] and, hm. the, it's part of, the system is [laugh], it's rotating. Well, if it's not, then I would think it's not part of the system, cause it'll scatter away from the system or something like that.
>
> I: Ok.

R: I'm not even really, think that's all too important. [I don't think it's very important.]

I: I just, cause you had only mentioned equations up to know, so I picked out something out of the paragraph, just to see, are you reading, do you go and read the words, do you hop from equation to equation.

R: Oh, I read the words and, I believe the book and say ok, [laugh] you are right, and I'm not really going to question it too much. Maybe I should, but then it will take too much time, and, I can't afford too much time.

I: Huh. Maybe you should, why do you say that.

R: Maybe I should

I: Question it.

R: Um, to understand it better [laugh]. Maybe I'll question it before the midterm.

I: When you say question it, do you mean question like maybe that's not true, or

R: No, just think about it more, and try to think in my head, for myself why it would work out. Try to prove it or, at least, yeah, go into it more.

(WCOH-1; I-5)

The passage contained, for one, an indication of *Weak Coherence* in Roger's statement that the answer to my question was not "all too important." But important for what was not clear: understanding or getting what he needed for the course. Roger went on to note that he "maybe... should" "try to think... why it would work out," which I took to indicate *Independent*, as it expressed a belief that he would need to develop his understanding for himself.

As with Daniel, it may be that Roger's various acknowledgements of what he could be doing reflect his limited goal of succeeding in the course. Still, that he did not consider conceptual or coherent understanding essential for the course is an interesting result in itself, and may as well reflect his sparse notion of what a deeper understanding would involve.

Numerical summary

ROGER	Total instances: 69 Total codings: 87	
Pieces 14	*Weak Coherence* 5	*Coherence* 3** 1*
	Apparent Concepts 15	
Formulas 17		*Concepts* 13* 1**
	Weak Concepts 3	

By Authority	*Independent*
10	3**
	2*

* = Reconciled
** = Unreconciled

Jill

Of all the subjects, Jill had the most impressive academic record, including 5's on both the Calculus and Chemistry Advanced Placement exams, and a score of 780 on the Math SAT. She was thinking of majoring in chemistry, and was concurrently enrolled in a third semester course, along with a course in differential equations and linear algebra. She was also the subject most favorable in her account of her high school physics course, saying that she did well in it until electricity and magnetism, when she became frustrated because she "couldn't understand anything."

Jill was a very diligent student, judging from her accounts in interviews as well as from her score on the problem sets (100%). She finished the course with the highest grade out of the original 5 subjects (A–, 82nd percentile). However, it was clear she retained a number of misconceptions about the material. For example, near the end of the course she was persistent in saying there is an upward force on a projectile moving upward.

In discussing her approach and understanding of the material, Jill seemed the most embedded in the context of the course. While Roger, for example, spoke of his own understanding in comparison to how he could or should understand in principle, Jill measured her understanding exclusively in relation to the course. When she said "we know about this formula," she seemed to be applying a convention: if the professor said it in lecture, students know it in an official sense and are entitled to use it.

In general, Jill's comments and behavior could be described quite consistently by *Weak Coherence*, *Apparent Concepts*, and *By Authority*. Jill's interviews took place on 2/6, 2/24, 3/6, 3/27, and 4/17.

<u>Open and semi-directed discussions</u>

Jill was very cooperative during the interviews, and she had a good deal to say in essentially all contexts, although she never initiated discussion.

In the first interview, I asked how she liked various parts of the course. She did not find the book very helpful[1]:

> J: The book, the chapters are not that clear. The book is generally talking about how the formula was derived and [unintelligible word] examples. I don't really like to read those chapters. [laugh] Because, um, I cannot really understand [inaudible]
>
> I: Really. So what's better about the lectures than the book?
>
> J: Better in lectures, I guess that's because he really does things step by step, not like books who kind of skip a part, they just say ok, what we did to a formula, and then we got this, as kind of, skip one part, and we have to really figure out ourselves.
>
> I: So in lecture, if he, if he does something to it, he shows which piece.
>
> J: Yeah, and the way he shows formulas and how to use them in examples. And like the book, the examples don't really work that closely to, I don't know, they don't really show us how to use formulas that well. It's like the author is talking at his position, and we are standing at the other position. Is [laugh]
>
> I: The author's talking at his position
>
> J: ...I think it's ok, it's not really that hard to understand, but um, sometimes they were talking about the formulas, and why they work this way, and sometimes they talk about, I guess, how scientist find out [unintelligible] But, that part is kind of like, we cannot really see it ourselves, so we don't quite understand why they're talking that way. (F-5; BA-4; WCOH-1)

Here Jill spoke of the material exclusively in terms of formulas and of the role of the teacher or the text as to 'show formulas and how to use them.' Moreover, at the end of the excerpt, she spoke of limited expectations for what students need to or are capable of understanding, in regard to "why [formulas] work this way... and how scientists find out." These were indications of *Formulas*, *By Authority*, and *Weak Coherence*.

Jill's description of the value of labs indicated *Apparent Concepts* (2/24):

> J: It was kind of just the main thing, because, we would do it ourselves, rather than professor show us how to do it. [laugh]
>
> I: Ok, ok. How much of it did they, did you, did you come up with the way to find out what the acceleration was, or did you
>
> J: Um, they, they kind of show us, um, like, on board, they put out some proofs, like saying why you graph like this, why the slope equals to acceleration, and they show you proof, and basically they want us to try it ourselves and plug in formula to see the way it works. So to show us those formulas are really like, correct. (AC-4)

Like Daniel, Jill thought students need to "try it ourselves" to learn, but to her the point was simply to show that the formulas are 'really correct.'[1]

---

[1] English was Jill's second language.

Similarly, she described the purpose of demonstrations in lecture as providing literal associations with observations, so that "you can... memorize it better" (3/27):

> I: How do you like demonstrations in lecture, are they helpful?
>
> J: Yeah, it's pretty helpful, because, it's better than just saying ok, I will turn faster if I pull the mass toward me, rather than, he just sit there and like show us, right away, so you can tell, you can kind of memorize it better, you can just remember what he did, other than remember in concepts. (AC-1)

Specific content

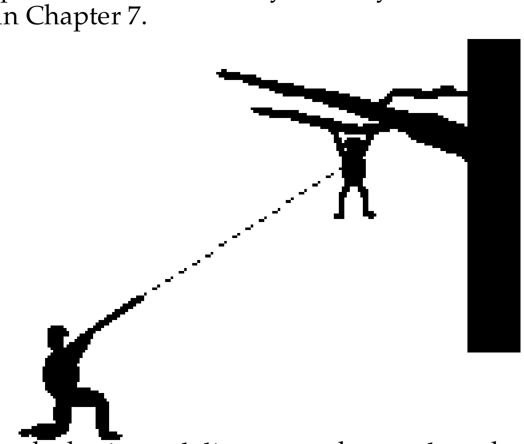
Jill's account of the monkey and hunter[2] problem provided further evidence of *Apparent Concepts* in her view of the value of demonstrations. She understood the point of the demonstration to be that the bullet and monkey hit, so that the professor could set their heights equal in a calculation. In fact, that the two heights were equal was the result of the calculation, not an assumption: the calculation was to show that, because of the nature of projectile motion, the two heights must be equal and so the bullet must hit the monkey. Jill, by focussing on a literal association with the result, missed the underlying mechanism the calculation was meant to elucidate (2/6):

> J: Oh. This is, well, I think he just try to figure out, since they can hit together, that means that this height will be the same for both of them. And he found out a formula for the height from the bullet, so he's going to find out the height in another formula from the monkey, so that is just h - one half gt^2. And they should equal, because they hit together. (AC-5)

---

[1] To be sure, by my observations, Jill was right: the point of labs *was* mainly to verify formulas. I will discuss possible effects of the course on beliefs in Chapter 7.

[2] The monkey and hunter is a standard demonstration in introductory physics courses: A hunter aims his rifle directly at a monkey hanging in a tree so that if the bullet were to travel in a straight line, it would hit the monkey. Just as the hunter fires, the monkey lets go of the branch. Does the bullet hit the monkey? The answer is that it does, as the demonstration shows.



Jill was describing the professor's explanation for why. He wrote algebraic expressions for the heights of the monkey and the bullet, setting the initial velocity of the bullet to be in the direction of the monkey, as shown in the figure. He then showed that at time T, the time for the bullet to move the horizontal distance to the monkey, the bullet would be at the same height as the monkey. In essence, the bullet hits the monkey because both fall the same distance below what their paths would be in the absence of gravity.

At one point in the first interview, I interrupted Jill's explanation of how she solved a problem to ask her about a formula she had used (2/6):

> J: That's the... formulas of the height....
>
> I: Why is, where does that formula for the height come from?
>
> J: That's kind of like the distance it displace. That's not really hard. It should be like x minus x oh equals to v oh h t plus one half a t^2, [$x - x_0 = v_{0h}t + \frac{1}{2}at^2$] that's how he derived in class, which is um when the acceleration is constant....
>
> I: How, how did he derive that?
>
> J: How did he derive that?
>
> I: Yeah, how do you get that?
>
> [J tries to explain the derivation but is not successful.]
>
> J: I'm not quite sure about this, because, well, I don't remember this. Um, this part's wrong. I'm not so sure how to derive this, but, um, I remember that's how he tell us about it in lectures. So, we know about this formula....
>
> (P-4; WCOH-1; F-2; BA-4)

Jill first interpreted my question of 'where does it come from' to mean 'what is it for.' She went on to say that she did not "know about" the derivation because she did not "remember" it, but she did not seem to think it was important. That she was told about it in lecture was sufficient for her to say "we know about this formula."

In the third interview, she spoke similarly about a formula from one dimensional kinematics (3/6):

> J: That's just the formula we memorized. And, well, he derived it in lecture, so we know we could use it.

Statements like these probably reflect in addition the importance of the educational context in Jill's beliefs. Here she seemed to be describing her perceptions of the rules of the game: 'if he tells us about it in lecture, we can use it.' The idea in physics was to find ways to take what is given and use it to find the answer.

$v = v_0 + at$

Jill worked on the two rocks problem at the beginning of her fourth interview (3/27). She answered the question immediately, with a qualitative argument, but she said that, if it were for homework, she would probably "do a calculation... because the TA would say, 'Ok you got to show it.'" I asked her to show me what she would hand in. In responding, she used $v_f = v_0 + 2at$ as part of an attempt to find the final speeds. When that led to some complicated algebra, she decided to try "to find the times another way" and wrote: $a = (v_0 - v)/2$.

Asked to compare this with the first formula, she said "I think I memorized the wrong formula" and wrote down a third: $v_f = v_0 + (1/2)at^2$. Challenged again, she was unsure, saying that she should "go back to the beginning, or... check the book." When I asked if there was any way to figure out which was correct, she derived $v = v_0 + at$ and $x = x_0 + v_0t + (1/2)at^2$, by integrating $a = dv/dt$. She explained that she had "remember[ed] the wrong thing... combined formulas together."

Jill's taking $a = (v_0 - v)/2$ as "another way," without considering how it related to what she had first tried, $v_f = v_0 + 2at$, indicated *Pieces* (P-5); her inability to decide between the alternatives for the formula, which I have assumed to have accessible conceptual content, indicated *Formulas* (F-3).

<u>Problem solving</u>

Jill described problem solving as a process of finding equations with the right variables (2/24):

> I: So why do you want to know t?
>
> J: Um, because I want to put it into the next one, the next formula, or equation so I can find out at least one variable, otherwise I have like three variables, or two variables. (F-5)

In other words, Jill explained, she wanted to find the time because she would then be able to substitute it into the next equation, so as to reduce the number of unknowns. This was an indication of *Formulas.*

In explaining a solution to a midterm problem, Jill described rejecting one method because it did not give her an unambiguous answer. She was solving for the time it would take a tossed rock to reach a height of 32 feet. Her solution involved a quadratic equation, which she found to give two solutions. She dropped this technique because she could not decide which answer to pick (3/6):

> I: So you got t is 1.48 seconds or 1.35 cause you solved this quadratic formula,

J: Right.

I: and since these are so close together it would be tough to decide which one.

J: Yeah, and from this, this way should get about the same answer, and this way I don't have to try to find out square roots or anything.

I: Ok, tell me what the two different ways are? What way is this?

J: Um, this way is looking from distance, I look back to the vertical motion again, and try to find out how many seconds it takes for the ball to go up to 32 feet high. And try to find out that time and put it back to, to how I find out the horizontal distance it travelled, because I can find out the horizontal velocity, I know the initial velocity already, I can find it like an angle, I can find out the component, and then, but, later I was thinking, those two times so close, maybe because the ball was travelling a horizontal distance already at that time, so maybe there is like an error in there. I can't really figure it out myself.

I: Maybe there is an error in where?

J: In the time. Like, maybe it reaches 32 feet high, and like, from here already, and it's going horizontally, so there's a range.

I: Oh, I see what you're saying, so maybe these are like endpoints?

J: Yeah.

I: So you're saying maybe it first reaches 32 feet at 1.35 seconds and then goes horizontally until 1.48 seconds.

J: Yeah. (AC-3, AC-5; BA-1)

The problem statement had specified that "the ball is moving horizontally: at the moment when it reaches 32 feet. One of Jill's thoughts about her first attempt to solve the problem was that there might be "a range" over which the ball travels horizontally. This episode indicated *Apparent Concepts* for two reasons. The first is based on an assumption that a conceptual argument why the ball could not travel horizontally should have been accessible. Independent of this assumption, Jill did not examine the mechanism of the first method to understand why it might have failed when it produced a result she did not trust.

There was also an indication of *By Authority*, in Jill's statement that she "can't really figure it out" herself. In addition, that she interpreted the result of her first calculation as possibly reflecting horizontal motion seemed at least partially to result from her literal reading of the wording of the question.

airplane

Unfortunately, the tape recorder malfunctioned during the last interview, so there were no transcripts for Jill's solutions to the airplane and ring problems, nor for her responses to direct questions. The evidence came entirely from her written work during the interviews and from the notes I made afterward.

Jill's solution to the airplane problem involved choosing a formula by the symbols it contained. When I asked her how a plane travelling in a straight line could have an angular velocity, she drew a circle around the earth, as had Daniel. As with him, this behavior indicated *Apparent Concepts*.

ring

Jill solved the ring problem using the formula for rotational kinetic energy. She did not think the linear formula could apply, and she did not refer to the material from the text passage she had explained earlier in the interview. Her work on this problem thus indicated *Pieces*.

<u>Counter-indications</u>

Out of 88 initial codings, only 13 were inconsistent with the characterization I have presented. Of these, half were codings of *Concepts* easily reconciled with *Apparent Concepts*, as in the case of Jill's explanation of an examination problem, which asked for what the acceleration would be if a film of a falling stone were played backward (3/6):

> J: Ok, that's a motion picture is the free fall, and if you play upward, the stone will just go upward, but, um, its velocity is upward because it's moving upward, but its acceleration is moving downward, because I can see the stone like moving slower and slower.

The excerpt indicated *Concepts*, for the evident use of conceptual argumentation. However, it could be reconciled as consistent with *Apparent Concepts* because the reasoning involved only a literal picturing of the phenomenon (REC-5), to connect to a known case.

In the same interview, there was a coding of *Concepts* based on Jill's account of the solution to the problem of the two stones dropped one second apart. As with Roger, the coding could be reconciled because her explanation was the same as one presented in lecture.

There were 2 indications of *Independent*, both of which could be reconciled as consistent with *By Authority*. In describing lectures, Jill said it is not sufficient merely to "copy everything," one must "think a little bit" (2/6):

> J: Can just, you know, copy everything he put on the boards, and then without really think a little bit.

> I: You don't have time to think about it.
>
> J: Yeah, well, maybe that's just for me, I'm kind of slow most of the time. But, um, actually I like to think about what the professor said and then I copy it down to make sure I understand what's going on. Otherwise I'm kind of confused. (I-5 —> REC-11)

This could be reconciled as consistent with *By Authority*, because it could be interpreted as saying that one needs to pay attention and to follow along with the lecture in order to learn, rather than as saying one needs to figure things out for oneself (REC-11).

Of 5 indications of *Coherence*, 2 could be reconciled. In two cases, Jill discussed taking care to follow presented derivations, as in the second interview, discussing the text (2/24):

> J: Um, actually I really did go through them, like, I really say like ok, here's r1 here's r2, so there's a change of angle, and then I will put this one here, ok, that's v1, that's v2, so that's change of angle cause they are the same thing, and here's the change of velocity, so I don't really say, ok, I understand this, I won't look at it. I really do go through and see what happens....
> (COH-1 —> REC-1)

Here Jill's concern for following the derivation may not have been as much for coherence in her understanding as for fidelity. That is, in a manner consistent with *By Authority*, Jill wanted to be sure to receive the information completely and accurately.

There were 3 unreconcilable indications of *Coherence*. One was Jill's explanation (2/6) that she "cannot just plug in the formulas and do it," but has "to go through it step-by-step" (COH-1). The other was her explanation of the difference between this course and her high school physics (3/6):

> I: Is this different from the high school physics? How does this compare?
>
> J: Um, it's pretty much the same, but like high school, they don't really go into calculus that much, because not all high school students study calculus. And, sometimes they don't really explain why there are so many equations or how to derive equations, it's kind of purely memorize sometimes.
>
> I: In high school. And here
>
> J: In here, it's like, you have to understand how to derive it, otherwise you won't understand what's going on, unless we have these, understand at the beginning how scientists thought about it, and that's why they have those kind of equations, and that's probably the difference. (COH-1)

These comments directly contradicted others Jill had made about memorizing formulas and the importance of understanding "how scientists thought about it."

Numerical summary

JILL	Total instances:	68
	Total codings:	88
Pieces 16	*Weak Coherence* 8	*Coherence* 3** 2*
	Apparent Concepts 16	
Formulas 19		*Concepts* 6*
	Weak Concepts 1	
By Authority 15		*Independent* 2*
		* = Reconciled ** = Unreconciled

Evan

Evan was one of the original five subjects. Of all the subjects, he had the weakest academic record. Like others, he did not think his high school physics course was very good, although his was an advanced placement course. Evan took the AP but only scored a 2, which is not considered sufficient for advanced placement. He was currently taking the second semester of calculus; his grade in the first was a B–. He "didn't do too well" in Chemistry 1A the previous semester.

Evan frequently acknowledged that he did not 'understand physics too well,' so he could not 'rely on what he thought.' He felt he "kind of" understood, well enough to survive in the course, which was all he expected. Of all the subjects, it was most clear with Evan that he did not consider real understanding accessible or essential to his work in the course. In the end, his was the lowest score of the subjects I interviewed (43rd percentile, C), although he put in a significant amount of effort (95% of problem set points).

His beliefs could be characterized consistently by *Weak Coherence*, *Apparent Concepts*, and *By Authority*. The interviews took place on 2/10, 2/24, 3/10, 3/31, and 4/21.

<u>Open and semi-directed discussions</u>

Evan had only a little to say in the most open discussions about the course. There were, however, several indications of *By Authority*. In the third interview, he said he was glad the lectures had slowed down (3/10):

> I: All right, are you, is the stuff interesting lately.
>
> E: I'm just glad he slowed down a little bit. Cause I wasn't going to go to lecture because after a while I go I better catch up and read, cause I haven't been reading before lectures, like, really hard to follow, but he's slowing down now, so I go and listen, and maybe something will go through my head and then I'll remember it when I do the homework or something. (BA-4)

Evan's hope that "something will go though [his] head and then [he]'ll remember it" indicated an expectation that learning involves taking in information.

Like Daniel, Evan did not think it was useful to spend much time on a problem he did not know how to solve (2/10):

> I: So, so this, so you worked with a friend, and you help each other on the homework.
>
> E; Yeah, which is better because sometimes, I'll be, I don't know like, cause I'll be working like on a wrong approach and I'll totally work it out and waste my time doing it, where, cause I overlook something simple, so

He reiterated this position in the third interview, and I challenged it a little (3/10):

> E: I can follow it kind of, the book, but if he assigns a problem it's like kinda hard to do. Plus, I don't want to waste my time doing it wrong too, so, and so, I'm like kinda, I don't know, I'm reluctant to waste my time trying to do it, cause I always end up doing it wrong, so I get help first, [unintelligible]
>
> I: Oh, so you get help first before trying it.
>
> E: Yeah, well I look at it and I start it halfway, but I always end up doing it wrong, so I look at how other people do it.
>
> I: Some people would say that it's good to do it wrong, cause then you find out that it's wrong and then you
>
> E: Learn from your mistakes?
>
> I: Learn from your mistakes.
>
> E: Hmm, I guess. But, I, if you have a lot of time [laugh] or if you can do it really fast. It takes me a long time to do the homework.

Both of these instances were indications of *By Authority*. Evan felt that working on a problem he was not sure how to do was a 'waste of time,' rather than that coming to understand such problems was part of learning.

<u>Specific content</u>

$v = v_0 + at$

Evan worked on the two rocks problem in the fourth interview. I interrupted him to ask about $v = v_0 + at$ and $d = vt$ (3/31):

> I: Ok. Um, if you were going to have to teach someone, say, $d = vt$, $v = v_0 + at$, could you do it, how would you do it?
>
> [...]
>
> E: [laugh] I don't know, it just seems like, you just know it, ok, let's see. Velocity equals acceleration times time, um
>
> I: You just know it.
>
> E: I don't know how to explain it. It's just like, I just go [pause] Or I'd go, ok, for sure, or, velocity is equal to distance over time, like when you drive miles per hour, so that's easy to memorize, I guess, or, it's just distance over time, that's velocity, so then if you know that, that's for constant acceleration.
>
> I: Velocity is distance over time is for constant acceleration.
>
> E: If you were to apply it to physics problems.
>
> I: If you were to apply it to physics problems.
>
> E: [laugh] Or, I mean like, you could still find velocity if you drove a car to San Francisco and you took your miles per hour and you're accelerating in the middle.
>
> I: Ok, so that's not really a physics problem, that's
>
> E: Everyday life, something you can relate it to.
>
> I: Ok, so this is something, velocity is distance over time is something from everyday life.
>
> E: Yeah.
>
> I: ...Ok. Um, and v equals at?
>
> E: I'm trying to figure out how I got that. Let's see. I don't know how to explain that, I don't know, I just like know it. It's harder to explain acceleration, but, I don't know.... (AC-1)

The excerpt indicated *Apparent Concepts*: Evan was able to associate $d = vt$ with his intuitions about speed from driving, but he described this as useful only because it made the formula "easy to memorize." As well, Evan considered applying the formula to "physics problems" as different from applying it to driving, showing he generally distinguished "everyday life, something you can relate it to" from physics. Furthermore, Evan did not have, and did not think it important to have, a conceptual understanding of $v = v_0 + at$.

I brought $v = v_0 + at$ up again in the final interview in the context of the direct discussion about using common sense. Evan said $v = v_0 + at$ was (4/21) "kind of like a calculus derivation or something... I understand that... but I couldn't explain it..." I then presented the national debt problem. He was able to solve it, after some initial confusion about whether the increase was multiplicative or additive.

> I: Ok, so is this, is this an easy question?
>
> E: Yeah, I'd say it's [unintelligible word] easy.
>
> I: Common sense? [pause] Ok. Now, what's the definition of acceleration?
>
> E: Velocity times time.
>
> I: Definition of acceleration.
>
> E: Oh, definition? What is the definition? Um [pause] it's ah, well it's like the change of velocity over time.
>
> I: It's the change of velocity over time. Ok, so if I have an initial velocity, and then I, and it changes, by a, a is the change per second, of this velocity, and t seconds goes by, what is the final velocity.
>
> E: v i plus a t. [unintelligible] velocity
>
> I: Now why is that?
>
> E: Because, this is like the rate it's changing per time, and this is the amount of time, so that would give you another velocity. Like the change in velocity would be the change, so you just add it to the initial amount to get the final.
>
> I: Ok. Is that hard?
>
> E: Um, not really.
>
> I: Is it the way you were always thinking about this?
>
> E: Um, no the way I thought it was just like, I just kind of like memorized it, it was like, I didn't really think, like, what it really meant, I just kind of knew it.
>
> I: Um, so is it, if I ask you now, is this common sense kind of thing, or is it a, formula kind of thing?
>
> E: Um, it's, I guess it's common sense kind, I mean it's more than just common sense to think about it, but I mean, you can explain it to someone and it's, yeah, without any calculus.
>
> I: Ok. So, is this pretty much the same as this?
>
> E: Yeah.
>
> I: All right. Um, ok, mm, so, is this kind of thing that you just did here, is that a kind of thinking you'd be using in the course?
>
> E: Um, not, not really.... (P-4; F-4)

Like Roger, Evan was able to solve the national debt problem with relative ease. He was also able to apply that reasoning to understand $v = v_0 + at$, although he needed a few prompts. This again supported my assumption that the conceptual content should be accessible. Most important, as had Roger, Evan went on to say that this was not how he had thought about the formula before: he had 'just kind of memorized it and didn't think about what it really meant.' As well, this was "not really" the kind of thinking he did much of in the course.

Problem solving

Evan's deemphasis of "learning from your mistakes," in an excerpt above, was also evident in his accounts of his errors on a midterm (3/10):

> I: Ok. All right, on this one what happened? Two rocks are dropped at the same time, one 1 meter above the other, and you said the distance, you said the separation between them decreases, and they said that's wrong it does not change.
>
> E: Yeah, actually it doesn't change, cause now I've thought about it, cause, if you just drop two rocks, they're going to go thump-thump [gestures, hits table] they are going to land the same, they're just going to have the same [unintelligible]. I just thought, if you drop two things, [pause] yeah, cause I just thought about dropping things, and it would just stay the same.
>
> I: Ok, that's since then. Do you remember what you were thinking while you were doing this that made you say decreasing?
>
> E: Um, I was thinking that one would have more time to accelerate, but then, acceleration is constant, so
>
> I: The one above would have more time or the one below?
>
> E: Yeah, the one above would have more time, so that it would cover more ground, that's what I was thinking.
>
> I: But that's wrong because
>
> E: Well I was talking to a friend, and he goes, like, just like, it just made sense that the time wouldn't decrease. (AC-3, AC-6)

Evan never explained why the distance between the two stones would not change; he only described substituting one mental image for another. Moreover, as with Daniel and others, it was not important for Evan to understand why his first way of thinking about the problem failed: when I asked why thinking that "the above one would have more time" was wrong, he could say only that "it just made sense that the time wouldn't decrease."

airplane

I asked Evan about the airplane problem in his fourth interview (3/31):

> E: Well I used this formula, um, angular velocity is equal to, [pause, flips page] angular velocity is equal to velocity, [pause] ok, yeah, it's equal to the radius times, no, velocity is equal to angular velocity times the radius.
>
> I: Velocity is equal to angular velocity times the radius. Ok.
>
> E: Yeah, and then, so they have the velocity and they have the radius, so then I just multiply together to find, ah, ok, they want the angular velocity, so angular velocity is equal to velocity divided by the radius.
>
> I: Ok, ok. Now see that one I thought might be kind of weird just because it's talking about, well, it's asking for an angular velocity but there's nothing rotating, [unintelligible] it's just a plane moving in a straight line.

> E: Yeah well I was confused too, but then it was like the only thing, I mean like looking through the chapter it was like the only formula that would work, so I figured, it'd go in. (P-2; F-2)

Evan chose the formula for its symbolic content. In response to my question, he acknowledged confusion, but he did not find, or consider it important to find, any explanation.

ring

Evan solved the ring problem using the expression for rotational kinetic energy. He did not connect my question, about the possibility of using linear kinetic energy, to the passage he had explained earlier in the interview (4/21):

> I: Ok. Um, this was, ah, somebody else tried to do this, and tried to use one half m v squared to solve it.
>
> E: [pause] Well so that wouldn't work cause it's not translational motion. Cause it's just, it's just spinning around in place.
>
> I: Ok, um, what she tried to do was say, but these things are moving. [pause] And so actually what she did was she just put one half times 5, times v squared, because all these things are moving at the same speed, and all together they made up 5 kg.
>
> E: But it's not, [pause] that's for, isn't that for linear motion, or, that's cause it's not really moving a distance, it's like going in a, well since it's
>
> I: I guess, go ahead.
>
> E: Since it's a circle, it would, well, since it's a circle, I have to use this, or, I use angular velocity. (P-5)

This was an indication of *Pieces*, in that Evan did not invoke the discussion earlier or expect a connection between translational and rotational kinetic energy. He thought of the two methods as unrelated and applying in unrelated circumstances.

Direct questions

Evan's response to my direct question about whether intuition is useful in the course explicitly indicated *Weak* and *Apparent Concepts* (4/21):

> I: Well so the question is then, um, is your intuitively, is your intuition useful in the course, or is it not useful?
>
> E: Um, well I don't know, if there's a test question, and it seems simple, if I think it's simple, then I think it would be something else, a different answer. Cause, like if it's too simple, and I think there's something mathematical behind it.
>
> I: So that's like this one.
>
> E: Yeah, so that's why I don't usually follow my intuition, and try to follow like the math part.... Well, I guess it helps, cause that way, um, [pause] I don't know, it doesn't really affect [laugh] how I, well to me it doesn't really affect

> how I do it, [pause] do the problems and stuff, I just like do it, I don't like, think about it.... I don't go, ok, the answer should be around this, like look at it, and estimate it before I do it.
>
> I: And when you say do it you mean
>
> E: Start working on it.
>
> I: You work on it with algorithms.
>
> E: Formulas [said overlapping]
>
> I: Formulas. Um, so intuition kind of stuff doesn't really affect how you work on it too much.
>
> E: Well, probably because I don't really have that good of a, like a, physics background thing, I mean I don't really understand it too well really, so I can't really rely on what I think. And things like um, like for instance like units and stuff, I don't really pay attention to units, I just kind of get the number right, so that's fine.
>
> I: That's like on the years and the
>
> E: Yeah, so I don't really, I mean I just kind of do it, I don't really understand it that well.
>
> I: Ok. You don't understand it that well.
>
> E: Um, well I kind of understand, but I mean like, if you start asking me questions like that, then I get messed up, they just confuse me more.
>
> I: Ok, but if I don't ask you questions like that, how about
>
> E: [laugh] I think I understand. (AC-1, AC-2)

Evan distinguished between different levels of understanding, intuition and formulas, saying he didn't "usually follow his intuition," but tried "to follow the math part." Moreover, he acknowledged that intuition could be useful in principle, but did not expect it to be useful for him. In fact, he tended to be suspicious of his intuition, often choosing not to follow it, when it seemed "too simple." At the end of the excerpt, Evan said he "kind of" understood the physics, except "if you start asking me questions like that," a sentiment he repeated elsewhere.

The indication of *Apparent Concepts* was in the distinction Evan drew between "the math part," which he tried to follow and seemed to feel he understood, and his intuition. The indication of *Weak Concepts* was that he did not expect to have an intuitive understanding himself, although he acknowledged its existence. Again, it is worth noting, there was also an intimation of Evan's having a goal of succeeding in the course, rather than of understanding.

<u>Counter-indications</u>

Out of 78 initial codings, 13 did not agree with the characterization. More than half of these were codings of *Concepts* easily reconciled with *Apparent Concepts*. One example of these was Evan's explanation of an exam problem (2/24):

> E: Yeah that part, it was like you drop a stone, like 100 meters, and 101 meters, right, [gesturing] and the time that they hit the ground is t1 and then you drop one from 200 and 201, and that's time 2, and then what would be, they go t1 greater than t2 like that, and so t2 is less than t1 because they're coming down faster, so they have greater velocities, so they cover more distance, so that, I mean they're going to be closer together when, or, it's just going to come down faster, it has less velocity. (CON-2 —> REC-7, REC-6)

For one, the use of conceptual knowledge here could be seen as apparent. More important, this was an example covered literally in lecture. In fact, Evan's explanation for the exam problem uses the same numbers as the lecture explanation, numbers not in the exam problem statement.

Another indication of *Concepts* reconciled similarly was Evan's reasoning on an exam problem (2/24):

> E: They just said you film a stone, you drop a stone, off a table or something, and then they play it in reverse, so then which way would the velocity be going, and I said up, and which way would the acceleration be going, and I said down, because the acceleration is negative. And I would think, cause, it would be, in the film it would be going really fast and then slowing down, so I said it was going down, and I got it right. (CON-2 —> REC-7)

As earlier with Jill, Evan may have been describing no more than a literal visualization of the event, to match with a situation that had been well discussed in lecture.

There were only 3 codings, 2 of *Concepts* and 1 of *Coherence*, that could not be reconciled. One was Evan's statement, in the second interview, when he was explaining what he had done on an exam question (2/24):

> I: So what does this mean, this is
>
> E: See I didn't, actually I didn't understand why, that's why I wrote it down, cause I figured, if mass was going down it should be negative, plus, negative mg to negative a, and this would, I was, this would be um, mv squared minus mg here, so I don't know. It confused me and I was getting tired, so I just wrote that down. [laugh] (COH-3)

This was an unreconciled indication of *Coherence*, because Evan seemed to be saying that he would expect to "understand why," but that in this instance he was "getting tired."

Evan's reasoning on hearing the two rocks problem was an unreconciled coding of *Concepts* (3/31):

> E: Well, the velocity of this one coming down would be, um, I guess gravity times time, of the horizontal one, and then, this one, let's see, the other on is thrown down, so it would be v nought plus gt, so the one thrown down is going to have, already the initial velocity added to it, so it's going to hit the ground first. And which one hits with the greater speed, this one, it's already starting out with more speed, so it's just going to, I mean, this one has more speed to start off with, and they're incrementing at the same amount, I think, so, it's just adding on to there so it's going to have a greater speed.
>
> (CON-2)

Evan's use of the word "incrementing," integrated with the formula, had me code this as indicating a conceptualization of mechanism not consistent with *Apparent Concepts*.

<u>Numerical summary</u>

<u>EVAN</u>	Total instances:	62
	Total codings:	78
Pieces 15	*Weak Coherence* 4	*Coherence* 1**
	Apparent Concepts 16	
Formulas 21	*Weak Concepts* 3	*Concepts* 10* 2**
By Authority 6		*Independent* 0
		* = Reconciled ** = Unreconciled

Ken

Ken was the second subject chosen for his high score on the first midterm. With the exception of Larry, he was the most uncomfortable being interviewed, especially solving problems. He had little to say in open discussions, but, perhaps because by the third interview he had relaxed, he was quite articulate in response to direct questions about his beliefs.

Ken scored a 700 on the Math SAT and was concurrently enrolled in second semester calculus. He also had advanced placement credit in chemistry. Thus, he had a strong academic record, but it was not as strong as either Roger's or Jill's. Nevertheless, from his explanations in interviews as well as from his final grade in the course (A, 92nd percentile), Ken fared better than either Roger or Jill. His score on the problem sets was 89%.

In some ways, Ken was the most interesting subject. As much as with anyone, it was clear he was mainly concerned with success rather than directly with understanding, and he seemed to think of the course as a matter of learning how to solve problems. In these respects he differed from Tony. However, unlike Roger and the other subjects, Ken clearly felt conceptual understanding was essential. On the other hand, again in contrast to Tony, Ken explicitly devalued knowledge of derivations, except insofar as they supported his conceptual understanding.

Thus Ken was a counter-example for several possible suppositions: 1) that success in physics is dictated mainly by mathematical ability or background; 2) that it is essential to have a goal of understanding; and 3) that it is important for students to follow the details of proofs and derivations.

Ken's beliefs could be characterized as *Concepts* and *Independent*, but I could not find a consistent characterization along *Pieces* ↔ *Coherence*, except to say "not *Pieces*." Ken's interviews took place on 3/31, 4/18, and 4/28.

<u>Open and semi-directed discussions</u>

Ken was the most reticent of all the subjects during open and semi-directed discussions. We did not spend much time going through lectures; but we had discussions about specific content, mostly as came up in the course of problem solving.

<u>Specific Content</u>

Like Tony, Ken found rotational dynamics to be "not intuitive," in contrast to linear dynamics (3/31):

> K: See now I can picture that, force acting this way, force acting that way... you can really see that, but when you get up here, this kind of stuff... like statics and torques... you can't picture it... But I'm sure that if I work with it enough, it will get easier. (CON-3)

This was an indication of *Concepts*, in the implied expectation that picturing should be involved in understanding.

$v = v_0 + at$

Ken's response to my question about $v = v_0 + at$ resembled Tony's (3/31):

I: And what's that, where is that from?

K: That's just kind of an, that's a very intuitive equation that says that the final velocity is going to be the velocity it starts out with, plus the acceleration, times how long its been accelerated. (CON-1)

As had Tony's, Ken's explanation essentially reiterated the formula in words. The indication of *Concepts* was simply his statement that this was "very intuitive."

Problem solving

Ken was generally quite tense about working on problems during interviews.[1] He was far more comfortable discussing problems he had already solved.

In the first interview we went over some questions from the first midterm examination. Ken's solution to one problem was marked wrong, but he had felt it was correct and complained. In fact, his solution was correct, but it was different from what had been taught in the course.

The problem involved two blocks, one on top of the other, with a certain amount of friction between them. It asked about the force between the two blocks, given certain conditions. In essence, Ken solved the problem in the accelerated reference frame of the lower block, with a fictitious force on the top block in the accelerated frame.[2] Ken did not use this terminology: these ideas had not been discussed in the course. Apparently the graders thought he had misunderstood the physics, that his correct answer was fortuitous (3/31):

K: I think they wanted to solve it some different way, or some slightly different way. And because a lot of people, it seemed, at least what I hear from the TAs... you can get this answer doing it the wrong way, apparently, because of just the way the numbers work out, so I think they just looked at it and said, oh, he set it up a little different.

I: So, well now, I guess I'm confused because you say there's no movement but there is an acceleration.

K: I think that's where they were confused too. It's just the way I thought of the problem. It's just because, well, there is no acceleration between these two blocks. So, there's a force pushing that way, and a frictional force pulling that way, and because it stays together here, there is no acceleration between the two blocks. The frictional force has to counteract the force that is, in a sense, that is pushing on the top.

I: Now where is that force?

---

[1] See below for a description of Ken's attempt to solve the two rocks problem.

[2] The term "fictitious" is often used to describe a force perceived by an observer moving in an accelerated frame of reference.

> K: That force is imparted by the acceleration of the system. The first thing you have to do is find the acceleration of the system, find it's moving 3 meters per second squared.
>
> [...]
>
> K: Relative to this block, when you push it forward, there's a force acting that way. It's like the ground underneath it is moving, that means there's a force on the block, that has to equal the friction force to keep it in the same position, relative to this block. (CON-2; I-6, I-7)

Ken's explanation of his solution was self-consistent and conceptual. It seemed likely he had invented it for himself. The conceptual level of his argumentation indicated *Concepts*. His ingenuity in the solution method as well as his confidence in his own reasoning as valid indicated *Independent*.

airplane

Ken was the only student I interviewed who did not apply $v = r\omega$ to solve the airplane problem (4/18):

> K: Ok, the way I think I did it was, it asked for angular velocity, so that's like radians per second, so then I just found how far the plane would go in a second, and then found how many radians that was...

His response to my probe showed *Concepts*:

> I: Well, the reason I've been asking people, one of the reasons I've been asking people about this one a lot is I thought maybe it would see kind of strange that it's talking about angular velocity, but the plane is moving in a straight line.
>
> K: Well, it's not really strange if you just think about it in terms of radians per second, omega is equal to radians per second, cause then it's clear that you're going to be traversing a theta there whether it's straight or not, you're still going to be going over this theta. So it's not strange in that way.
>
> (CON-2)

Ken's explanation of the situation, that "it's clear you're going to be traversing a theta," was a valid conceptual argument.

ring

Ken was also the only subject who immediately solved for the kinetic energy of the ring using linear velocity. When I asked him to justify the method, he invoked the argument from the text passage he had read earlier in the interview (4/18):

> K: Ok [sigh] kinetic energy of the ring. Well, the total kinetic energy is going to be, a summation, a total summation of all the little different points on the ring, and since they all have the same velocity, 3 m/s, basically I think it's an

integral. At least that's the way I would do it. So, let's see. Kinetic energy, one half m v squared, you want to sum up all these little guys....

[Ken solves the integral correctly.]

I: Now, a lot of people when they're given this problem solve it by finding the moment of inertia of the ring, and then the angular velocity.

K: [pause] could do it that way. Just different ways of thinking about it.

I: Um, ok, in fact, when I say to people who do it that way, that other people do it something more like this, they say well, you can't do it like that, cause this is rotating.

K: Well... all the particles, all rotation is is just, they have velocity this way, tangential, I think [those people are] missing that or something. So I mean at this point, at any time, it's just a bunch of particles, with velocities going off tangentially.

(COH-5; CON-2)

Ken's control of the relationship between rotational and linear kinetic energy indicated *Coherence*. That he explained the relationship with a conceptual argument indicated *Concepts*.

Direct questions

Ken was most articulate responding to direct questions about the role of common sense in his reasoning (4/28):

I: ...would you say is this something that you get, do you know this because you found it in the book? When I ask you how do you know this is true, um, two extreme answers, one is well it's sort of obvious that it's true, and another is it says so.

K: Well, it's probably a combination of the two of those. Basically, I mean, once you see a formula like this you go, well, what does that mean, velocity at some point is going to be the velocity you started with plus the change in velocity per unit time times time. You say, oh, well that makes sense. So I mean you just kind of, once you see a formula, if it's just a bunch of numbers, I mean, letters, whatever, it's not going to make that much sense to you, but if you, really sit down and look what it's saying, then it starts to make sense.

I: Is that something, is that a kind of thinking that you would say is important to doing this stuff?

K: Oh yeah, you can't just look at the formulas and not understand, and just try to plug in, cause you're not, there are so many things in physics that you can throw in, you know added complexities, and different twists and things, that you're not going to be able to do it just using formulas. You've really got to understand the formulas and what they mean, and how they apply to other things, not just to the example problems in the book.

(CON-1; I-1)

Ken expressed a general expectation that formulas should make sense, that they should be more than "a bunch of... letters." He also said that understanding a formula involves "really sitt[ing] down and look[ing] at what it's saying," showing, for one, a sense that formulas say something, and, for another, a sense of personal involvement in learning. The excerpt indicated *Concepts* and *Independent*.

Shortly later I introduced the phrase "common sense":

> I: Ok, so let me ask another question. How much would you say, um, your common sense is involved in this course?
>
> K: Well, common sense, I mean it plays a big part, just being able to reason out, but then your common sense is modified by what you've learned in here. Because I mean at first, it doesn't seem common sense that angular momentum is equal to r cross p, that's not common sense, the guy on the street, he's not going to know that. But working with it, and seeing it a lot, it almost becomes a part of your physics common sense, so I mean, yeah, it's obvious that this will be true, if you have this and this, that there's going to be an angular momentum. So I mean, having common sense helps, that you can see that the forces are acting this way, but it really has to be modified by what you've learned in the course, and added to. (CON-1; I-4)

Ken's response, that "common sense is modified by what you've learned," indicated *Independent*. The excerpt also, again, indicated *Concepts*, in that Ken felt common sense was involved in the course whether it was adequate to begin with or not: one develops a "physics common sense."

I challenged his position, that common sense is modified, to try to understand what Ken thought modification entailed:

> I: An example that I keep using is centripetal – centrifugal force, and a lot of people come in with the idea that it's out, everybody's been in a car or something, and the car turns left, and you feel thrown outward... And then you get in the course, and the course says no that's not true, there's a force going in... is it easy to do, to modify your common sense that way?
>
> K: Well, it's like anything, you just have to sit down and think about it, could that be true, using what I know from my common sense and what I've learned from my physics, could it not be a force outward, could it just be a force bringing it back in? I mean, we went through this pretty extensively, finding the tangential velocity, and we derived the acceleration going inward, and then I mean you think about it, ok, at any point you're going to be going outward like this, to keep you in a circle there's got to be a force bringing you inward, so you just kind of keep thinking about it, and then finally you say yeah, well there must be a force pointing inwards, otherwise how would you stay in the circle. So, the common sense is modified, and, I mean it's not something that you just accept. I mean, you can, for a class you can write it down and, ok, I know that for the test, but then not really, not really put a whole lot of weight in it, or something, you could do that, but if you really want to understand it, you have to sit down and think about it.
>
> (CON-1; I-4, I-5)

Ken described the process of modification as involving more than simply replacing wrong ideas. He distinguished a shallow understanding, without a "lot of weight," from "really" understanding, which was what he expected for himself.

Counter-indications

Out of 42 indications in Ken's protocol along *Formulas* ↔ *Concepts*, 8 were inconsistent with *Concepts*, all of which could be reconciled. Most involved Ken's discussing how he used the text, either in answer to my general questions or in going through the passage on rotational kinetic energy (4/18):

> K: I just use the book basically for examples.
>
> I: So do you read the text part?
>
> K: I read it, but I don't know, I don't really get much out of it, just basically for formulas and examples it's pretty good.

(4/28):

> K: Anything with summations I immediately ignore.
>
> I: Wow. Anything with summations you ignore. Why?
>
> K: [laugh] Just because, from reading this book, I know that you're never going to have to use a summation in a formula, they always bring it down to some simpler formula, so I just skim right to here. Right this is K equals one half I omega squared. That's a formula, I pay attention to that. This is a formula, but that's a summation formula. I don't pay attention to that.
>
> (F-5 —> REC-9; WCOH-1)

These indicated *Formulas*, for the implication that the literal formulas are the content of physics knowledge. Based on Ken's explicit discussion of the meaning of formulas in the final interview, it was at least plausible that he used the word "formulas" to refer both to the symbolic expression and to their underlying conceptual content.

Another indication of *Formulas* similarly involved Ken's description of how he had solved a problem (3/31): "That was really easy. Basically plug and chug equations." By this, however, Ken could well have meant that the problem did not require much thought (REC-8). This reconciliation was supported by Ken's later description of another problem, saying (4/28) "it wasn't even really a physics problem... it's just like formula manipulation, you don't even have to think about anything."

There were 4 indications of *By Authority*. 3 of these were statements Ken made in the course of trying to solve the two rocks problem in the first interview. He had a great deal of difficulty with it because, he explained, he had not seen this type of problem before (3/31):

> K: ...even in the first section, where it had problems like this, he had said those exact questions in lecture, so I knew how to think, but this one, I don't really know how to think about the problem. (P-5; BA-1)

The statement indicated *By Authority*, as well as *Pieces*, because Ken thought he needed to have seen the same type of question in lecture to know how to solve it.

However, Ken was extremely tense in solving this and other problems during the interviews. While he was working on the problem, Ken said he felt it should be easy, that he was missing something obvious:

> K: I don't know, it just seems like a simple problem, like something I should be remembering from chapter 4... I guess I'm really rusty.

When we had finished with the problem, he accounted for his difficulty:

> K: And I think I also got stuck on this one just trying to find the quick and easy way, instead of just trying to just grinding it out... cause I get discouraged by equations, big equations, well, there's got to be an easier way.

That Ken felt it should be an easy problem probably contributed to his embarrassment over not being able to solve it. As he described, his wanting "to find the quick and easy way" probably did inhibit his ability to solve the problem.

I decided to reconcile the inconsistent indications as reflecting Ken's tension, an effect of the context of the interview, rather than his beliefs about what was involved in learning in the course. This rationale was applied only to Ken, and only to his solution of this problem. One might object to such special treatment, but I believe that, although it is important to try to delineate consistent criteria for such judgements, it is also important to be flexible to other factors that seem obvious and relevant. Certainly it is reasonable to suppose that, for a particular subject, discomfort in the interviews may have a dramatic effect.

There were also 8 indications of *Pieces* during Ken's attempt to solve the two rocks problem, compared to 2 indications of *Pieces* in the rest of the protocol. I considered these 8 as reconciled in the same way. The remaining 2 could also be reconciled, but by the same rationale used with other subjects.

It was, however, not possible to characterize Ken's beliefs consistently along *Pieces* ↔ *Coherence*. It might be argued his beliefs could be characterized as "not *Pieces*," but it was not clear that a characterization *Weak Coherence* or *Coherence* would suffice, either.

He was certainly not always concerned with understanding proofs or derivations, except insofar as they supported his conceptual understanding. This was clear in his explicit comments on the value of derivations as well (4/28):

> K: You know, you can probably derive this formula if you wanted to, or you could show mathematically why that's true, and this one you probably couldn't do that, but the important part in problem solving is realizing what this means, and how it affects the situations and the problems that you're working with, and how it applies to those problems. And to really be able to understand that. (WCOH-1; CON-3)

In reading the passage from the text on rotational kinetic energy, he explained that he generally ignored derivations (4/18):

> K: I don't know, that's just all it's saying, [laugh] basically is, cause of all the little summation formulas, and, in this book they always try to derive, you know, their formulas, they always try to show you how they got to them. It's usually by summing up a lot of things and then they get to the real formula, so if I already understand that they've summed up a bunch of little things, I don't really need to know that, all I need is the formula.
>
> I: I see. You pay attention to the summations enough to know that they're summing a bunch of little things.
>
> K: Right, but I don't really need to know what, cause I already understand the concept. If I didn't know, you know, at all what's a moment of inertia, I don't understand how they do that, I might read it... (WCOH-1; CON-2)

> K: Basically, well, here, they're talking about total moment of inertia, which is just the sum of the moments of inertia of a bunch of parts, basically you can define it as the mass times the radius squared, away from the axis of rotation, or the resistance of something to rotational motion, or the rotational analogue of mass. It's all those things.
>
> I: Ok. Why would it be mass times radius squared?
>
> K: It's because of this, um, derivation that they do here. They're summing up all the kinetic energies and then converting them into that, and they get the summation one half m i r squared
>
> I: Ok. So would it be fair to say that you don't pay attention to this because you already know it?
>
> K: [brief pause] Yeah, pretty much.
>
> I: Or would it be that you don't pay attention to it because it isn't important.
>
> K: Combination of both. I already, I already understand the concept, and, I understand it well enough to know that it's not important.
>
> (WCOH-1; CON-1)

Ken was saying that the derivation itself was not important; he needed only the conceptual summary that it's "summing up all the kinetic energies." This indicated *Concepts* and *Weak Coherence*. In his response to the ring question later, it was clear he did make use of the derivation. There, the coherence of the rotational and linear expressions was prominent in his understanding.

One might argue, then, that he could be characterized by *Weak Coherence*, in that he was aware of the derivations, but did not consider them generally of use. That, however, does not seem to capture his beliefs, contrasted with, for example, Daniel. For Daniel, proof and derivations were simply other pieces of knowledge that did not make up a functional part of his own understanding.

Like Daniel, Ken's judgement of the importance of proofs and derivations also seemed mediated heavily by his goals. It was clear on a number of occasions that his main purpose was success in the course, and that understanding was important to him only inasmuch as it served that goal. However, he did seem to believe that coherence was often functionally important in serving that goal, especially in supporting his conceptual understanding.

I would argue that Ken would be best characterized by *Coherence* in regard to his conceptual understanding and by *Weak Coherence* in regard to the formalism and derivations themselves. Thus he paid attention to the conceptually central content of the derivations, but not to the details of the manipulations. However, I do not believe there was sufficient evidence in the transcripts to support this characterization.

<u>Numerical summary</u>

<u>KEN</u>	Total instances:	58
	Total codings:	78
Pieces	*Weak Coherence*	*Coherence*
10*	5	4
	Apparent Concepts	
	0	
Formulas		*Concepts*
8*		34
	Weak Concepts	
	0	

By Authority 4*	*Independent* 13

* = Reconciled
** = Unreconciled

Larry

The interviews were not very successful at eliciting information from Larry. He was uncomfortable with the entire process, to the point that I did not ask him to return for the fifth interview.

My general impression is that he his beliefs were consistent at least with *Concepts* and *Coherence*, but that he never felt he understood the material, and so he was continually embarrassed at having that revealed. There were not enough indications in his protocol, however, either to support or to argue against any particular characterization. For this reason, I have omitted his protocol from the analysis.

Summary

This chapter has shown that:

1) <u>It is possible to construct characterizations of students' epistemological beliefs, consistent across interviewing contexts and across physics content.</u> With the exception of Larry, for whom the interviews were not effective at eliciting information, and of Ken along *Pieces* ↔ *Coherence*, clear characterizations were possible for all subjects.

To be sure, the characterizations were not entirely consistent, but complete consistency should not be expected. First, this analysis considered only beliefs about knowledge and learning. Thus it left out a number of aspects of subjects' reasoning, such as their goals and perceptions of the requirements of the course. Several of the subjects' goals evidently shifted between *understanding the material* and *succeeding in the course*.

Second, the framework I have chosen provided only one perspective on beliefs. In its initial state, it was not even adequate, as discussed in Chapter 4; there is no reason to expect the modifications to make it complete. Certainly further iteration is possible, if not significant reorganization. For example, the framework distinguished whether students thought of learning as taking in information or as reconstructing for oneself, but it did not consider what subjects thought constituted reconstruction. Daniel felt it was important for students to work on problems without too much help; Roger felt he should "question" the material, but each seemed to have had a limited sense of what questioning the material or resolving conceptual difficulties should involve.

Nevertheless, it is impressive how consistent the characterizations of subjects' beliefs could be. There were patterns evident in each subject's problem solving, explanations of physics content, use of the text, and in explicit comments, both spontaneous and in response to direct questions. The patterns were evident as well across physics content: note that the indications discussed above were drawn from discussions related to a range including one-dimensional kinematics (eg $v = v_0 + at$), rotational kinematics (*angular velocity*), linear dynamics (*force*), and rotational dynamics (*moment of inertia*). There were clear differences between what the subjects said and did; these could be described as differences in epistemological beliefs.

<u>2) Students differ significantly in their beliefs about knowledge and learning in physics.</u> There appeared to be two general patterns: Daniel, Evan, Jill, and Roger could all be characterized by *Apparent Concepts* and *By Authority*; Tony and Ken by *Concepts* and *Independent*. The following table summarizes the results to the standard problems highlighted above, another illustration of the consistency of the indications of subjects' beliefs. A "I" denotes indication of *Weak Coherence*, *Apparent Concepts*, or *By Authority*; a "II" denotes indication of *Coherence*, *Concepts*, or *Independent*.

subject	**v = v + at**	**airplane**	**ring**	**direct Q**[1]
Daniel	I	I	I	I
Evan	I	I	I	I
Jill	I	I	I	*
Roger	I	I	I	I
Ken	II	II	II	II
Tony	II	II	II	II

*Data not available.

[1] The choice of I or II for the direct questions was based on extended conversations, usually containing a number of indications, and not on the immediate response. In Tony's case, for example, there were two unreconciled codings of *Apparent Concepts* associated with his response to the direct questions. Nevertheless, the overall tenor of the discussion was strongly oriented toward the Group I characterization.

In the next chapter, I will discuss the involvement of students' beliefs in their work in the course, as was evident in the interviews. The emphasis will be on comparisons between the two general patterns identified here.

Chapter 6: Effects of beliefs on learning and problem solving

Many of the indications cited in the previous chapter were based on the evident involvement of beliefs in subjects' reasoning. There the focus was on establishing the possibility of consistent characterizations. Here the focus is on the involvement itself, in subjects' learning and problem solving.

There was evidence of involvement in correlations between subject's understanding of the material and their beliefs, as characterized in the previous chapter, and there was evidence in the form of specific instances in which beliefs appeared to play a significant role.

It is important to note that this discussion is not to imply a one-way causality. One would certainly expect beliefs to be affected by students' knowledge and resources, such as through experiences of success and failure with different approaches. A student may fail repeatedly in attempts to use common sense, due to lack of content-level resources or strategies for applying and modifying this knowledge. Such experience might lead to a belief that common sense is of limited relevance. In this sense, beliefs may serve as a route by which the student's experience can feed back on itself (diSessa, 1985). The purpose here, however, is only to discuss the effects of beliefs on students' learning. Analysis of the origins and development of beliefs is an area for further research.

The chapter is organized as follows. First, I will present the correlational evidence between subjects' responses to three qualitative questions and the characterizations of their beliefs. Then I will discuss mechanisms by which beliefs could be observed or inferred to have had an effect on the subjects' learning or problem solving, citing examples from the protocols. I will refer to Daniel, Evan, Jill, and Roger as "Group I," characterized by *Weak Coherence*, *Apparent Concepts*, and *By Authority*; and to Tony and Ken as "Group II," characterized by *Concepts* and *Independent*.[1]

---

[1] As I have noted, Tony was quite articulate in solving problems during the interviews, while Ken was uncomfortable and reticent. Consequently, it was easier to draw inferences concerning the involvement of beliefs from Tony's protocol than from Ken's. Most of the examples of "Group II" in this chapter are from Tony's interviews.

Correlation

As discussed in Chapter 3,[1] I asked three qualitative questions of each subject as a measure of their understanding of the material. Each question was designed to probe for a specific misconception commonly found among introductory physics students.

toss

The first question was a probe for the misconception that motion implies force (Clement, 1982; Viennot, 1979). I asked each subject to identify all of the forces on an object thrown straight up in the air, after it is released but while it is still moving upward. As a follow-up, I asked for an explanation of what happens to all of the forces as the ball rises and falls. I coded a subject as showing a misconception if she or he described an upward force in responding both to the initial question and to the follow-up. Evan and Jill were the only subjects to show the misconception on this problem.[2]

merry-go-round

The second question involved tossing a ball on a merry-go-round, to probe for the misconception that an object could move in a curved path without a force acting on it (Halloun and Hestenes, 1985ab; McCloskey, Caramazza, and Green, 1980). I coded a misconception if the subject thought the ball, as seen by a stationary, overhead observer, might move in a curved line. Daniel and Roger both showed the misconception.

juggler

Finally, the third problem asked whether a 98 pound juggler juggling three one-pound balls would make it across a bridge that could hold no more than 100 pounds. This problem was to probe for misconceptions about force: I coded a misconception if the subject thought a ball

---

[1] See page 27 or Appendix B.

[2] Ken, in answering the toss question, at first identified an upward force (4/18). When I asked him to "describe what happens as it goes up and comes back down," he amended this, explaining that "as it goes up, it loses its initial velocity... This is just, actually, this isn't a force, this is an initial velocity." I challenged his amended explanation, but he stayed with it: "You give it an initial force, it gives the acceleration that you want, but after it leaves your hand, there is nothing, there is no more force on it acting upward." Daniel's response was similar. Following the discussion in Chapter 3, page 28, I did not code these as misconceptions. In contrast, Evan and Jill both maintained their original assertion through several probes and challenges.

being thrown or caught would not exert more force on the juggler's hand than its weight of one pound. Daniel and Roger again both showed a misconception.

That a subject did not think there would be an additional force on the juggler's hand could be attributed in several ways. It could represent a failure to recognize acceleration in the one-dimensional motion of the ball (Trowbridge and McDermott, 1981). It might also be a failure to realize that the upward acceleration of the ball requires an upward force. Or it could involve a sense of asymmetry between the force of the hand on the ball and that of the ball on the hand (Maloney, 1984). I did not, however, try to classify the misconceptions for this problem in any particular way; for the purpose of this study, it was only important to note that there was some kind of fundamental misconception involved.

In sum, the only subjects who showed the misconceptions for which I probed were those in Group I. The results are summarized in the table.

subject	**toss**	**m-g-r**	**juggler**	**grade/%-ile†**
Evan	misc	no misc	*	C+ / 43
Daniel	no misc	misc	misc	B– / 59
Roger	no misc	misc	misc	B– / 64
Jill	misc	no misc	no misc	A– / 82
Ken	no misc	no misc	no misc	A / 92
Tony	no misc	no misc	no misc	A+ / 100

* For no good reason, I failed to give Evan the juggler problem.
† %-ile refers to percentile ranking in the course, as determined by a weighted total of exam, problem set and lab scores.

The correlation can also be seen in the subjects' performance in the course, from the grades listed in the right hand column.

Certainly it was not surprising that Tony and Ken had higher grades and showed fewer misconceptions: they were chosen for their high scores on the first midterm examination. What was interesting was not that they understood, but that they could be characterized by beliefs different from those of the subjects who showed misconceptions.

Observations

More significant than the correlational evidence, there were many instances in the protocols in which beliefs appeared to play a significant role in subjects' learning or problem solving. A number of these have already been mentioned, as indications of beliefs in the previous chapter. I will describe three overlapping aspects of the involvement of beliefs in subjects' learning and in problem solving, in regard to

1) how these students formed or broke conceptual associations;

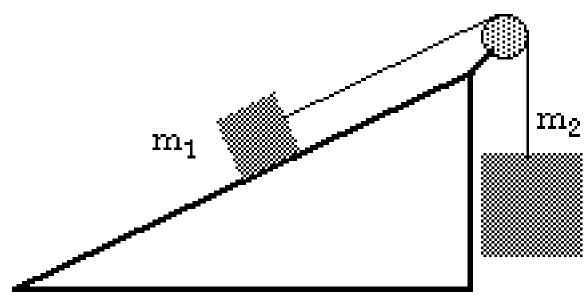
2) how they used information presented in the course or acquired in solving problems;

3) their attitudes and goals in the course.

Forming and breaking conceptual associations

Subjects in the first group were quite casual about making and breaking associations between different aspects of their knowledge; subjects in the second were much more careful about building and modifying their understanding.

One problem from the text (Ohanian, 1985, p. 150) involved two masses connected by a cord. m_2 (3.0 kg) was suspended; m_1 (1.5 kg) slid on an inclined surface, and the problem was to find "the acceleration of the masses." The problem also specified the angle of the ramp and the coefficient of friction between m_1 and the ramp.



It happened that Roger and Tony made a similar mistake[1]: they both solved for the total force on the pair of masses but applied it separately to each using $F = ma$. This resulted in the two masses having different accelerations.

[1] I should note reasons for caution in comparing the two mistakes. For one, Tony solved the problem much later in the course than Roger. For another, the solution paths were different: Roger's calculation involved tension from the outset, and he did not notice the difficulty on his own; Tony discovered the discrepancy, which led him to think about the tension in the string.

I pointed this out to Roger (2/16) :

> I: And let's see, so, it looks like, m one is, which, m two is bigger than m one?
>
> R: Yeah.
>
> I: So, if I divide this by a bigger number for m two, then that means m two has a smaller acceleration than m one. [pause] Is that right?
>
> R: From what I put, I guess that's right, but is that right? Oh geez. How could one be accelerating faster than the other one? Shouldn't they be equal? [pause] They should be equal. [pause] Shouldn't they be equal? Oh, no, that's right, yeah.
>
> I: So this one accelerates more than this, ah, slower than this.
>
> R: [pause] That would mean that the velocities would have to be different. Is that right. [pause] Yeah, I guess so .
>
> I: Ok.
>
> R: Well, I don't know, I'd check, and see if I got the right answer. I'm 90% sure.

Roger chose to reject his notion of plausibility rather than the solution, at least pending further consultation with an authority, without analyzing why his common sense might be wrong. As a result, he did not discover the conceptual error in his calculation.

Tony, on the other hand, immediately abandoned his solution technique (3/29):

> T: Yeah, but I wonder if this term is correct now. Cause, if there were no friction, it would simply be, m two, yeah the force would still be m two g, hmm. [pause] Ok, I'm starting to think I did something basically wrong in the problem now, I'm starting to wonder.
>
> I: And what's making you think that?
>
> T: Because, the way I have it set up now, if there were no friction, the force would just be m two g, so then the acceleration of this would just be m two over m one g, but that's not right.
>
> I: Why?
>
> T: Because the acceleration here is just going to be 9.8 meters per second squared, here it can't be higher, it can't pull this along, they have to move at the same speed.... Ok, now what I'm doing is, before I said that this force [indicates the expression for the total force on the pair] was going to be [acting] right here [on m_1], and now I'm saying that's not true. And actually now that I think about it that is right, cause this force, the gravity here is trying to, accelerating more than just this box, it's also accelerating this box.... See, the force isn't going to be equal, the accelerations are, the accelerations are going to be equal, ok, that's what it is.

A contradiction with common sense prompted Tony to reject his calculation and to reconsider his understanding of the situation. This led him to discover the conceptual error in his solution.

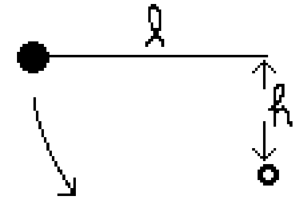
It is at least arguable that these two students showed similar familiarity with the content at the outset of their problem solutions. However, they differed significantly in how they handled what each perceived as a contradiction with common sense.

For Tony, coherent, conceptual content was part of understanding physics, and so it was necessary for him to resolve the contradiction. Rather than simply reject his solution technique, he felt it was important to identify the error in what he had done: "before I said that this force going to be right here." (It is worth noting that he had never actually *said* where the force would be, except in how he had performed the calculation, an indication that he thought of the algebra as an expression of conceptual content.) Tony's discovery of the error helped him construct a correct solution to the problem. Moreover, having identified the bug in his reasoning, he would likely be less susceptible to it in the future.

For Roger, conceptual understanding was ancillary to physics understanding, separable and not essential. To him, the contradiction with common sense simply meant that in this case his common sense did not apply. Because he did not question his solution technique, having followed what he thought to be the correct procedure and having made no algebraic errors, he saw no need to look for conceptual errors. Not only was Roger unable to solve the problem, but he missed an opportunity to resolve a point of confusion in his understanding.

Daniel's solution to one problem (3/13),[1] which I gave him as part of our conversation about the difference between kinetic energy and momentum, showed a similar casual abandonment of a connection with common sense:

A ball is tied to a string of length l, *attached to a pivot. A post is placed a distance* h *directly below the pivot. If the ball is released from the position shown, with the string straight and horizontal, how large must* h *be so that the ball will swing completely around the post?*



Daniel tried to solve the problem using conservation of kinetic energy: he set the initial potential energy equal to the kinetic energy at the bottom of the swing, and equal to the potential

[1] See page 65. I describe the problem again for convenience.

energy again at the top of the swing.[1] After some computational errors, which he discovered after an answer did not have a simple algebraic form, he found $h = \frac{l}{2}$ from

$$mg\,(2l - 2h) = \frac{1}{2} mv^2 = mgl\ .$$

He recognized that this meant the ball would have "zero velocity" at the top of its swing:

> D: Well, I'm assuming that, um, if h is l over 2, right here, I'm assuming that it's going to go up here, it's going to up here, you know here you will have, um, zero.
>
> I: It will stop.
>
> D: It will, yeah, it will have zero velocity, so I'm also assuming that it will drop this way, instead of back down, or, straight down. [Daniel sketches the options: the pendulum bob reaches the top of its circle, and from there either drops straight down or continues along a circular path.] [...]
>
> I: You're assuming it will do that. Is there any way to show that it will do that?
>
> D: [pause] I think so, yeah. Something to do with [unintelligible] velocity. [pause, page flipping] Yeah, the velocity at the top, of the circle, has to equal, [page flipping] that's the first time I saw that [teaching assistant] ...that specific case was like on the second homework assignment or something, and, I forget how to do that. I['m not] really good with remembering how to do problems....
>
> I: So you say it will come up here, gonna fall, and then it stops right there.
>
> D: It will have zero velocity, I didn't say it was gonna stop.
>
> I: Ok, it will have zero velocity, you didn't say it was gonna stop.
>
> D: I'm saying all the kinetic energy will be converted to potential energy.
>
> I: Is it moving at this point? You didn't say it's going to stop, so now say, is it going to stop or not?
>
> D: [pause] Is it going to stop or not. [pause] Good question....
>
> I: Ok. So you know, you know that the velocity's zero,
>
> D: Mm-hm. [Yes.]
>
> I: but you're not sure if it's going to stop.
>
> D: Um, [pause] I'm not sure if it's going to stop. See I'm always confusing myself, I'm thinking about too many things at once, because, it seems to me that momentum will carry it this way, or inertia, will carry it that way, but I'm also thinking about critical velocity of whether it will have enough speed at this point to make it go around or not.... I think all the information is given, I just have to remember which, concepts to use.

Daniel arrived at the result that the velocity of a pendulum bob equaled zero at an instant when his common sense indicated the bob should still be moving.[1] Rather than make use of his

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[1] Daniel set the height at 0 at the bottom of the swing, so he had the initial potential energy = mgl and the final potential energy $mg(2l - 2h)$. Both of these terms were correct, but Daniel's expression omitted a term for the final kinetic energy. In addition, a complete, correct solution would have involved an expression of the condition for the ball to have enough speed to loop: the centripetal acceleration at the top of the swing should equal the acceleration of gravity.

intuitive knowledge, he chose to abandon at least one aspect of his association of *velocity* with motion. He was confident that the bob "will have zero velocity," that "all the kinetic energy will be converted to potential energy," but he was "also assuming" that it would keep moving along the path he expected.

As with Roger, Daniel had enough information available for him to decide there was an error in his solution, but he did not think to look for it. Nor did he think to examine his common sense to try to understand why it failed. He was unable to solve the problem, and he missed an opportunity to improve his understanding. Worse, in casually abandoning his association of zero velocity with no motion, Daniel could have developed a misconception that would encumber him later.

Roger's and Daniel's behavior in these instances are examples of a pattern in Group I subjects. They showed little inclination to resolve conflicts between different aspects of their knowledge and were often quick to abandon connections to common sense that might have been useful. This seemed partly a reflection of their beliefs about knowledge in physics: if one expects only casual, apparent associations between physics and conceptual knowledge, one would have little concern for maintaining associations in circumstances where they do not seem apparent. Moreover, if one thinks of learning as receiving information, rather than as modifying one's understanding, resolving conflicts for oneself would not be of obvious value.

In contrast, Group II subjects seemed particularly motivated by contradictions. They thought of conflicts as important to resolve, which often resulted in modifications of their common sense. This seemed to reflect, in part, their expectation of an intimate association between physics and conceptual knowledge, and, in part, their belief that discovering and repairing inconsistencies in one's understanding is an essential part of learning.

It also happened that Group I subjects rejected erroneous solutions, but they did so in a similarly superficial manner, one not likely to be of general value. In her solution to a projectile

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[1] Daniel's common sense was correct; his result of zero velocity was not.

problem, discussed in Chapter 5,[1] Jill considered but rejected a solution that would have had the projectile travel horizontally for a length of time. Her reason was not that this result contradicted her understanding of projectile motion; it was that the question had implied a single number response. Jill did not try to understand why her solution technique might have failed or why the idea that the projectile would travel horizontally would be wrong. In this way, she missed an opportunity to learn, and, it is reasonable to assume, she retained a potential misconception about projectile motion.

Tony's reasoning about a pair of examples from lecture displayed a deliberate attempt to build connections between intuitions in the process of trying to resolve his confusion. The examples, which came up in Chapter 5 as well, involved an open train car rolling without friction on level tracks. In the first case, the train was filling with water from rain falling straight down relative to the ground; in the second, the car was full of water that drained, straight down relative to the car, from a hole in the floor. Tony incorrectly remembered the professor's having said that the car would slow down in both cases, and I asked him to explain.[2]

Tony was comfortable with the first case, which he understood and explained in both conceptual and formal terms. In explaining the second case, however, he decided that he still did not understand it. Rather than simply deciding his intuition did not apply, or that it was not a matter he could sort out for himself, Tony tried to understand the situation through various analogies (4/25):

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[1] See page 94.

[2] In fact, in the first case the car would slow down, but in the second it would not. In the first case, one way to understand this is in terms of the forces on the train car. The water falling straight down has to be accelerated to the car's speed as the car fills, so the car has to exert a force forward on the water. The water must then exert a backward force on the car. Since there are no other horizontal forces, the car slows down. In the second case, the water does not exert any horizontal force on the car as it drains. Since there are no other horizontal forces on the car, it continues with constant speed.

Another way to understand these examples is in terms of momentum, thinking of the car and the water inside as a system. In the first case, the system gains mass but not momentum. With greater mass, the car must have less velocity to have the same momentum ($p = mv$). In the second case, the car loses mass and momentum proportionally, since the water draining moves forward at the same speed as the car.

> T: Ok, this took me a long time to figure out. Ok, um, thing that you say, like, the tank and um, instead of thinking it as a tank and the water, think of it as like if, two things are tied together and they're thrown, and, um, without, say there was like a timer on one of them so that it would all of a sudden, no, that's impossible to do, um, hm hm hm. Just think of it as like, as two particles going along, with some momentum mv...

Tony compared the situation to a pair of objects of equal mass separating, then to a collection of objects bound together with only one object separating:

> Well, I mean, I see it in terms of this one here, you have like, um you have like, you know, say five particles, each moving along at a velocity v, and you have, well, that's doesn't make sense either, because if you have, like here is v 6, and [unintelligible] one of them drops off, and momentum is 5 mv, but the mass is 5/6 of what it was before. Wait a minute. The mass of this is 5/6, ok, you have 6 mv, and one of them drops off, the total momentum is going to be 5 mv, I don't know why that would make it go slower.

His next analogy was to a ballerina spinning and holding bricks in outstretched arms, trying to understand whether she would speed up if she dropped the bricks:

> If I assume this problem is truth [the car draining water slows down], she should slow down, but if I ignore that that's truth, I don't know what she would do.

He went on to say that, although it "instinctively" made more sense to him that the ballerina might slow down, he could not understand why she would.

Tony was clearly looking for a mechanism by which the car would slow down. Expecting connections with his intuitive knowledge, he looked for analogies to try to elucidate a mechanism. In the end, none of the analogies helped him understand why the car would slow down; in fact he said it made more sense to him that it would continue at the same speed. His reasoning had him question what he thought the professor had said.[1]

In this case, as in the example of the two blocks and the ramp, Tony's beliefs about what kind of knowledge to expect, and about what was of value to learning, had him examine his understanding at a conceptual level. What he discovered in the process, that he had been thinking of the total force as acting only on the smaller block, and that draining water can be

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[1] Much of Tony's difficulty derived from his confidence in what he believed the professor had said. However, it was clearly a separate question to him whether he thought the car would actually slow down and whether he thought it was something he understood. He did not consider an appeal to the professor's having said it to be of value as explanation or justification.

thought of as separating masses, could well have been of value in making him aware of a general point of confusion and of a useful way to resolve similar conflicts in the future.

Group I subjects also generated connections between different aspects of their knowledge, but the process involved much less deliberation, and the connections they generated were often superficial. Jill, in reviewing a lecture, spoke about spring forces, saying that the force is proportional to displacement. When I asked her to explain, she said it was like the situation "if you drop something, then you need to have a certain force to stop it.... when you drop something in a log." This was a reference to a lecture demonstration showing that the faster an object is moving, the greater it penetrates an arresting medium. What seemed to lead Jill to make the connection was that both situations involve a proportional scaling of something with distance.[1] She formed a connection based on an apparent but superficial similarity, and she did not question the comparison further.

Asked to explain why an airplane flying in a straight line would have an angular velocity, Daniel and Jill both came up with the apparent but inappropriate explanation that the plane was really moving in a circle around the earth. Again, they did not expect this conceptualization to be of significance in their understanding, so they did not examine it for validity. It was only convenient to satisfy a particular need, in this case that of answering my question.

Tony, in contrast, treated conceptual associations as an integral part of his understanding, and he expected them to be useful. That he compared draining water to separating masses and to a spinning ballerina dropping bricks showed he was not forming connections based on superficial similarities but on a feature of the physical mechanism.

In one respect, students in Group I had an advantage in solving problems. Not needing to build a conceptual understanding or to find intuitive connections, they could simply invoke a

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[1] In the case of a spring, the magnitude (strength) of the force is proportional to the distance the spring is stretched or compressed. The stopping distance of a projectile is proportional to the kinetic energy of the projectile, when the stopping force is constant.

relevant formula to solve a problem quickly. Students in Group II sometimes had more difficulty, because of the additional burden of building intuitive understanding.

This was evident in different solutions to the airplane problem. Ken's was only approximately correct, and he did not receive full credit for it in the course. He found the angular velocity of the plane by solving for its linear displacement over 1 second, and then finding the angular displacement from the perspective of the observer on the ground. Thus he actually solved for the average rather than the instantaneous angular velocity. Subjects who applied $v = r\omega$, based on its literal content, found the correct answer.

Jill, in fact, was the only one who gave the correct answer to the juggler problem without equivocating. Her solution was quick and valid: she said the juggler and his balls made up a single system that weighed more than the bridge could support. She did not justify or elaborate on this solution, nor did she look for alternative ways to think about the problem that might have conflicted. Tony made the same system argument, but he spent a great deal of time questioning it because he could not think of a mechanism by which a force greater than 100 pounds would be exerted on the bridge.

Ken's failed attempt to solve the two rocks problem[1] may have been another example of the inefficiency of a conceptual approach. He spent a good deal of time and effort trying to find an easy, conceptual argument by which to solve the problem without having to do any calculations. In the end, this did not further his solution of the problem.

Using information from lectures and reading

Subjects in the first group were quick to decide that they understood new information, while those in the second were more reflective and questioning.

The examples above of Group I subjects' casual formation of connections with other knowledge can also be described as examples of the subjects' having decided prematurely that they understood the material. Evan's explanation of centripetal force was another (2/10):

E: It's inward.

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[1] See page 113.

> I: Ok, how do you get that?
>
> E: Um, I just thought it, I don't know, it says here that it's inward, and it's just, I don't know, I always thought of it inward, cause when you, I mean when you go in a circle, you go inward, so, I don't know, that's how I thought of it.

It is unlikely Evan had actually "always thought of it [as] inward." Regardless, his understanding that it was inward was simply that it seemed right, and not that he had examined an inward force for consistency with what he knew otherwise or against the possibility of an outward force.

Daniel's explanation of an example from lecture, in answer to my request, involved a misconception that motion implies force (3/3):

> D: The concept of in outer space, where, the only way a rocket can go forward is to, to shoot off all this hot gas, in the opposite direction, does make sense. Um, why that makes sense, um, [pause] the way I, for me that just, if I picture it in my mind, I can't think of any other way to explain it. I just accept that. I mean, of course it makes sense [laugh] to me, I don't know why, if somebody asked me that, yes, it does make sense.

He felt that he understood the example because he could "picture it in [his] mind," but the picture in his mind had it that "the only way a rocket can go forward is to... shoot off... hot gas."[1] By his own account, he 'just accepted it,' without checking to see if the conceptualization he used was valid, if it fit in with what else he knew or had heard in the course.

Jill similarly misinterpreted another example from lecture (2/24):

> J: Here is the pendulum problem... there's a force, going like horizontally, because it's moving that way, so that there must be a force somewhere around here to move it upward...

Roger, who accepted the new formula $\frac{1}{2}I\omega^2$ because it "reminded" him of $\frac{1}{2}mv^2$, did not question the relationship between the two.[2]

In each case, the subject described understanding new material, but she or he could not explain it and had not, apparently, questioned it beyond a familiar interpretation. Evan's sense of familiarity, while not based on a misconception, gave him no reason to consider the matter more carefully. If he did have some latent notion of centrifugal force, he would not have discovered it. For Daniel and Jill, the sense of familiarity was based, at least in part, on a misconception. Had they questioned their understanding of the situation more thoroughly, they might have

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[1] In fact, the only way the rocket can *accelerate* is to shoot off hot gas.

[2] See page 83.

discovered the involvement of the misconception, or the conflict in their reasoning with what they had learned elsewhere in the course. Had Roger been more critical about deciding he understood the formula $\frac{1}{2}I\omega^2$, he might have learned more about the conceptual relationship between rotational and linear kinetic energy.

In this way, epistemological beliefs might account for a complaint often voiced by students of introductory physics, and expressed by all of the Group I subjects: they understood the lectures but they could not solve the problems:

> Daniel (3/3): I think I never really had much problems with his lectures, I mean, basically, everything made sense, when I'm sitting there and looking at it, because, um, it just came to me, ...but, if you give me a problem, how do I know where I'm going to start.
>
> Roger (4/5): I understand perfectly in lecture, cause I'm all interested, it's all new stuff... but then [laugh], then I come home and try to do the homework, and it's nothing like what he's doing... well not nothing like it, but, it's just different...
>
> Evan (2/10): I understand when he said it, but when I go do my homework, I'll probably going to have to like look over it and keep looking over it.... I mean, when I was there, when he explained it I, I just go, yeah that makes sense, but it didn't like, get drilled in my head or anything.
>
> Jill (2/24): ...when he explains I can follow pretty well, but I'm not sure if I'm going to do it myself...

These subjects' beliefs about what constitutes understanding in physics allowed them to decide prematurely that they understood new information, based on apparent connections or on having followed the formal manipulations. Lectures thus provided a sense of understanding, but this left the students with only a superficial grasp of the material that did not help them in solving problems.

All of these subjects did express a vague sense of not understanding things as well as perhaps they should. However, they found other ways to attribute their difficulties, rather than to lack of conceptual understanding, such as to not knowing "what the problems are asking" or to the lectures not having covered necessary material:

> Evan (3/10): I didn't even know what [the problem] was asking.
>
> Roger (4/5): ...that's a problem with a lot of these questions, I don't fully understand what they're asking.

> Jill (2/24): ...maybe we don't quite understand what the problem is asking for, um the TA will just graph it out, kind of analyze how we should think about it...
>
> Daniel (3/13): ...there's not much correlation between what he says in lecture, and how to approach this problem. It's kind of like there's a missing gap between them, and we're supposed to fill that in by ourselves?

Such explanations may be counter-productive, because they direct students' attention away from their own understanding of the material.

Neither Ken nor Tony made such complaints or attributions, but they had no reason: both seemed genuinely to understand the material and were able to solve the problems. The contrast, with Daniel and the others, was more striking in the remarks of one subject from the pilot study (Hammer, 1989), whose epistemological beliefs would put her in Group II with Ken and Tony. This student, Ellen, was not able to develop a good understanding of much of the material. However, while Daniel and the others complained that they could understand the lectures but could not solve the problems, Ellen complained the inverse: she could solve the problems, but only by "pretend[ing]," by "do[ing] it the way... it's supposed to be done" without "know[ing] why for real." She felt she did not understand lectures:

> Well things like friction... I'm trying to imagine.... in my mind it would make the force on this car less, but according to the equation that was on the board, it doesn't affect the amount of force...

The gap Ellen noticed was not between lectures and problems but, in lectures, between what she called "theory," the presented formalism; and "reality," her own sense of physical mechanism.

Ellen felt she understood the material at the beginning of the course, and by my observations she genuinely did. After the first few weeks, however, she had difficulty. This could be attributed perhaps to her lack of content-level resources, and perhaps partly to a lack of support she needed from the course for building coherent, conceptual understanding. In any case, she was aware that she did not understand lectures and readings.

The first group subjects in the present study did not have this advantage. As I have argued, because they believed they understood the lectures, they had less motivation to examine their understanding as a possible source of difficulty. Moreover, because they thought of understanding the lectures and readings as separate, they were hindered in applying theoretical ideas to solving problems. This was also evident in the ring problem: none of the Group I students connected it with the text passage, which each had read earlier in the same interview and had claimed to understand.

Attitudes and goals

Subjects in the first group were reluctant to spend time working on problems they did not know how to solve, while those in the second seemed to consider these the most interesting. In part, this appeared to reflect different goals: subjects in the second group appeared to have a goal of understanding the material, while subjects in the first did not always consider understanding important.

Subjects in Group I, on meeting a difficulty with a problem, were much more inclined to give up, to decide that it is not something they could solve without consulting authority. Daniel and Evan, in fact, argued explicitly that to try to solve such problems would be a waste of time:

> Daniel (4/27): Sure, I understand that maybe doing the homework problem will help me understand the concept, but if it becomes so trivial, such that I can have someone explain it to me, without me having to spend 3 hours... wasting 3 hours just to get extremely frustrated, it's not worth it.
>
> Evan (3/10): I can follow it kind of, the book, but if he assigns a problem it's like kinda hard to do. Plus, I don't want to waste my time doing it wrong...

It is at least arguable that Group I beliefs about learning as receiving information from authority contributed to this attitude.

In contrast, Ken and Tony both described enjoying the problems:

> K (3/31): I'm enjoying [the course], I like the problem solving aspect of it, you get a problem and then you find a way to get to an answer...
>
> T (3/29): ...I like the sort of problems, are really, you have to sit down and think about them...

The problems they did not know how to solve seemed generally the most interesting to them. With Ken, who, as I have noted, was shy about solving problems during interviews, this was evident only indirectly, in his description of problems he had solved for homework or on an exam. It was directly evident with Tony, who was quite tenacious in solving problems during interviews. In our discussion about the ring problem, Tony became confused and disturbed, but he was persistent. At the beginning of the interview, he had told me he needed to leave early to get to crew practice. At the end, while he was working on the problem, I reminded him of practice. He answered, "That's ok, coach can wait. I have a problem here."

In part, the subjects' approach may have reflected their different goals. Although this study did not include analysis of goals, it was clear that most subjects were trying to succeed in the course more than trying to understand physics:

> Daniel (4/27): I didn't think about it for, I mean, I know that, if I want to really go into details and be real trivial about it, but again, that's not what they want, so.
>
> Roger (4/5): I read the words and, I believe the book and say ok, [laugh] you are right, and I'm not really going to question it too much. Maybe I should, but then it will take too much time, and, I can't afford too much time.
>
> Evan (4/21): I don't really understand it too well really, so I can't really rely on what I think.
>
> Jill (3/6): That's just the formula we memorized. And, well, he derived it in lecture, so we know we could use it.

With Daniel and Roger, there appeared sometimes an element of cynicism in this decision, as with Daniel's "that's not what they want" resignation, or Roger's ambivalent "I believe the book and say ok, you are right." Both of these students at least acknowledged the possibility of probing more deeply, although neither did. Evan said he did not "really understand," but he did not seem to think it was an option. Jill measured understanding only in terms of functionality with respect to the course: she understood if she could solve problems using the officially given information.

It is important to note that one might have success in the course as a goal and still consider understanding as a useful means for achieving that goal. This appeared to be the case with Ken, who almost always discussed what was important in terms of whether he would need it to be able to solve course problems:

> Ken (4/28): I do a lot of problems to prepare for the test, because I figure, if you've seen a lot of problems, and a lot of different twists and tricks that you need to do in the problems, then you'll be ready for more of the twists and tricks that the professor wants to put on the test.
>
> ...You know, you can probably derive this formula if you wanted to, or you could show mathematically why that's true... but the important part in

> problem solving is realizing what this means, and how it affects the situations and the problems that you're working with...

It is reasonable to assume that some of the differences in subjects' behavior could be attributed to differences in their goals. Still, if Group I subjects did not take understanding as a goal, or as an important means by which to achieve success, this might have derived in part from their epistemological beliefs. Conceptual understanding, as they thought of it, would probably not have made an especially satisfying goal, nor would it have been very useful for succeeding in the course.

Summary

There was a good deal of evidence in these interviews in agreement with results from research in physics cognition, as reviewed in Chapter 1: novices tend to solve problems by purely symbolic manipulations; they often have weakly organized knowledge from the course and fragmented prior intuitions; they often retain misconceptions after instruction. However, in these respects there were substantial differences between the subjects in this study, all of whom were introductory level students.

It would be difficult to attribute such differences entirely to content-level aptitude or background. Jill and Roger had the strongest records in mathematics, including scores of 5 on the Calculus Advanced Placement Exam, higher scores on the Math SAT than either Ken or Tony, and concurrent enrollment in a course in differential equations and linear algebra. Roger and Evan both seemed able, with little prompting, to arrive at an intuitive understanding of $v = v_0 + at$, a formula they had both already known only by its symbolic content. These facts would all put pressure on any claim that Tony and Ken were simply "more intelligent" or better students than the Group I subjects.

There was also no evidence to support the supposition that Ken and Tony began the course with a more substantial familiarity with the content. All of the subjects had had one previous high school course in physics.[1] Moreover, Ken and Tony each made a fair share of

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[1] Halloun and Hestenes' (1985a) statistical analysis, in fact, showed no effect of high school physics background on students' performance in introductory University Physics.

conceptual errors in the course of solving problems. Tony made essentially the same conceptual error in working through the blocks and ramp problem as Roger.

Certainly it is reasonable to assume that there were some differences in the content-level resources these subjects had available, but it would be difficult to support a claim that these were enough to account for the substantial differences evident in the subjects' understanding and approaches. Explanations based on content-level ability or knowledge are not sufficient to account for how evidently intelligent students like Daniel and Roger could decide that a moving ball has zero velocity, or that the blocks in the ramp problem,[1] connected by a taut cord, could move with different speeds.

In Chapter 5, I gave evidence to show that these subjects could be characterized as having general beliefs about what knowledge and learning in physics entail. In this chapter, I have argued that these beliefs could be seen as involved in the subjects' learning and problem solving.

From this perspective of beliefs, it is not surprising that some novices solve problems through purely formal manipulations: they think physics knowledge is made up of purely symbolic expressions. That their understanding remains fragmented and disorganized can be explained by their not considering it important to build or to maintain connections between various parts of their knowledge. Students may retain intuitive misconceptions because intuition is not involved in a substantive, intentional manner in their work in the course.

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[1] See page 122.

Chapter 7: Summary and implications

Conclusions

Beliefs as a theoretical perspective

The main purpose of this dissertation was to study whether students can be understood as having beliefs about knowledge and learning that affect their work in introductory physics.

To study this question, I developed interviews consisting of a variety of tasks and conversations closely tied to the context of the course, in which subjects might reveal something about their beliefs. Through iterative development of an analytic framework, I was able to assemble information from the interviews into characterizations of subjects' beliefs, with exceptions as discussed. The characterizations satisfied a criterion of consistency: the indications of beliefs were consistent across physics content and across interviewing contexts.

In addition, many of the indications revealed mechanisms by which beliefs appeared to affect learning and problem solving. Beliefs could be seen as affecting how subjects formed and broke conceptual associations, how they used new information, how they worked on problems, and how they approached learning. Beliefs could be seen to be involved in whether subjects were able to solve problems and in whether they developed a coherent, conceptual understanding of the material.

Both aspects of the investigation, establishing consistency and showing involvement, were essential to support beliefs as a valid perspective for understanding student reasoning. If the indications of beliefs had not been consistent across physics content, they would have been better interpreted as indications of content-specific knowledge; if they had not been consistent across interviewing contexts, they could have been interpreted as merely artefacts of the interviewing tasks. Evidence of the involvement of beliefs in learning and problem solving was necessary to show the relevance for understanding student reasoning.

In sum, this dissertation has shown that beliefs can be a useful perspective by which to understand student performance in an introductory physics course. In particular, they can provide insight into the differences between students who succeed in learning physics and those who do not.

Framework

The second contribution of this work is the analytic framework itself.

To the extent that the framework evolved through iterative application to the data, it represents results of the study. As described in Chapter 4, preliminary analysis of the protocols showed that the range of beliefs about physics from absolutism to relativism was not relevant for understanding subjects' performance. Nor did it seem important whether subjects thought of the course as a matter of learning to solve problems, as opposed to learning about the physical world.

As well, it was not useful to distinguish whether subjects believed the professor fallible or not, that is, whether they accepted what he said as fact. Instead, what emerged as important was whether they believed hearing what the professor said constituted learning (*By Authority*), or whether they thought they needed to go through some process of creating understanding for themselves (*Independent*).

In this way, the three dimensions of the "initial framework," *Pieces* ↔ *Coherence*, *Formulas* ↔ *Concepts*, and *By Authority* ↔ *Independent*, already reflected aspects of the data. Further refinements were necessary for consistent characterizations: *Weak Coherence/Concepts* and *Apparent Concepts*.

The category of *Apparent Concepts* should be highlighted as an important result. The distinction between *Apparent Concepts* and *Concepts*, between thinking of physics knowledge as incidentally associated or as essentially integrated with conceptual knowledge, was critical for consistent characterizations of the protocols and for distinguishing between Groups I and II. The limited, familiar use of conceptual understanding consistent with *Apparent Concepts* was prominent in many of the instances in which beliefs appeared to play a causal role in Daniel's, Evan's, Roger's, and Jill's work in the course.

As a tool for discovering and validating patterns in the transcripts, the framework was successful. However, there are a number of ways in which it might be usefully recast. In constructing the framework, I made no attempt to divide beliefs into orthogonal dimensions. It might be advantageous to recast it as a set of approximately independent primitives (diSessa 1985), as a perspective on the beliefs' underlying structure, or as a sequence of stages (Säljö, 1982; Pramling, 1983), as a tool for understanding their development.

Unaddressed questions

There are, in fact, a number of general questions this study did not address:

Breadth

Because the analysis was restricted to the context of an introductory physics course, it did not reveal whether the beliefs are specific to physics, learning in school, or whether they might be attributed more broadly. It is possible that many of the beliefs described would apply to a range of disciplines, such as to the physical sciences and mathematics, or to formal education in general.

Development

Related to the question of extent is that of how these beliefs arise and develop, whether they are engendered by instructional practices (Schoenfeld, 1988), school culture (Lave, 1988), or by other aspects of the larger culture, such as from popular images of physics and physicists or from general views of the nature of knowledge. A conception of physics as made up of formulas, for example, may be related to a Western predilection for knowledge as stateable and precise. Conceptions of knowledge as coming in pieces and provided by a source may derive from an understanding of knowledge through a metaphor to material substance (Lakoff and Johnson, 1980). Tracing the development and finding possible influences is clearly an important task for future work.

Goals

As I have noted, there remain issues about the extent to which goals affect beliefs and vice versa. The arguments of plausibility in the previous chapter should be investigated with studies specifically focused on characterizing goals in addition to beliefs.

Strategies

Often in this paper I have attributed a subject's failure to invoke relevant content-level knowledge or to resolve a contradiction to the subject's beliefs. For example, I assumed that a conceptual understanding of $v = v_0 + at$ would be accessible to students who thought to look for it.

In more complicated situations, it is reasonable to assume that content-level knowledge and beliefs would not be sufficient to account for whether or not students succeed in solving a problem or in developing their understanding. They would probably also need strategies for using their current knowledge to develop the understanding they believe they should have.

Tony, for example, in trying to solve the problem of the two blocks and the ramp,[1] decided to consider what would happen if there were no friction. This was an indication of his expectation of coherence in physics, because he looked for a relationship between the two situations, but it probably also reflected strategic knowledge about how to use his understanding of related situations to find the coherence he expected. Tony's decision could have reflected a general strategy, "try looking for simpler cases," as well as a more specific strategy applicable in problems involving friction, "consider the same situation without friction."

Schoenfeld (1985) noted the value of general heuristics in mathematical problem solving, but, he argued, specific knowledge about how to apply those heuristics in particular circumstances is essential. For this reason, he emphasized the importance of situating problem solving heuristics in a context rich in the content of mathematics. General beliefs may support notions of general strategies, but these must be tied to strategies in content-specific contexts in order to be of use. Identification of general and specific strategies in physics may be important

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[1] See page 123.

for further understanding of the nature of beliefs, how they function, and how they might be addressed in instruction.

Instructional implications

I see the potential value for instruction of the perspective of beliefs as two-fold. First, it may help with decisions in the classroom by contributing to teachers' informal models of their students. More generally, it may provide insight for instructional design: I will discuss issues regarding the course I observed as well as possible considerations for designing instruction to support useful beliefs.

The relevance of educational research for instruction[1]

First, it is important to emphasize that this work should *not* be understood as an attempt at definitive identification of properties inherent in students.

At a broad level, I have argued that students can be characterized as having epistemological beliefs, and that this perspective can be useful in understanding their performance in a physics course; more specifically, I have presented a particular framework for constructing these characterizations. As the researcher, I made a number of choices at both levels: to separate out beliefs as distinct from other aspects of cognition, to attribute beliefs to individual students rather than to the students and setting as a whole, to focus on beliefs about knowledge rather than about reasoning, and so on. To the extent that the framework represents such choices, it constitutes an analytic tool, a conceptual lens through which student reasoning might be examined.

This point has direct practical significance. It is customary to think of research as providing a *reliable basis* for instructional design. When results are accepted as "valid," they are taken as accurate accounts of how students reason.[2] In this way, they can come to *supplant* a

[1] See also the discussion in Appendix C.
[2] The other side of this view of validity is that conflicting results are often seen as reason for rejecting one account or the other. Constructivist arguments, for example, have led many to reject behaviorism outright. The claim here is that both accounts, although they are inconsistent, may be useful as alternative perspectives. Neither has been so demonstrably successful to justify disregarding the other entirely.

teacher's independent judgement. So, for example, a teacher may take various research programs as having demonstrated that young children are only capable of "concrete" reasoning, that students learn better in groups, that tangible rewards are essential, or that lectures are ineffective. I believe this represents a narrow view of the potential relevance of research for instruction, and it can have significantly deleterious effects.

I have tried to stress that what I have presented is a possible *perspective* on student reasoning, one a teacher may sometimes choose to adopt and find useful. It should serve as only one of many conceptual tools, to *support* the teacher's judgement and not to replace it. Not all perspectives are worthwhile: what I have shown in this dissertation is that the perspective of beliefs can be useful.

This, I assert, should be a central purpose of education research: to provide conceptual tools for teachers, as opposed to stable foundations. In this regard it is important that teachers understand the status of the results as such, so they do not make the mistake of suspending their judgement, having misinterpreted results as definitive.

Informing local decision making

Instructors are always generating informal models of their students. For most, this takes place automatically; often the models are extremely sparse. A model may be no more than an expectation, based on no data, that students are familiar with the body of knowledge presented in the previous lecture. It may be based on a student's inability to answer a question, consisting of the conclusion that the student does not understand the content. Or it may be quite specific, based on an extended dialogue, such as an idea that a student does not remember a particular fact or has some misleading notions from everyday experience.

These models inform intervention. Much of a teacher's ability to respond effectively depends on the ability to develop adequate informal models. In developing these models, however, most instructors seem to focus on specific content knowledge: whether students understand the material, are aware of particular facts, have intuitive misconceptions that need to be addressed. At least partly for this reason, the interventions tend to focus on the level of content as well. Thus teachers often handle student difficulties through careful explanation, through a convincing demonstration, or by assigning supplementary readings and problems.

The perspective of beliefs may provide additional insight to support teachers' informal models. Part of the difficulty for students may be understood as arising from inappropriate beliefs about what understanding in physics means, and how it is developed. Thus a teacher may understand a student's behavior as possibly reflecting counter-productive beliefs, rather than a lack of content knowledge, and choose an intervention accordingly.

For example, in some situations, rather than explain the content it may be more appropriate to suggest to students that "This is something you already know how to do," or to ask "What would you say if you had never taken physics?" A number of studies (Brown and Campione, 1984; Brown and Kane, 1988) have shown that similarly general suggestions lead to substantial improvement in the problem solving ability of very young children.

On the other hand, it may often be appropriate to present information. From a constructivist perspective, lectures may never seem warranted, because they put students in a passive role. If, however, a teacher has reason to expect the students know to engage themselves in a process of construction, a lecture or a reading assignment may be a good choice.

Informing instructional design

More broadly, consideration of beliefs can provide insight for instructional design, helping teachers and developers to identify aspects of a course that may unintentionally support certain beliefs and to develop methods to encourage others.

<u>Issues regarding conventional instruction</u>

The style of instruction in the physics course I observed may have contributed to or at least reinforced some counter-productive beliefs. Comments from subjects describing their perceptions of the professor's intention supported this conjecture:

> Daniel: I didn't think about it... but again, that's not what they want...
>
> Roger: I don't fully understand what they're asking. And so, I just assume that that's what they wanted me to say.

There was a strong emphasis in lectures, readings, and problems on algebraic manipulations. The textbook, in fact, offered the following "advice to students" (Ohanian, p. 39):

> Most of the problems in this and the following chapters are applications of the formulas derived in the text. If you find it difficult to decide what formula to use, begin by looking at formulas that are valid under the given physical conditions of the problem and make a list of known and unknown quantities. Then try to spot a formula that expresses the unknowns in terms of the known quantities....
>
> When you have finished your calculation, always check whether your answer is plausible. For instance, if your calculation yields the result that a diver jumping off a cliff hits the water at 3000 km/h, then somebody has made a mistake somewhere!

For some students, the practice described would comprise a useful tool to apply in difficult situations; for others, it could support an idea that physics knowledge consists entirely of formal expressions.

Moreover, as in most physics courses, the flow of information was almost exclusively from the professor and text to the students. The students applied the theoretical concepts of the course, but they were not involved in or even, for the most part, witnesses to the formation of those concepts. There was no extended, deliberate attempt to involve the students' prior knowledge. The laws were simply provided and explained, and students were to become familiar with them through practice in solving problems.

In laboratory as well, students were involved in verification more than experimentation. The teaching assistants I observed introduced assignments by describing procedures and reviewing a large number of equations:

> Jill: ...they show [us the] proof, and basically they want us to try it ourselves and plug in [to the] formula to see the way it works.

The professor did try to have students participate in lectures by asking multiple-choice qualitative questions and polling the class for answers. However, once students had responded, by raising their hands, he simply told them which answer was correct and why. He did not, for example, engage the students in a process of trying to determine the answer or spend time trying to discover why other answers were wrong.

For some students, the information provided in lecture could serve to challenge their current understanding and lead them to modify it. By my judgement, the lectures were very well organized, clear, and correct. For those students with productive beliefs about how to use the information, they were quite effective. Both Ken and Tony were very happy with the lectures and felt they learned a great deal in them. For other students, the style of instruction might well have supported a view of physics as a collection of correct answers and a view of learning as receiving information provided by authority. Daniel, Evan, Jill, and Roger all described feeling that they understood lectures, but, they said, this did not help them solve problems.

Alternative methods

Certainly the question of how to improve on conventional pedagogy depends critically on what are the goals of physics instruction. Training students to be able to solve a class of problems reliably should be very different from teaching them to reason independently. As noted in Chapter 6, students in Group I were sometimes at an advantage in solving problems correctly because they applied the formulas without question. One approach to instruction might be to improve upon the procedures students are to follow, taking care to ensure that the rules are clear, complete, and coherent.

Labudde, Reif, and Quinn (1988) taught students a procedural definition of the concept of acceleration, one that directly reflected the conceptual content: find the velocity of the particle at the time of interest and at a short time later; find the difference by vector subtraction, and divide by the time interval. For a sufficiently small interval, the result is the acceleration. Labudde *et al* found this to be successful at helping students develop a coherent understanding, substantially improving performances on qualitative problems. Students who had studied acceleration in an introductory course had answered an average of only 40% of the questions on a pre-test correctly. After only two half-hour sessions of instruction in the procedure, they were able to answer correctly an average of 95% of post-test questions.

The approach was extremely effective at enabling students to solve a class of physics problems. It may, however, support views of learning as receiving information supplied by authority, and it would probably not challenge beliefs that understanding physics means knowing correct procedures. The conceptual associations students form may not be integrated with their intuition otherwise, except as anticipated explicitly in the instruction. Providing such procedures may be counter-productive for the purposes of developing an integrated understanding of physics and enabling students to reason independently in a broad range of situations.

Similar concerns apply to approaches based on qualitative explanations or models. Hewitt (1985) is a textbook that offers clear, compelling intuitive explanations. Arons (1990), a resource book for physics instructors, includes many qualitative models for topics throughout the introductory curriculum, based largely on results from research in physics cognition. A number of research programs have focused on using computers to provide students with conceptual models or representations (Frederikson, Spoehr, and White, 1990; Pea, Sipusic, Allen, and Reiner, 1990; Roschelle, 1990).

Compelling explanations and representations can be very effective at engaging useful components of students' intuitive knowledge. The risk is that, if students interpret these intuitive associations as *apparent*, they may decide they understand without examining and modifying their other intuitive knowledge. In other words, an explanation or representation might be so transparent or convincing that some students would not pay serious attention to their conflicting intuitions. In addition to useful conceptualizations, students need to be able to resolve contradictions with equally compelling alternative conceptualizations.

Consideration of beliefs thus points to the importance of involving students in developing and critiquing models, representations, and procedures for themselves, as has often been suggested from constructivist perspectives (eg Karplus, 1979; Strike and Posner, 1985; Thornton, 1987). diSessa, Hammer, Sherin and Kolpakowski (in press) described a class in which sixth grade students designed their own abstract representations of motion. Minstrell (1989) described a discussion in which student ideas play a substantive role in the development of theoretical ideas.

Schoenfeld (1985) designed an approach for teaching mathematical problem solving based largely on his studies of students' beliefs. It addresses beliefs explicitly, through class discussions, and implicitly, through the structure of activities. Palincsar and Brown's (1984) method of "reciprocal teaching" was partly intended to promote productive beliefs about what is involved in reading.

Many students have a great deal of difficulty learning physics. To think of student reasoning only at the level of content presents for teachers the daunting responsibility of anticipating and correcting a myriad of conceptual faults. Certainly it would help if students could learn to find and correct some of these faults on their own. The perspective of beliefs offers reasons for optimism: students may have knowledge and abilities they do not use because of what they believe about physics. If instruction were to address their beliefs, students might bring those resources to bear.

Epilogue

I recently came across a striking example of the potential power of intervention at the level of beliefs. To help teach drawing, Edwards (1989) designed exercises specifically to address the counter-productive beliefs she had found many non-artists to hold (although she used popular right/left brain terms). One of the first exercises was to copy by freehand a sketch, presented *upside-down*, of a man in a chair. The instructions were to pay attention only to the way the lines of the sketch met and formed spaces and *not* to what part of the man or chair the lines made up. It was a relatively minor intervention, but for me the result was a *substantial* improvement over my drawing of the man right-side-up.

I had always thought drawing was a matter of reproducing objects (eyes, nose, mouth...). I had never questioned this belief – I was never aware of it. My previous attempts at learning involved practice on simple objects; I never had much success. I thought I just wasn't good at drawing, that I didn't have the dexterity, or the perceptiveness, or some other ability.

Evidently I had abilities I didn't know to use. Edward's book encouraged me to think of drawing as reproducing lines and shapes, rather than as reproducing objects. I found that while I was not very good at drawing faces *as faces*, I was pretty good at copying the shapes of curves and at arranging lines to intersect in the right proportions. I learned that drawing involves seeing objects differently from how I usually see them, and this enabled me to tap into my large store of knowledge about shapes. In reproducing the sketch, I found myself thinking "this line is almost a parabola," and the like.

Of course, the general belief that drawing is a matter of reproducing lines and shapes would probably not have been sufficient. I also needed strategies for using that belief and for avoiding old habits: turn the drawing upside-down; avoid thinking about parts of the man; pay attention to lines and how they intersect. To continue to improve, I'll need to start seeing shapes like parabolas in physical objects, not just in sketches; and I'll need to practice, develop techniques, and add to and refine my repertoire of familiar shapes. With enough experience, I may internalize new beliefs, so that I won't have to keep reminding myself how to think.

Still, with very little instruction, my drawings have already improved. Most important, I'm now in a better position to learn.

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Appendices

Appendix A: Participant selection process

At the beginning of the semester, I chose 20 names from the course roster. The selection was arbitrary except that I allowed no more than two subjects from the same laboratory section, in order to avoid having subjects who would discuss the tasks of the interviews with each other. I drafted a letter to each student on the list stating the purpose of the study, making it clear that participation would in no way affect their grade in the course, and offering $5/hour compensation. I delivered these letters to the students during their first lab meetings, where I also answered questions and, usually, arranged the first meetings. I stopped when 5 students had volunteered, which was after I had delivered 10 letters.

Later in the semester, in order to ensure a diverse pool of subjects, I invited 3 students from the top scores on the first midterm. 2 of these students accepted.

Letters of invitation

David Hammer
Group in Science and
Math Education
4533 Tolman Hall
(date)

Dear (name),

I am looking for some students in Dr. ____'s 7A lecture to spend some time speaking with me individually during the semester. Your name was chosen at random from the course roster, along with about twenty others.

This is entirely voluntary. Participants will be paid $5 per hour. There will be about five meetings over the semester, each lasting an hour, during which I will ask you some questions about what you've learned, what you think of the course, and so on. The meetings can be scheduled at your convenience and will take place in Tolman Hall.

The purpose of these conversations, along with my observations of lectures and the rest of the course, is to help me study what students get out of introductory physics. Hopefully the results, along with information from other courses, will be useful in the evaluation of physics instruction, as well as in the development of future courses. I am a graduate student in the Science and Math Education Group; this work will form part of my doctoral research.

Again, this is by no means required of you, and your participation or non-participation will have no effect whatsoever on your grade in the course. In fact, your name will not be used in any way.

If you are interested, or if you have any questions, please let me know as soon as possible. You can leave a message by phone at 658-3849, or by campus mail at 4533 Tolman Hall.

Thank you,
David Hammer

Dear (name),

I am looking for some students in Dr. ____'s 7A lecture to spend some time speaking with me individually. Your name was chosen because you did well on the first midterm.

This is entirely voluntary. Participants will be paid $5 per hour. There will be at most 3 meetings, each lasting an hour, during which I will ask you some questions about what you've learned, what you think of the course, and so on. The meetings can be scheduled at your convenience and will take place in Tolman Hall.

The purpose of these conversations, along with my observations of lectures and the rest of the course, is to help me study what students get out of introductory physics, why some do well while others do not. I am a graduate student in the Science and Math Education Group; this work will form part of my doctoral research.

Again, this is by no means required of you, and your participation or non-participation will have no effect whatsoever on your grade in the course. In fact, your name will not be used in any way.

If you are interested, or if you have any questions, please let me know as soon as possible. You can leave a message by phone at 658-3849, or by campus mail at 4533 Tolman Hall.

Thank you,

David Hammer

Appendix B: Description of interview tasks

Open and semi-directed discussions

Purpose: to have conversations in which students might bring up relevant issues, to create opportunities for unsolicited comments about their beliefs.

Open discussions

Description: The open discussions were of the course and physics at a very general level. They usually started from questions listed here, but they also began from within other tasks. Several subjects often initiated these discussions.

How's the course going?

How do you like the course / physics / the lectures / the labs / etc?

How was your high school course; how did it compare with this one?

How will you study for the midterm?

What advice would you give about how to approach this course?

Examples of probes: *I don't understand. / So you're saying (repeat or paraphrase last statement)? / Ok, but someone else might say (present another position).*

Semi-directed discussions

Description: The semi-directed discussions involved tasks, closely related to the course, with somewhat more structure than in the conversations above. They were semi-structured (Bernard, 1988) in the sense that, although they set a context for several minutes of activity, the main task could be abandoned for a time as interesting side issues arose. In this way, the main task served as a baseline of activity, from which could arise other useful activities of either greater or lesser specificity.

Lecture: *What went on in lecture today / last time?*

Text: *Could you go through the book, say starting on this page, and explain what you're thinking about as you read, show me what you pay attention to?*

Lab: *What was this week's / last week's lab about?*

Midterm: *Let's go through the midterm. Tell me how you did each question, if you thought it was hard, if you made any mistakes.*

Examples of probes: *Does this stuff make sense? / How do you tell when you understand something? / How did he get that? / Did you read that paragraph or just skim it? / How did your results come out? / Do you think they graded you fairly?*

Discussions of content

Purpose: to provide opportunities for indirect indication of beliefs and to provide evidence of subjects' understanding of the material.

Description: These were discussions specifically concerned with particular pieces of the material, during semi-directed tasks or problem solving, when concepts came up and it seemed worth probing the subject's understanding. They were not entirely a separate category of discussions, as they took place within the other contexts.

What is (concept - eg torque*)?*

Where did (formula) come from?

How do you know (formula)?

Why does it work out like that? (regarding some result, eg that acceleration in uniform circular motion is toward the center)

Example probes: *Could you say that again? / Does that make sense? / If you had to teach this to someone, how would you do it?*

$\underline{v = v_0 + at}$: The most important discussion of specific content involved the formula $v = v_0 + at$. This came up during the solution to the two rocks problem, described below. I asked subjects to explain how they knew the formula and how they would teach it to someone. Based partly on an assumption that the relation should be conceptually accessible to students, I coded as indications of *Formulas* subjects' descriptions of it as something they would not be able to explain or to understand other than as a mathematical equation.

Problem solving

Purpose: As above, to provide evidence of subjects' understanding of the material, as well as to provide indirect indications of beliefs.

Description: There were three sorts of problem solving activities. In the first, subjects explained quantitative problems they had already solved in the course. The idea was to minimize performance anxiety, as well as to find the students' perspectives on what a constituted a complete solution. This overlapped considerably with the midterm review, the main difference being that here I asked students to recreate the solutions, while I initially asked only for descriptions of midterm solutions.

The second type of problem solving was on course problems not previously solved, or on other quantitative problems similar to those in the course. I chose these problems sometimes at random and sometimes for their potential conceptual relevance.

The third type of problem solving involved qualitative problems I presented to evaluate conceptual understanding. I will describe these as a separate category below.

From the first two categories, there were three problems that were particularly useful. I asked these with all subjects:

Airplane (previously solved course problem, Ohanian, 1985, p. 265): *An aircraft passes directly over you with a speed of 900 km/h at an altitude of 10,000 m. What is the angular velocity of the aircraft (relative to you) when directly overhead? Three minutes later?*

Follow-up question in the interview: *Why is there an angular velocity if the plane is travelling in a straight line?*

This question distinguished between subjects who looked for a formula with the right variables and those who had a conceptual understanding. I coded as *Formulas* or *Weak Concepts* subjects who described finding a formula with the right variables and could not answer the probe. Those who showed a conceptual understanding of the relation I coded as *Concepts*. This was based in part on my assumption that the conceptual content of angular velocity, at least to answer the follow-up question, would have been accessible to students who thought to look it.

Two rocks: *Two rocks are thrown with speed v_0 from a cliff of height* h. *One is thrown horizontally and the other straight down. Which one hits the ground first, and which one hits with greater speed?*

This problem, originally from Sternheim and Kane (1986), was from a course assignment in the pilot study (Hammer, 1989). In presenting it, I included a simple sketch. The main value of this problem in the present study was to provide a context for discussion about $v = v_0 + at$: as described above.

Ring: *This is a ring. It has a mass of 5 kg, a radius of 2 m, and it's rotating so that a point on the rim moves at a speed of 3 m/s. I'd like you to find the ring's kinetic energy.*

Follow-up: *Would it be possible to use the relation for linear kinetic energy to solve this problem?*

Earlier in the same interview I had asked subjects, as a semi-directed task, to explain a passage from the book deriving the expression for rotational kinetic energy from that for linear kinetic energy. All subjects described the passage and claimed to have understood it. The theoretical content of that passage was directly applicable to this problem. I coded as an indication of *Pieces* or *Weak Coherence* a subject's failure to cite or apply the theoretical content of the passage.

Misconception probes

Description: The qualitative problems, listed below, involved Newton's Laws, which were covered early in the course. I saved these problems for the later interviews, in order to avoid influencing the students' attitudes by the introduction of a type of problem different from those they generally encountered in the course.

In all cases, I coded *Misconception* or *No misconception* based on extended discussion and not on the subject's immediate response.

Merry-go-round: *This is a merry-go-round, rotating this way, and these are two people playing catch. There are two parts to this question: first, how should this guy throw the ball to get it to the other guy; and second, if we were watching all of this from a bridge that isn't rotating, what path would we see the ball follow?*

Follow-up: *What if this guy wants to catch the ball himself, how should he throw it then? Suppose he catches it over here (indicate point about 45 degrees away).*

This problem was intended to reveal misconceptions involving curvilinear motion. I coded *Misconception* if the subject was not able to decide whether the ball's trajectory would be in a straight line as seen from the bridge.

Pen toss: *If I throw a pen in the air, like this, what are the forces on it just after it leaves my hand, but while it is still moving up?*

Follow-up: *Explain what happens with each of those forces as the ball rises and then falls.*

This problem was intended to reveal the misconception that there is a force in the direction of the ball's motion.

Juggler on bridge: *This is a bridge, and this is a juggler with three balls. The juggler weights 98 pounds, and each of the balls weighs one pound. But the bridge can only support 100 pounds, so if the juggler carries the balls across, the weight would be 101 pounds, and the bridge would collapse. What if he juggles as he crosses, so that two balls are always in the air. Will he make it?*

This problem was intended to reveal misconceptions regarding Newton's Third Law. I coded the response as a misconception if the subject's explanation described a ball being thrown or caught as exerting no more force on the juggler's hand than its weight.

Appendix C:

Some thoughts about theoretical frameworks in qualitative research

The purpose of this appendix is to address some issues regarding the nature and meaning of theoretical frameworks in qualitative education research. To me, with a background mainly in physics, the subjectivity of qualitative research is striking. To what extent is it merely the perspective of a researcher, and to what extent does it reflect aspects of student reasoning? What kind of validity should be attributed to the results, and how should they be applied?

In any discussion, whether an interview or an everyday conversation, all participants interpret each other's thoughts, as expressed or implied. What distinguishes the perspective of the researcher as not merely the interpretation of any participant is, for one, the repeated examination of the conversation through taping and transcribing, and, for another, the involvement in that process of a theoretical framework of analysis (Cicourel, 1964, p 51). The latter is in part specified in advance and in part developed through iterative application to the data.

Different accounts place different emphases: Cicourel (1964) stressed the importance of specifying the framework in advance; Marton (1986) argued for the value of *not* specifying the framework in advance so as to allow the unprejudiced *discovery* of categories in the data. In either case, the process is subjective throughout, meaning it involves the judgement of the researcher. For this study, I made a number of choices in advance: I chose to study individuals[1] and to look for beliefs that might affect how students apply their intuitive knowledge. I thought of the beliefs in terms of dimensions of variation, rather than, for example, in terms of a developmental sequence, as did Säljö (1982), or as a set of loosely connected primitive, as diSessa (1985) suggested. I explicitly specified in advance that the descriptions of beliefs should be consistent across physics content and interviewing contexts.

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[1] The assumption that reasoning can be attributed to an individual has been challenged forcefully (eg Lave, 1988).

On the other hand, there was much in the development of the framework I did not specify in advance, as I described in Chapter 4: the dimensions of the framework underwent significant modification in the course of applying them to the data, in pilot studies and in the present work. These modifications were guided both by the choices I made in advance, and by my perceptions of the suitability of the framework in accounting for the data.

In the end, I arrived at a set of categories I claim others will be able to recognize in the data as well, as partially verified by a second coder and as the reader can judge from the excerpts and transcripts provided. This is not to say that others would have discovered the same categories on their own: there is no reason to expect another researcher to make the same choices, nor to "discover" the same aspects of the data:

> Would other researchers find the same conceptions or categories if they were doing the study for the first time?... The original finding of the categories of description is a form of discovery, and discoveries do not have to be replicable.
>
> [However,] once the categories have been found, it must be possible to reach a high degree of intersubjective agreement concerning their presence or absence if other researchers are to be able to use them. Structurally, the distinction I draw here is similar to that between inventing an experiment and carrying it out. Nobody would require different researchers independently to invent the same experiment. Once it has been invented, however, it should be carried out with similar results even in different places by different researchers. [Marton, 1986, p. 35]

To the extent that a framework represents choice or invention, it is in a sense a analytic tool. In other words it is only one of many possible perspectives by which to interpret the data. That the number of possible perspectives is large reflects both the subjective nature of interpretation and the complexity of the phenomena involved. To the extent that the framework can be shown to be useful for understanding student reasoning, and that the categories can be recognized and used consistently by other researchers, it constitutes in itself an essential result of the research.

To summarize: 1) the process involves the judgement of the researcher, both in advance and during development; 2) one would not expect different researchers independently to develop the same framework; but 3) in the end, other researchers should be able to recognize and apply the categories of the framework. In these respects, in fact, the development of the framework is quite similar to the development of a measurement technique in the natural sciences. To elaborate on Marton's comparison, I suggest an analogy in physics research.

To study light, for example, a physicist would make a number of choices in advance, eg: to think of light as existing in itself,[1] to think of it as made up of particles, to measure how the positions of these particles are affected by various possible influences. These choices would affect the kinds of information the research could discover, as well as the meaning one would attribute to the results.

As well, a technique would develop as a result of early applications to the data. The study might begin looking for the effects on the path of light on the proximity of various substances. In the course of analyzing the results, the study might find that the kind of substance did not seem to have much effect, but that the shape of the substance did.[2] This would lead to modifications in the measurement process as well as in the researcher's conceptualization of light.

Because other researchers might make different choices, they might develop different techniques, associated with different conceptualizations. For example, if they were to use transparent objects, they might find that the kind of substance does affect the light's path, and refine their ideas and methods accordingly.

Finally, once the technique is described, it must be possible for other researchers to obtain similar results in order for the measurement to be considered useful. If a technique can be shown to be useful, then it in itself constitutes an essential result of the research.

---

[1] It is possible to assume otherwise. Wheeler and Feynman (1949) chose to think of light as simply a way to describe the interaction of two particles at a distance. Instead of seeing light as emitted, travelling, and then received, they took the light to be emitted and received as a single process between two specific particles. This perspective allowed them to solve what is a problem from the conventional perspective.

[2] I am thinking of a situation in which, to study the effects of substances on light, a researcher shoots a narrow beam of light very close to various objects. If the objects are opaque and non-reflective, only diffraction around sharp edges or narrow openings would affect the path.

It is important to note, however, that the fact that a measurement of the position of light can give useful results does not mean that position is a property inherent in light.[1] Similarly in qualitative research, the fact that a framework can be used to describe and account for student reasoning does not mean that the categories of the framework represent properties inherent in students. They represent a perspective by which researchers (and hopefully teachers) can understand student reasoning.

Thus there are several respects in which a theoretical framework, such as the one I developed for this dissertation, can be compared to a measurement technique in physics. There are also two essential differences:

1) One cannot expect the same kind of "objectivity" in qualitative research as in physics, because in the former the construction and application of the framework involve the same kinds of reasoning the framework is meant to describe. Even to converse meaningfully with subjects in interviews, the researcher must share to a large degree aspects of the subjects' knowledge. Thus the object of study shapes the framework both from the outside, as data, and from the inside, as a necessary part of how the researcher interprets that data (Cicourel, 1964).

A measurement technique in physics, once developed, is objective in the sense that it can be made entirely explicit – in fact it can usually be automated. Given a particular apparatus, anyone can use it in exactly the same way to give exactly the same results, although certainly all may not agree on the meaning of those results. The application of a qualitative framework, however, cannot be made entirely explicit, because it involves much of the same knowledge it is meant to describe. That is, in qualitative research, *human judgement is a necessary component of the "measurement" process*.

One implication of this is that "intersubjective agreement" (Marton, 1986, quoted above) may depend on culturally specific knowledge. It is easily conceivable that a researcher from

[1] In fact, we know position is not an inherent property of light: we can also measure momentum, but our models do not allow us to think of a "piece" of light with position and momentum both precisely defined at the same time.

another culture might not recognize the categories, even in the same set of data. This is a second reason not to attribute the properties of even a successful framework to subjects' reasoning, but only to the interaction between subjects' reasoning and researchers' understanding.

2) In physics, there are many areas that are well enough understood to allow the specification of a "complete" set of measurements. In other words, the number of useful perspectives has been distilled down to only a few, and these have been applied to a very wide range of phenomena. Such is the case with light. This is not the case in education research. There are certainly a number of useful perspectives, from constructivist models of development that have been successful at explaining and predicting children's performance in various kinds of tasks (Piaget, 1977), to models of human behavior as conditioned response (Skinner, 1976), which have been successful in a different class of circumstances. As yet, however, there is little coherence among these models: cognitive science in this sense is very young.

Analytic frameworks have often been misused as attempts at definitive classification of students' abilities. While it is often possible to base designs of physical devices on the results of a given set of measurements, it is not reasonable at this point to do the same for instruction. As I have stressed in the dissertation, the framework developed in this study should be interpreted as only one of a variety of possible perspectives a researcher or a teacher might adopt in trying to understand student reasoning.

Appendix D: Coding justifications

This appendix provides lists of the types of indications in each category and of the types of reconciliations. The lists should not be considered exhaustive or definitive: I compiled them *ex post facto*, from coding and reconciliation justifications used in the analyses.

	Beliefs about structure	
Pieces	*Weak Coherence*	*Coherence*
	Beliefs about content	
Formulas	*Apparent Concepts* and/or *Weak Concepts*	*Concepts*
	Beliefs about learning	
By Authority		*Independent*

Summary of final framework

Initial codings

Pieces: Physics is thought of as a collection of separate pieces. There is no expectation of coherence, and no need to look for it. To know something is to remember it; one either knows a piece or does not.

P-1: A statement that one would not be able to proceed without a particular fact, formula, or procedure, ie that if the piece of knowledge is not available, there is nothing one can do. Eg: 'I couldn't solve that problem because I didn't know the formula.'

P-2: A statement implying that knowing physics means remembering information. Eg: 'I can usually remember how to solve problems if I've solved them before.'

P-3: In solving a problem, shopping for solutions until one arrives at what one believes to be the correct answer, without concern for why previous solution attempts failed.

P-4: Recognizing a difference between two clearly related situations without trying to understand why there is a difference. Eg: noticing that frequency does not depend on mass for a pendulum but does for a spring, without concern for explaining the difference.

P-5: The failure to consider a connection between two pieces of knowledge when (by the coder's judgement), the connection would be accessible if one thinks to look for it. Eg: The subject tries $v = v_0 + 2at$, which leads to difficult algebra, then tries $a = (v_0 - v)/2$ as "another way" without considering how it relates to the first.

Weak Coherence: Students with a mainly *By Authority* view of learning may think there is coherent content to physics, but that it is the responsibility of experts. It is in principle knowledge that exists, but it is not accessible or essential for a student taking introductory physics, except to the extent it is explicitly "covered" in the course.

WCOH-1: A statement to the effect that it is not important to be familiar with derivations. Eg: 'I never pay attention to the derivations.'

WCOH-2: A statement describing a contradiction as discomforting but not essential to resolve, without an attempt to reconcile the contradiction. Eg: 'Yeah, that kind of bothered me, but I figured I probably got it right.'

Coherence: Physics is expected to make up one coherent system. There is a need to perceive that coherence, to resolve apparent conflicts and to find key ideas from which others derive. If some part of one's knowledge is missing, one expects it may be possible to deduce.

COH-1: A statement describing or evidently taking as implicit that proofs / derivations are part of one's understanding. Eg: 'That proof took me a long time to figure out.'

COH-2: A spontaneous effort to reconcile or to explain perceived inconsistencies between different situations. Eg: on noticing the difference in the dependence of frequency on mass between a pendulum and a spring, trying to figure out the reason for the difference without prompting.

COH-3: A statement implying that inconsistencies can or should be resolved. Eg: 'I really need to find out why those two are different.'

COH-4: Statements indicating an expectation that one is able to rederive or to reconstruct content. Eg: 'I can usually rederive equations on an exam if I forget them.'

COH-5: Evidence of coherence in subject's understanding , as part of explanation of a problem solution or of specific content. Eg: The subject shows a coherent understanding of how a plane flying in a straight line can have an angular velocity.

Formulas: Physics knowledge is thought to consist of symbols and rules for manipulating them. An entailment of this position is that one solves problems through purely symbolic methods, by finding the appropriate formulas and manipulating them algebraically.

F-1: Reference to a particular formula as something to be understood or remembered purely by its symbolic content. Eg: '$d = \frac{1}{2}at^2$ is easy to remember, because of the two 2's.'

F-2: The use of formulas as symbolic expressions without reference to conceptual content, in cases where (by the coder's judgement) conceptual content should be accessible. Eg: 'I knew r and v, and the problem asked for ω, so I used $v = r\omega$.' (Exception: where the subject spontaneously and explicitly distinguishes this use from "physics.")

F-3: Inability to remember, explain, or decide between alterative versions of a formula that (by the coder's judgement) should have accessible conceptual content. Eg: The subject cannot decide whether $x = x_0 + v_0 + \frac{1}{2}at^2$ or $x = x_0 + v_0t + \frac{1}{2}at^2$ is correct.

F-4: A statement explicitly distinguishing physics content from common sense / intuition, either as a general rule or in specific situations. Eg: 'You couldn't explain $v = r\omega$ intuitively.'

F-5: Description of physics content as equations or of the process of problem solving as the manipulation of equations. Eg: 'I should do well, because I know all the formulas.'

Apparent Concepts: Physics knowledge is thought to be made up of symbols and formulas loosely associated with conceptual content. One thinks of the conceptual content as *Pieces*: a conceptual association is either right or wrong, and one either knows it or does not. There is not a general expectation that physics is conceptual or that such understanding can be developed: one makes these associations when they are apparent, or, as Daniel put it, "convenient." They serve to help one remember facts or to recognize the validity of formulas and calculations.

AC-1: A statement describing common sense as ancillary and not essential to physics understanding, as useful for remembering physics rather than as physics itself. Eg: '$d = vt$ is something I could figure out from driving, so it's easy to memorize.'

AC-2: A statement describing common sense as having limited application, applying only in familiar circumstances or only to results. Eg: 'I guess I use common sense sometimes to check results, or on things I have a lot of experience with.'

AC-3: Failure to try to explain recognized contradictions with common sense, in the course of problem solving or explaining content. Eg: A subject says that centripetal force is inward, notes that this does not agree with common sense, but says simply that the common sense is wrong without trying to resolve the contradiction.

AC-4: Accepting that a formula or solution technique works if it gives acceptable results, without an examination of the solution process itself to decide why it would work. Eg: 'I think that formula works here because the answer seems reasonable.'

AC-5: In explaining content, the lack of conceptual understanding in a case where 1) (by the coder's judgement) such an understanding should be accessible and 2) limited use or acknowledgement of conceptual understanding is also evident. Eg: In answer to a question of how a plane flying in a straight line overhead has an angular velocity relative to an observer on the ground, the subject explains that the plane is actually moving in a circle around the Earth.

AC-6: A use of superficial and inappropriate similarities to associate different aspects of one's understanding. Eg: The subject explains the proportionality of a spring force with distance as 'just like' the proportionality of an arrow's kinetic energy with the distance it penetrates wood.

Weak Concepts: Students with a mainly *By Authority* view of learning may think there is conceptual content to physics, but that it is the responsibility of experts. It is in principle knowledge that exists, but it is not accessible or essential for a student taking introductory physics, except to the extent it is explicitly 'covered' in the course.

WCON-1: A statement to the effect that 'deeper' / intuitive/ conceptual understanding is not essential or accessible. Eg: 'I guess if I had a lot of experience with physics, these things might be intuitive for me.'

Concepts: Physics knowledge is made up of concepts, often represented by symbols and formulas. One expects problem solving to be guided by conceptualization and measures understanding by one's ability to explain in qualitative terms.[1]

CON-1: A statement describing specific formulas or physics knowledge in general as common sense / intuition / experience. Eg: 'Physics is mostly just putting common sense into equations.'

CON-2: The use of conceptual argumentation in problem solving or in explaining formulas. Eg: A subject explains angular velocity by waving an arm to show that the hand moves with greater speed than the elbow, but they have the same angular velocity.

CON-3: A statement indicating an expectation of mechanistic / intuitive / conceptual understanding in regard to specific content or a specific situation. Eg: 'I'm pretty sure that's the right answer, I just can't understand how it actually works out that way.'

CON-4: In problem solving, looking for a physical mechanism by which to understand the situation. Eg: A subject says that a moving cart draining water will slow down, then looks to figure out what actually pushes on the cart to make it slow down.

By Authority: One expects physics knowledge to come from authority. Teaching is telling, and learning is a process of remembering what one has been taught. This position might be considered a naive 'tabula rasa,' but it is also a kind of abdication of responsibility for learning to the instructor or text.

BA-1: A statement, in the course of problem solving or explaining content, that one cannot proceed without consulting with an authority or because one cannot remember what one had been told. Eg: 'I don't know, I'd have to look in the book.'

BA-2: An appeal to authority as a principle and valid justification for a fact, formula, or procedure. Eg: 'Yeah, I'm happy with my solution, because that's how they did it.'

BA-3: Resolution of some uncertainty by factual verification with authority, without further explanation or consideration. Eg: A subject checks the book, verifies the information, and reports satisfaction.

BA-4: Statements implying that the professor or text can provide understanding, such as through clear explanations. Eg: 'I have trouble solving the problems, because he doesn't explain them very well.'

Independent: One thinks of learning as a process of applying and modifying one's own judgement. It is necessary to make sense of or to recreate the ideas for oneself. This position

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[1] The label may be misleading: by "concepts" I mean informal knowledge, to include intuition (evolved from experience) and conceptual knowledge (based on a qualitative sense of principles or structure). (Recall the discussion at the beginning of Chapter 2.) These could, perhaps, play different roles in students' beliefs. Several subjects, in fact, showed an articulate sense of different kinds of informal knowledge. For this paper, however, I treat conceptual and intuitive knowledge as a single category.

might be described as a naive constructivism, but it is also an assumption of responsibility for one's own understanding.

I-1: A statement of an expectation that physics can be made to "make sense." Eg: 'Usually I can figure it out and it just makes sense.'

I-2: A spontaneous effort to explain or make sense of a surprising result or confusing content. Eg: A subject looks for a way to explain the surprising result that a rock thrown horizontally and one thrown straight down with the same initial speed hit the ground with the same final speed.

I-3: An extended, self-motivated effort to explain or make sense of a surprising result or confusing content, whether the initial recognition of surprise or confusion was spontaneous or resulted from interviewer questions. Eg: A subject spends a great deal of time trying to understand an example from lecture prompted only by a request for an explanation from the interviewer.

I-4: A statement to the effect that common sense / intuition is modified (as opposed to irrelevant or discarded) in the course of learning physics. Eg: 'Your common sense gets modified by the course.'

I-5: A statement of expectation that one needs to work through difficulties oneself in order to learn. Eg: 'The trouble most students have is that they get spoon-fed the solutions to problems without working them out for themselves.'

I-6: Contradiction of an authority, in problem solving or in discussing content, thus displaying a reliance on one's own sense-making. Eg: A subject describes the grader of an exam as not having understood his explanation properly.

I-7: Evident inventiveness - the use of methods not discussed in the course, when 1) the methods are correct and conceptually sound, and 2) the subject is able to explain them convincingly. Eg: A subject solves a problem in an accelerated reference frame, when that method was never discussed in the course.

Reconciliations

Consistent with *Weak Coherence*:

REC-1: Indications of *Coherence* (COH-1) based on accounts of attention to derivations in the text or lectures. For example, in describing a lecture a subject might focus on the derivation of a formula, which as an isolated incident would indicate a concern for derivations. However, a subject may focus on derivation in the interest of accurately recounting the lecture rather than because of a sense that derivations are important.

REC-2: Indications of *Coherence* (COH-1) based on a reluctance to use unfamiliar formulas, when elsewhere the subject described similar reluctance for lack of faith in the formulas as unfamiliar tools rather than for concern for understanding their derivations. A subject might be reluctant to use an unfamiliar formula because he does not know how the formula was derived or how it fits with his other knowledge. Alternatively, he might be reluctant to use it simply because he had not used it before.

Consistent with *Coherence*:

REC-3: Indications of *Pieces* (P-1, P-2) based on unelaborated description of knowledge as made up of "facts" or of "remembering" information. A subject might note in describing work on an exam that she did not "remember" a formula, so she could not solve a problem. If later, for example, she were to say that with more time she could have derived the formula, this would support a reconciliation of the indication.

REC-4: Indications of *Pieces* (P-5) based on separation of ideas that should be linked, when the subject subsequently elaborates on the separation and tries to account for it. The subject's notion that the ideas are separate, even if incorrect, might be problemmatic or the result of some reflection rather than the result of a general expectation.

Consistent with *Apparent Concepts*:

REC-5: Indications of *Concepts* (CON-2) based on use of conceptual argumentation in cases where the association with conceptual knowledge might be superficial, even if it is correct. For example, a subject might argue that objects move in circles because they are pulled in toward the center. Although the explanation is "correct" and conceptual, it may be only superficially held, such that the subject could not elaborate or support the explanation in response to challenges.

REC-6: Indications of *Concepts* (CON-2) based on use of conceptual argumentation that was presented in lecture. This is essentially the same as REC-5, with the interpretation of the conceptual understanding as possibly superficial specifically supported by a correspondance to a presentation in lecture.

REC-7: Indications of *Concepts* (CON-2) based on statements of involvement of conceptual knowledge in physics that, although not indicating limitation to that involvement, do not describe an involvement beyond that consistent with *Apparent Concepts*. For example, a subject might say that he uses intuition often in the course but later elaborate this statement to describe a use of intuition restricted to checking results against plausible expectations.

Consistent with *Concepts*:

REC-8: Indications of *Formulas* (F-1, F-2) based on application of formulas independent of their conceptual content when the use of the formulas is 1) correct and 2) routine. That a subject uses the formalism could certainly be consistent with a belief that the formalism only represents underlying conceptual knowledge. In standard cases where the subject uses the formalism correctly, she might not articulate the conceptual content simply because she takes it as understood or unproblemmatic.

REC-9: Indications of *Formulas* (F-1, F-5) based on casual, unelaborated reference to physics content as "formulas" or "equations," when elsewhere the subject evidently used those words to refer to the conceptual content along with the literal, symbolic expression.

REC-10: Indications of *Formulas* (F-4) or *Apparent Concepts* (AC-4) based on description of specific content as not common sense or intuitive when, 1) the subject described the content as an exception in this regard and 2) by the coder's judgement, the conceptual content should be difficult to grasp.

Consistent with *By Authority*:

REC-11: Indications of *Independent* (I-5) based on a statement to the effect that students need to do more in lecture than just take notes, without elaboration, ie when the statement could be interpreted as expressing only a concern for accuracy in receiving information. For example, a student may describe it as a problem if he goes into lecture and just copies everything down. Alone this would indicate *Independent*, but it might also be interpreted as expressing a sense that one needs to pay attention in order that the information can "sink in," rather than an idea that one needs to "make sense" of the information.

Consistent with *Independent*:

REC-12: Indications of *By Authority* (BA-4) based on casual, unelaborated description of lectures as providing information. That the subject describes a lecture as providing information may not necessarily imply she thinks that receiving the information consistitutes learning.

Appendix E: Transcripts and analyses for two subjects

This appendix provides the "complete" transcripts and analyses for Tony and Daniel. I have chosen these two subjects because of their contrasting views and their articulate descriptions of their beliefs. As will be clear, I have left some material that was not involved in the characterization of beliefs out of the transcripts. In each case, I have noted the nature of the material omitted.

Short comments in parentheses, eg (yeah), denote minor interjections, by myself when the subject is speaking and by the subject when I am speaking.

Subject: Tony

<u>First interview - 3/29</u>

I: I will start out just asking you some background questions. Did you take physics in high school.

T: Yeah, we had a notoriously bad teacher, but yeah.

I: So, did you not learn much physics in high school, or

T: Um, kind of everything we, he gave us, were the kinds of things we already knew but had never actually formalized, if that makes any sense.

I: Huh.

T: You know, if we had sat down and thought about I realize ok a ball falls off [unintelligible] stuff, but we never actually sat down and thought out the equations and everything, but it was common sense type stuff. Just kind of putting together thoughts you already knew.

I: And now is different from that, or

T: Actually it's a lot more of the same. I don't know, everything we do in physics seems like, it, simply, you know, it makes, you think about it and that's what should happen and it's just a matter of putting it, putting common sense into equations. *1*

I: Hmm, ok. So that's, so your high school, I'm not sure if you're saying, your high school course was not all that helpful now, or

T: No.

I: Do you remember what book you used?

T: No, if I saw it again I would remember, but, the book itself wasn't that bad, it was the teacher. He was really funny, I love the man, but me and some friends ended up teaching the class more or less.

[Details about background – Tony took physics his senior year. He says he had also taken, in his senior year of high school, a college math course on differential equations that "emphasized physics a lot too, that did help a lot." He never took the Calculus Advanced Placement exam, so he was in a first year calculus in a self-paced course. He had chemistry for

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1 *Concepts*: Tony explicitly describes equations as expressions of common sense.

the first time the previous semester, which he "took a long time to get used to," but he ended up with an A. Discussion about high school scheduling, why Tony had not studied chemistry before college.]

I: Ok, how do you like this course?

T: Physics? Oh, I like it, I like physics, I love physics in general, I just really enjoy it.

I: All right, good to hear. Ok.

T: You'd have to really work to make it, if it was like, if I thought it was busy work and stuff,then it would be really annoying, but I like the sort of problems, are really, you have to sit down and think about them, and just some abstract problems I really like, and stuff like that. It's not just busy work. *1*

I: And how do you like the lectures?

T: Oh, it's fine. I really like the, he does a lot of examples, that I think is very important, I like, he'll sit there and he'll take what I consider to be an interesting problem, a lot of people say oh, it's so boring, but he'll sit there and take what I consider to be a legitimately interesting problem, sit there and work out stuff, so something that you go, oh yeah I've never thought about that before, and he'll go through and figure out all the abstractness to it and everything, I like it. I like the problem oriented lecture.

I: Ok, when you say problem oriented you mean

T: Like, he'd give you the theory and then he'll give you a lot of examples to go with it.

I: Um, and labs?

T: Well, the labs are pretty basic, but they're fine. We had a nice computer lab yesterday, it was computer oriented, ha, ha, ha.

I: Is that good or bad, ha, ha ha?

T: That's good. I like goofing around with computers, but yeah the labs are fine. I mean they're, it's a lot different, they're not that important, so you can kind of go, grade-wise, they're only like 5 points a piece or something, whereas in chemistry the labs were super, you know, super important. Plus these labs were tied in more with the lecture, whereas the chem labs you're always 2 weeks ahead in lab than you were in lecture, so you never have any real clue what's going on in the lab, whereas the physics labs are spaced with lecture pretty well.

I: Ok, and the book.

T: The book's fine, it's good.

I: So you just

T: I like it, I really have no complaints, I enjoy it.

I: Well that's great. Do you by any chance have lecture notes with you? [T find them] so one thing we'll do, maybe not too much, but a couple of times, just to find out what you're getting out of lecture, you're go in and see a lecture, what did you

T: Um, a lot of the thing I get out of lecture, the theory you can read in the book, and in lecture he'll tell you the ways of using them. Like, I can find some example. I know in here, at a certain time he'll like give us a theory, and we just, the rest of the class was all examples, and it was just great. Ok, one point you have, is just that the TAs are just awful.

I: The TAs are awful.

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[1] *Independent*: Tony identifies as interesting the problems "you have to... think about," as opposed to "busy work."

T: They have no clue.

I: Really? Ok, so tell me more about that? Do you go to discussion sections regularly, or do you

[Tony complains about teaching assistants being unprepared and disorganized. He turns the discussion back to what he likes about the lectures.]

T: Ok, here's what I'm thinking of, well, maybe not. The general pattern is he'll give us the [1] equations and what-not, and he'll start off on an example, and he'll give us another example, how to use that, and each example brings in something new, not just doing the same example over and over again. Each example uses something new, a new way to use the theory, or bringing in an older theory. He uses examples very well. The demonstrations are fun too.

I: So were you at lecture last time?

T: Yeah.

I: So let's go to that. Do you read the book before you go to lecture?

T: Um, it depends, on how much time. I usually save my reading for the weekends, I read through to wherever, sometimes yes, sometimes no.

I: So you have these lectures notes, do you go over them regularly or do you take them then, or

L: Um, I usually, I take them, I [unintelligible] usually go over them, except right before a test, I'll go through and I'll remind myself of the stuff.

I: You usually don't go over them

T: Until the test. And usually I don't sit there and study them, I usually go through them [2] once, once I've read through it I kind of remember what he did in class. Like one of the problems on the last midterm, was kind of like one he did in class, only it was reversed. It was ah, the problem was the amount of work done pulling a string onto a table, and in class I remember he had done one pulling off the table, and I had read over my notes beforehand, and I remembered seeing that, and I actually remembered the lecture where he did that, I just went through and reconstructed it.

I: So how did you do it?

T: Oh, um, you had to figure out, at any given time how much string was still off the table, take that, and then the density would be the length divided by the mass, and then you, and then each, I wouldn't say dx, but each ah, each little bit of work you had to do was the amount of length divided by the mass, so it was the density, times how much there actually was there, times gravity, then times how far you're moving, which is the same thing as how much, which was inversely related to how much was left. What it ended up being, was, mgl/8 was the answer, but you had to integrate from, it started out halfway, you just had to integrate from l/2 to l, and then, just totaled up all the little work done. [looks for example in lecture notes] I can't find it now.

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[1] *Formulas*: Tony refers to the content as "equations," in saying "he'll give us the equations." Reconciled: Elsewhere it is clear Tony uses the word "equations" to refer to the conceptual content of the formalism, and not just to the literal symbolic expressions. It is reasonable to assume this is how he is using the term here.

[2] *Uncoded*: This paragraph is worth discussing, because I went through some deliberation before deciding not to code it. Tony's use of the word "remember" might be taken to indicate *Pieces*, but he also says that he "went through and reconstructed it," an indication of *Coherence*. Because of these opposing indications within the same thought, I did not code this excerpt.

I: That's ok. Let's look at, this was yesterday's? (yeah) Um, actually, you know, if you can remember to the week before, I don't know if that's going to be too hazy for you or not.

T: I'm sure I can. [unintelligible] Ok, so yeah, so it was, it started with rotational motion.

I: Ok, so let's start with this. What I'd like you to do is just go through the lecture and tell me what you got out of it, what didn't make sense, what made sense

T: Well, the um, the rotational motion what he's doing is he's showing us a lot of the *1*
analogues between translational and rotational motion, and I'm looking at my notes and I see that I screwed up this table but that's ok. That's was most definitely me not him. It was basically, he gave us the basic equations and derived some other ones that were direct analogues for the ones that we got with translational motion. And then right away, after he'd given us the analogues, he goes ok, well let's give us a problem right away here. The problem was, a flywheel that starts off at 1.5 radians per second turning, and it takes, let's see, and if it takes 40 revolutions to stop, find how long it takes to stop, and what the deceleration was. So basically it was just utilizing these formulas here and we went through and showed us just how to use the formulas we had just given, in this application. Basically I like that that, because it just gives you, that's how it usually goes, he'll do something new and then show you right away how to use it, that's what I really like about his lecture style. And then he went on to, what's this, ok then he's saying, ok now suppose acceleration isn't a constant, suppose it's a function of time, and then he went one and he showed us how to figure out, he showed us, basically more analogues, that the acceleration is just the derivative of the angular velocity over time, and then that's the derivative of actually position and so on. And then ah, then he showed us, ok, something that's rotating, and it's accelerating, then there's two components to the acceleration, one has to be centripetal to keep it going in a circle, and one has to be tangential to make it actually be going faster, and he was just showing us what that would mean, and the centripetal and the tangential add up and they end up being a vector that's diagonal to what's going on. I don't know why he showed us that, but I'm sure it will come in handy sometime. He's actually just figuring the total acceleration.

I: So what would happen, say, if ah, if the tangential acceleration were 0, what would that mean?

T: That means it's going in a circle but not getting any faster. It's going around in a circle in the same speed.

I: Ok, and if the centripetal acceleration were 0, what would that mean?

T: It's not going in circle any more. It has to be a centripetal acceleration to keep it going in the circle, um, if it's not there, it's just going to go off in a straight line.

[Tony describes more of the lecture, in particular the derivation of moment of inertia. He mentions a demonstration he found particularly convincing, and I ask him to explain it.]

T: There's a turntable here, and around the post was a loop string that went over under a 2
pulley and then over another pulley here and those had real weight, so as it came down the turntable had to turn. So you put the disk on the table, and let go, and the weight slowly accelerated because this was turning. And then he did the same thing, but then he put the cylinder on it, and you could see that the weight accelerated much more slowly by these theories had to be much larger, so it was harder to get to move again, so it was a very physical demonstration of what we had just been doing mathematically, and you could really tell the difference, even though they had the exact same mass. And basically the rest of this is all

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1 *Formulas*: "he gave us the basic equations"; "basically it was just utilizing those formulas... showed us how to use the formulas..." Reconciliation: elsewhere, Tony uses the work "equations" to refer to the conceptual content.

2 *Concepts*: Tony describes the demonstration as a "very physical demonstration of what we had just been doing mathematically."

moment of inertia, ways of, there's a little inertia hint, here' a little helpful hint when you're trying to find moment of inertia, and basically a couple of shortcuts, [unintelligible] problems. And a little bit at the end here of angular momentum which we got into more later.

I: So what's angular momentum?

T: It's ah, if something is rotating around a point, hmm, it's really hard to explain. If 1
something is rotating around a point, it's distance times it's momentum, well, not times it's momentum but cross with momentum vector, which, if they're directly at right angles with each other it's distance times its momentum it will be constant. So like if you're on a chair spinning with your hands way out and you bring your hands in you'll go faster.

I: Now why is that?

T: The, have you ever seen like when a skater is whirling around like this, (right, ok), because 2
out here, the momentum is directly related to speed, so how fast you're turning is related to your momentum, if you're out here spinning around, and you suddenly, and the angular momentum is the distance out times the speed, times the momentum which is related to the speed. So as you bring, the distance out, as you bring this distance closer, the angular momentum has to stay the same, so as this comes, as this becomes smaller

I: Why does it have to?

T: Because it's just a law of nature, that angular momentum is conserved. (ok) It's ah, like 3
[unintelligible] prove mathematically, but it's really hard to, it's, you can see it mathematically, but it doesn't explain common-sensically why it should do that, it just does. You can take that as a given, angular momentum is conserved, it's just a given.

I: It's a given. Can you explain, what do you mean by that? Well this difference, you can prove it mathe, how would you prove it mathematically?

T: Well, I'm sure he gave the proof right here actually, maybe on the next page [flips pages]. Ok, basically what we did was, we proved that, ok, angular momentum as a general definition doesn't, you have to be rotating around something, you can take a point, and any object anywhere can have an angular momentum with respect to that point. And what we proved was, we went through and proved that the rate that that changes is gonna be, let's see, tch tch tch, ok the rate that that changes, the dl,what is it, dl, yeah, dl dt, l being the angular momentum, turns out to be the same thing as the same point, the same distance, crossed with how much force is on that point. (ok) Now by definition of a cross product, if they're, at right angles, wait, I have to [unintelligible] real quick here. Shoot I just confused myself, how did I do that. The, the amount the angular momentum changes is, r, r cross, ok ok, that's right, um the amount angular momentum changes is the distance out crossed with the amount of force

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1 *Apparent Concepts*: Tony describes angular momentum in purely symbolic terms, including an example regarding a plausible result of conservation of angular momentum, but with no indication of consideration of mechanism. Reconciled: From subsequent conversation, Tony thinks of angular momentum as a particular topic for which he has no common sense explanation, which, he feels, is an unusual circumstance in physics.

2 *Concepts*: Tony uses conceptual argumentation to justify the formula. (This indication would also be consistent with *Apparent Concepts*, however.)

3 *Apparent Concepts*: Tony describes the mathematical proof of conservation of momentum as something that cannot be understood in terms of common sense. Reconciled: Tony later describes this as an exception to the norm. *Coherence*: Tony considers the mathematical derivation to be part of his understanding, and he goes on to give an explanation. It is important to note that, although he looks in his lecture notes for the proof, he does not follow them. Most of the time he tries to regenerate the proof himself. He has a little trouble, but he gives a good summary in the end ("But the important thing is..."): the change in angular momentum is $\mathbf{r} \times \mathbf{F}$, so if $\mathbf{F}$ is directed radially, as when a skater pulls in her hands, $\mathbf{r} \times \mathbf{F} = 0$ and there is no change in angular momentum.

on that point, so if it's rotating around this point here, the, if it's going in a straight line this way, with no force on it, the cross product would be 0, but r would be changing, so there would be, actually, let me take that angular momentum out [unintelligible] cause it would be 0 the whole way cause there is no force on it. But, the important thing is that, the change is r cross F, and the only time that has any kind of significance is if something is circling around a point. Cause if something is circling around a it constantly, the force is directed toward that point, always, that's how it gets it to go around, and anything crossed to something parallel is automatically 0. So since r cross F is 0, always, with something circling around a point, r cross F is always going to be 0, since the amount of, the change in angular momentum is r cross F, the change is going to be 0, so it has to be a constant.

I: But if, now, if I pull in my hands, is it still

T: The force is, you're still applying a force this way, so the r cross F, since the force is parallel to the displacement vector, the actual position

I: Well isn't it moving like this, in a circle?

T: Moving and force are two different things. You're moving in a circle because [unintelligible] whereas if you're actually pulling your hand in you're applying an acceleration. But since r cross F is zero, this force crossed with this displacement vector is 0, the amount of angular momentum changing is 0, which makes angular momentum a constant. Which also, but since r is still changing, r is getting smaller, the momentum has to go up, at this point, which means it has to go faster, that's the only way it can do that, or it could heavier, but that's impossible. *1*

I: So, that was the mathematical.

T: Yeah.

I: You said before you can say, you can prove it mathematically, but it isn't common sense or something.

T: It's very easy to prove mathematically, but it doesn't, did that make sense to you, what I was talking about, the r cross F?

I: Yeah...yeah...

T: It kind of makes sense, it's hard to talk about it in a non-mathematical way. It's hard to say, well the amount angular momentum changes is directly related to the amount of force on the on the point, and if that force is parallel to the actual position then it's going to be 0, and that kind of thing, it's easier to prove mathematically, it's like really high level differential equation or something, you can't

I: Is that pretty much all of physics?

T: Hmm?

I: Is that pretty much the way physics is, or

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[1] *Concepts*, *Coherence*: Tony's ability to distinguish "moving and force" as the answer to my question was evidence of coherence in his understanding.

T: No, not really, not at all. A lot of times you can very easily relate the equations to, angular momentum you can too, you can sit there and you can see that the skater gets faster as she pulls, and you can sit there and you can see that, and I can find equations for that and stuff, it's just the proof that is one exception, the proof of why it's conserved is kind of tough, and that's, it tends to be more of a given in the proof, it's more of a, how to say it, the fact that it's conserved is more a law of nature and you're kind of just confirming it. Actually it's more confirming our own mathematical conventions than it is the law of nature. [1]

I: Huh.

T: In most cases you can usually, like in any kind of translational motion you can sit there and say, oh yeah, it's that because it's slowing down, or whatever. And in most cases you can change it to map to real life.

I: But this is a case where it's tough.

T: Well, here you can, it's just hard to explain unless the person you talk to knows a lot about the math and stuff, involved, otherwise you have to go through and reprove all the math to begin with. Cause like in a total mathematical sense, it really does make sense, but, cause what it comes to is saying that, since it's not changing it's staying the same, that's basically the whole mathematical proof. All the rest of it is proving why it's not changing. It's proving what the change is, and then the change has to be zero, so it's not changing, so it's constant. [2]

[We move on to Problem 35, Chapter 6 (Ohanian, p. 150). I ask Tony to solve the problem: "*Two masses,* m_1 *= 1.5 kg and* m_2 *= 3.0 kg are connected by a thin string running over a massless pulley. One of the masses hangs from the string; the other mass slides on a 35° ramp with a coefficient of kinetic friction* μ_k *= 0.40. What is the acceleration of the masses?*"]

T: You want this pretty structured? [inaudible]

I: Well, however, tell me how to solve it, that's all.

T: First thing I always do is draw a picture. For me that's mandatory, because it helps you visualize everything. In problems like this, where you're dealing with forces and stuff, free body diagrams help a whole heck of a lot. Um, ok, the first thing, in the free-body diagram is just simply identify all the forces. Let me draw this box there. There's going to be a force up this way, a normal force, ok. [drawing] Now

I: Now is this something that you just came up with doing, or that he instructed in the course, or

T: He, he showed us in the course how to do it. The book stressed it more than he did, but, another class of mine, statics stresses it a whole lot.

[The other class is Mechanical Engineering. Tony flips through that textbook to demonstrate that there are many free-body diagrams in it as well.]

T: Um, ok, and it's a matter of identifying what all the forces are. It's going to be, mg, ok, for this to balance out, this is gonna, the normal is going to be equal to this, the normal is going to equal mg cos theta, [mumble work] I'll check that in a second. That's what I usually do, I usually go through once and then I check everything. I'm notorious for making mistakes the first time through, but I usually [unintelligible] the mistakes I make, so 9 times out of 10 I

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[1] *Concepts*: Tony identifies angular momentum as an exception to the rule, which is that usually you can "change it to map to real life." Moreover, he explains that with angular momentum, "you can see that the skater gets faster as she pulls," that is you can see that the *results* correspond, but the proof is an "exception." In other words, proofs are usually 'common sense' as well. (This indication of *Concepts* would not be consistent with *Apparent Concepts*: Tony distinguishes a result as intuitive from a derivation as intuitive, and he says that this derivation is unusual in being non-intuitive.)

[2] *Coherence*: Tony evidently thinks of the proof as part of his understanding.

catch them. [mumble work, pause] Then ah, ok [pause]. Yeah, ok that's right. Um, the one thing they emphasize in the course is always to go through and do everything abstractly at first and plug in all the numbers at the end, and that's kind of a very good thing to emphasize.

I: And why is that?

T: Because if you go through and plug, well, I 'll show you in the problem here. [mumble-work, "cosine theta, etc] Like if I went through here and started plugging stuff in, and then I went into some other equations, it's a lot easier to make mistakes with numbers than with just moving a letter around someplace. All right, so the total force [mumble-work, pause] *1*

I: You have F = m 2 g minus

T: Ok, since you know it's going to end up moving this way, the total force

I: How do you know it's going to end up moving that way?

T: Actually you don't, but if, I'm assuming it will move this way, and if this turns out to be a negative quantity, then I know it's moving the other way, but there's really almost no chance of that.

I: Now why do you say that, that there's no chance

T: Um, first of all, the problem asks, the way the problem is set up, hmm, it's common sense I guess. First of all, if there's a, mass 2 is heavier, this mass is heavier than this mass, this mass is being pulled straight down, this is only going up a ramp, so gravity has less of an effect on this one than it does on this one to begin with. (ok) If m 2 were much heavier, it would be the other way around, but since, yeah, I like that, m 2 is, and **2**
if were wrong, I mean that's one thing, if this were a negative number then it would just mean my direction is wrong, but the number actually get is still right. And here you just plug everything in. Another little check is, usually when we've done problems more or less a number of times, the other advantage of abstractness is, it looks right. If I had plugged number in back here, I would have, this would be a bunch of numbers, whereas now I plug this, and I've seen this quantity here a number of times in problems of the same sort. So in general that's a good hint, it doesn't mean it's right.

I: So what you've written out is the forces on this object. So this m 2 g is from the cord, minus

T: Ok, this is the frictional force.

I: The frictional force, ok

T: That's mu sub k, frictional force is proportional by this constant to the normal force is this here

I: And you're using minus because

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1 *Formulas*: Tony expresses the advantage to doing things "abstractly" as making it easier to avoid making algebraic errors. Reconciled: Tony is expressing a legitimate advantage to using formulas, which does not necessarily mean he thinks the formalism is all there is.

2 *Concepts*: Tony bases part of his solution on "common sense," and he feels it is legitimate to do so.

T: Because the direction's opposite. And then gravity slows it down a little bit too, 1
because it's going up, and then just plug everything in with m 2 is 3kg, [mumble-work] Well, actually, if you want to be totally abstract about it the whole way, it asked for the acceleration, since F = ma acceleration is just F over m, the thing that's moving, in this case, it's going to be F over m 1, so acceleration is going to be m 2 over m 1 g, minus just g, mu sub k, that's just dividing through by m 1, and that's going to equal [pause, works]. And I can also go through and check that the units are all right, cause these two kg will cancel, m/sec squared, m/sec squared (ok) [pause, works] Hmm, but the answer doesn't sound right at all.

I: Why, what did you get?

T: 22 something.

I: 22 something.

T: I got 22 m/sec squared, it doesn't sound right.

I: And that seems high because why. 2

T: Um, hm. That's kind of hard to say, actually. Well, for one thing, this number, I could have just plugged into the calculator wrong. 9 times out of 10 if I get number like that wrong, it's just I plugged into my calculator wrong, this is a long formula. This number here, I mean this is, 2, this is like 2 times 9.8 is only 18, 19.9, but it shouldn't be over, and then since this is a smaller, this should be a positive number here, if it's not something else is wrong. So
that's what I'm going to check, make sure this is a negative, this is a positive number, because this is the total force, these are the total forces acting up this way, so I'm subtracting them, if I 3
were to add these, then it should be a negative number. Cause then all my minus signs would have worked out in reverse.

I: So how do you know the sign of this number?

T: The reason it should be positive is I'm taking it's vector in the current direction, and in all the work previously I've done the signs as if this were its direction. So the number should come out positive.

I: But as you were saying before, if you thought it was this direction and it really turned out to be this direction then wouldn't you just get the opposite sign?

T: Then I'd get a negative number here and I'd get a negative number here too. Like I said before, on this problem here, if I were unsure, then I wouldn't be as questioning, but here I'm sure that this should be a positive number. Let's see.

I: So you're just trying the same expression again.

---

1 I note the misconception in Tony's solution, because this marks the beginning of an important example discussed in Chapter 6. At this point, Tony has written out

$$\begin{aligned} F &= m_2 g - \mu_k m_1 g \cos\theta - m_1 g \sin\theta \\ &= m_2 g - m_1 g(\mu_k \cos\theta - \sin\theta) \end{aligned}$$

and then

$$a = F/m_1$$

There are two problems here. One is a sign error in factoring out the term $m_1 g$. The second indicates a misconception: Tony found the total force on the two blocks treated as a single object, but to find the acceleration of the first block he divides its mass alone into that total force. He should have divided F by the total mass, $m_1 + m_2$.

2 I note an interviewing error. Tony did not say the number seems high. In fact, it does not seem to bother him that the number is high, despite my slip.

3 *Concepts*: Tony looks inside his calculation at an *intermediate term* in his calculation (the total force on m_1) and checks it against his informal knowledge of the direction of the acceleration. Moreover, he expresses confidence in relying on his intuition.

T: And here's the problem, that should be negative. Let's see if I can find a mistake here some place lese. [pause, works] Hmm, that looks right. [pause] Hmm, but it can't be that, because if there weren't any friction, it'd only have 19.9 m/s^2 going that way, and now it's telling me that it's 22. 1

I: If there weren't any friction, what would happen?

T: If there weren't any friction, this would just be straight, it would be 19.9 m/s^2, because nothing would be here to slow this box down, it would just go the exact same acceleration this was, one thing that's slowing this down is just this box here, and it would go down, at a speed of 9.8 m/s^2, and then just drag that along, well wait a minute. Hmm. I am not amused [pause]

I: So what just happened?

T: At this point, I don't know. I was trying to reprove my own theory, and I see something that wasn't right. Because if it weren't, if there were no friction, it wouldn't be 9.8 m/s2.

I: If there were no friction, this would be the only term that survives?

T: Yeah, but I wonder if this term is correct now. Cause, if there were no friction, it would simply be, m 2, yeah the force would still be m 2 g, hmm. [pause] Ok, I'm starting to think I did something basically wrong in the problem now, I'm starting to wonder. 2

I: And what's making you think that?

T: Because, the way I have it set up now, if there were no friction, the force would just be m 2 g, so then the acceleration of this would just be m 2 over m 1 g, but that's not right.

I: Why?

T: Because, that's the acceleration I mean, because the acceleration here is just going to be 9.8 m/s2, here it can't be higher, it can't pull this along, they have to move at the same speed. So I'm starting to wonder if I have to bring tension into the problem. However, if that is the case, I think it is the case, as a matter of fact. As a matter of fact, I am sure that's the case. The reason I'm thinking that is because I know, I'm looking at it abstractly, and abstractly it's just not right.

I: Ok. Actually you could do me a favor by not erasing that and starting anew down here.

T: Ok, [mumble work] so, the amount, the box acceleration is going to be [mumble work], and the other box is going to have the same T, [mumble work]

[tape side ends]

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[1] *Coherence*, *Concepts*: Tony checks his solution against what the solution would be in the same problem with no friction. In doing so, he discovers a difficulty in an inconsistency with his conceptual knowledge: the acceleration comes out greater in the case with friction than in the case without.

[2] *Concepts*: Tony finds another problem with his solution for the case of no friction: the two masses come out as having different accelerations, which does not agree with his conceptual knowledge that they must move at the same speed. *Independent*: He says that he "did something basically wrong," and proceeds to try to reconceptualize the problem. Thus Tony believes he can and should rethink the problem.

T: [mumble-work] Ok, now what I'm doing is, before I said that this force [$F = m_2g - m_1g(\mu_k cos\theta + sin\theta)$] was going to be [unintelligible] right here [on m_1], and now I'm saying that's not true. And actually now that I think about it that is right, cause this force, the gravity here is trying to, accelerating more than just this box, it's also accelerating this box. So now I have to go through and create a, ah, what the tension in the string is, and then, so basically in this equation here I'm replacing m 2 g which is what I assumed this thing was putting on this one with, that's not what, it's putting on with some amount T, which is also pulling back on the box, and actually I have to go through and ah, do the simultaneous, the only trick is I have to decide if, ah, the force on them is going to be equal or not. See, the force isn't going to be equal, the accelerations are, the accelerations are going to be equal, ok, that's what it is. [1]

I: Ok, the force isn't going to be equal, the accelerations are.

T: Yeah, the forces on these two boxes won't be equal.

I: Now is that the total force or the tension force,

T: The total force on that box won't be equal, but the accelerations will be equal, because whatever this accelerates at, this is also going to accelerate at. So that's how I'm going to do it, since F = ma, a is going to equal m 2, I think this is going to be g, [mumble-work] T over m 1 minus mu sub k [etc] Ok, since the accelerations are going to be equal, I can go ahead and place these two equal to each other. [works]

[I ask Tony to stop to move on to another problem. Tony summarizes how he would finish the solution.]

I: Two rocks are thrown from a cliff of height h. (ok) Two rocks thrown, each with speed v0 [writing this], one is thrown horizontally, and the other straight down.

T: Ok, but with the same initial speed. I mean, that v sub, the one is thrown v sub 0 horizontally, and one is thrown v

I: Yeah. (Ok) Right. And then from height h, two questions a, which hits the ground first, and b, which with greater speed.

T: Hm. [unintelligible] Ok. Well, which hits the ground first is going to be the one thrown down, because, the um, horizontal and vertical components are independent of each other, and the one thrown straight down is starting off with a vertical component, we'll call them A and B. B has [writing] initial velocity in the y direction, and y = v sub 0, where A has initial v y equals 0, and that's actually build up from there, so if you were to bring in the actual translational equations you get x is going to equal v sub 0 plus g t squared, [mumble-writes] and then here x is going to equal gt squared. At any given point, it's going to be, actually v sub 0 t, um this one's going to be farther down by a factor of v sub 0, what is was thrown by times how long it's been falling. And with greater speed, this one requires a little thought. I'm going to say this one, I'm going to say B again, but I have to think about it, just to make sure. Ok, A is gonna, let's see, v sub y, is going to equal, simply, actually 1/2 gt squared, 1/2 gt, no it's accelerating, so it's going to be gt, [mumble-work] [2]

I: So you had A = v [sub] y = 1/2gt squared. [$v_y = \frac{1}{2}gt^2$]

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[1] *Coherence*: Tony explains why his previous approach had failed, showing a concern for reconciling inconsistencies in his understanding. *Concepts*: Tony had never *said* that the "force was going to be right here." He seems to be referring to what he was 'saying' by performing the calculations as he did, an indication that he thinks of the formalism as expressing conceptual content.

[2] *Concepts*: Tony answers immediately, based on informal knowledge, and then constructs a proof showing a conceptual understanding of the formalism, saying v_0t is how much farther down rock B will be compared to rock A.

T: Where?

I: Well, you had it right here for a second, and then you erased it.

T: Oh, yeah, I was thinking, I had the wrong, I had the tran, I had the actual positional equation. This is, I'm equating speeds now. 1

I: Oh, so you have a = v sub y = gt.

T: This is of rock A.

I: Oh, rock A.

T: Yeah, I labeled them rock A and rock B here. It's going to equal, v sub 0, plus gt, now the question is, see on this one the t is the same, because ah, they were thrown at the same time, here, the t is dependent on how long it takes it to fall. So the question is, hmm, it's tricky. Ok, height h 2

I: Wait, can I ask you, where are you, where did you get, say this, $v_x = v_0 + at$.

T: Oh, that's just like a basic equation. Um, you know, like one of the first, few weeks ago that we derived equations for actual position of an element, and speed, and it's acceleration, well, this is assuming acceleration is constant, and since acceleration is due to gravity, it is. But that's just an equation we learned right off the bat, we derived. Sometimes they require a little thought to recover though. 3

I: Does this one require thought to recover?

T: Hmm? Yeah, because we haven't used these in a while. Or not in this way, anyway. But, doesn't, it requires thought, but it's there. Um, ok, so here t is going to be how long it took to drop. For it to fall a height h, it's going to be, now, it's a little side note, the fact that this is thrown horizontally really doesn't mean anything. Cause, the whole problem is up and down, you could have just said it's dropped, cause with respect to the vertical motion it's the same thing.

I: How do you know that?

T: That's another, that's a given you kind of have to prove with examples and stuff, that if you throw something this way and let it drop, and you just drop something, they drop at the exact same, they're always at the exact same level the whole way, cause, um, orthogonal, that's not the way he said it, but this is like, vertical and horizontal motion are independent of each other. (ok) If something's accelerating up, at the same time being blown this way, it will be the same thing as if it's going straight up, there's no, it doesn't affect, it's kind of a common sense thing, it doesn't affect, motion in one direction doesn't affect the other direction at all. (ok) Ok, let's see, so h is gonna be 4

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1 *Formulas*: Tony had written $v_y = \frac{1}{2}gt^2$, which he erased. He explained that he "had the wrong" equation. Reconciled: the use of the formalism was routine and correct. Tony may have considered his mistake an uninteresting typo that did not warrant conceptual explanation.

2 *Coherence*, *Concepts*: Tony checks his calculation plan to be sure that it is conceptually consistent, rather than just proceeding with the calculation to see whether it gives a reasonable answer.

3 *Formulas*: Tony describes the formula as "just... a basic equation." Reconciled: Tony later describes this formula as just an expression of common sense. As well, it is reasonable to assume he does not consider the conceptual content worth making explicit at this point.

4 *Concepts*: Tony says that it is "kind of a common sense thing," that horizontal motion doesn't affect vertical.

I: Well, let me ask you something about that. So this is an example where, before you had this angular momentum which was an example where it wasn't really common sense, you could do it with the math.

T: Um, angular momentum itself is common sense, it's proving that, actual proof that would [1] be considered a proof in scientific circles isn't common sense, it's ah, more mathematical, but the actual concept itself makes sense to me.

I: Now is this a case where, is this a case where it is not

T: Common sense?

I: Yeah, where you're

T: Here you really can't prove it mathematically, well, you probably can if you get into [2] orthogonal, what ever, but, um, it's just kind of, how to explain it, I mean it just makes sense, in that, if you take a ball, and you drop it, and you thrown a ball this way, they're both going to be accelerating down with the same speed, because nothing is affecting their vertical motion.

I: Ok. But this isn't, this is an equation.

T: Yeah, see, in mathematical terms, that just means I don't need to take the horizontal motion into account here, that's why I have these sub y's everywhere, that's just means I'm taking total vertical motion into account,

I: But for example, $v_x = v_0 + at$, you know that from an equation.

T: Yeah, that's a basic, that's a given equation. What that's saying is the velocity at any given point here is going to be the velocity it started with, plus it's acceleration times how long it's been accelerating.

I: Ok. So that's not common sense, that's an equation.

T: Hmm? Well, it's common sense too, if you think about it. I mean, all it's saying is that if [3] something starts off at a certain speed, and it gets faster for a certain amount of time, the speed it ends up with is going to be what it started with plus how much it accelerated, and, how, what it's accelerating at, for how long.

I: Ok. What about these? [$x = v_0t + \frac{1}{2}gt^2$]?

T: More or less the same thing. The only one that's kind of hard to grasp, totally common- [4] sensically is the $1/2\ gt^2$. What this is saying now is, actually the total equation is $x = x_0 + vt + 1/2\ gt$, or at^2, its acceleration in general. It's saying, something, its position, is going to be, where it started at, plus how fast it was moving times how long it was moving for, and if it's accelerating it's going to then be plus 1/2 its acceleration times how long it's moving for squared. This term right here is hard to, grasp mentally, because you can't really envision time squared, you know, what a square second is, but this is what it works out to be, and you

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[1] *Concepts*: This is a reiteration of what he had said before: the results and properties of angular momentum are common sense, but the proof is not.

[2] *Concepts*: Tony motivates the idea with an intuitive argument.

[3] *Concepts*: Tony describes the equation as common sense.

[4] *Concepts*: Here again, Tony describes the formula as common sense. His articulation of the different status of the t^2 factor, in fact, strengthens the interpretation of this statement as reflecting his beliefs and not the context of the interview. That is, Tony's finding and explaining a meaningful exception shows that he is not simply saying the 'right things' in the interview. *Coherence*: In describing the status of t^2, Tony remarks that one "can prove it mathematically 20 different ways."

can prove it mathematically 20 different ways, that's something you really can't grasp, what a 1/2 acceleration times time squared is, cause you really can't envision a squared second.

I: Ok.

T: Ok, so this problem, let's see. [mumble-work] Hmm, here's where my basics equation's more off, I've got to get more on the ball here. Since it started with, hmm. See the only reason that I going to have to work with this one here is because the second rock B started with more velocity, but fell for less time, whereas A started with no velocity, but it fell for a greater amount of time, so the trick is was the amount of time enough to overcome that initial velocity. Actually, you mind if, can I use my notes for these, or would you rather I not, it just for the ba,

I: Um, sure, tell me what you're looking for and

T: Cause there's other, there's four basic equations, like, there's two of them that are easy to know and the other two are less, are used much less often. These are the equations here, the x = x nought plus v nought, and then this is the v = v_0 + at, and these two here [$x = x_0 + \frac{1}{2}(v_0 + v)\, t$, and $v^2 - v_0^2 = 2a(x - x_0)$] you rarely use, if it were a test I would memorize them, and stuff. Those are equations you can't just recall off the top of your head, even though, you kind of can, but this one here is basically just saying, these two are both derived from these two, but they also have their common sense, this one's [$x = x_0 + \frac{1}{2}(v_0 + v)\, t$] saying x is the position of the object is where is started from plus the average of what it started at and its final velocity, their average times time. Hmm, I'm not sure if any of these, 1

I: And that one

T: This one's a little hard to grasp. This is actually, is derived from these two, and it actually is this one, if you modify it some more, it turns into energy, it's an energy, not an energetic equation, [laugh] it ends to equate um, kinetic energies together.

I: But these two, if you haven't memorized them you can't get.

T: This one kind of. I mean, you can derive, the trick is you can derive either one of them, from these thing, well if I remember, this one was pretty hard to derive, I can't remember, but yeah these two aren't, you just can't call from the top of your head, they're kind of obscure, in my opinion, some people may consider these obscure, but it just seems to me you use these two a lot more, than the other two. Alright. This is actually a harder problem than it looks. My money is probably on rock B. Well, actually, wait a minute. Hmm, I looked at it, I'm not sure if the problem is doable. Well, let's see. When this hits, ok, yeah let's find out how long these things are falling for. Here, [mumble-work, all algebra] Break out the old, there must be a better thing to use to solve for t. Could be wrong. [works] Hmm. Interesting. Ok, so putting that back in here, [works]

I: What did you say?

T: Oh, um I have to solve this, I'm looking at this um, abstractly again. These two equations here should be more or less the same if v sub 0 is zero, and for some reason they're not, and 2

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1 *Formulas*: Tony felt it was difficult to remember $x = x_0 + \frac{1}{2}(v_0 + v)\, t$. The coding is based on my assumption that the formula should not be difficult to conceptualize: it is simply the statement, for constant acceleration, that the change in position is the average velocity times time. *Weak Coherence*: Tony thinks the formulas should be memorized, although he is aware they can be derived from the others. These codings are unreconciled.

2 *Coherence*: Tony looks at a limiting case in which the initial speed is 0, saying that the equations for the two cases should look the same.

I'm trying to figure out why that is. I'm thinking maybe I made a mistake over here someplace. But, I don't see where I did, so I won't worry about it just yet. Um, ok, v sub 0 plus, minus v sub 0, these will cancel out, plus or minus, it's going to be close, because, can't have a negative sign,

I: Why can't you have a negative sign?

T: Oh, um, this is a mathema, this is a quadratic equation, and it's going to give you two 1
values, but one of the times will be negative, and you can't have a negative time, so you kind of throw it out. [algebra] So it looks like, [mumble work] One more question left is why is there a g there? Cause I don't get one here. Hmm. [pause] Over g, that's why. Hmm. Yeah, this would definitely be, 2h/g, [works] which is the same thing, as 2hg, ok, ok so that works out fine. Ok, so then, so at the bottom these are the two speeds. So B will be faster, anyway, B will hit faster [writes].

I: Um, if you were going to do this for homework, would this be what you'd do?

T: Um, more or less. Here I probably would have just said, um, B.

I: Here you would have just said B without any explanation?

T: Probably. Simply because it's, it really is common sense. It's something, something thrown 2
down, and something just let drop, it's going to hit first. I don't know, if you have two stones and throw one and you just drop the other, the one you throw is going to hit first, I don't know, that's just

I: Um, on this one, you have B will hit, so the horizontal motion you don't care about?

T: No, because if you were to, if you threw this horizontally, it would still hit at the exact same vertical speed. Hmm. Technically, oh man, I don't know, if you're going to think of it, by greater speed, do you mean greater vertical speed, or do you mean greater speed total?

I: I mean greater speed total.

T: Ok, I guess I have to go through and, I was assuming vertical speed. Total speed is going to be a vector. [works] See, this is like, where did you get this problem from, did you like create this, or, was it from a textbook?

I: Um, this was from another book.

T: Huh. It's odd that, this is worded kind of, greater speed is kind of ambiguous, usually they would say greater total speed or

I: Well, maybe they did and I just remembered it wrong.

T: That's what I, they go through it and they're very, if they're the least ambiguous in this, in a class of 300 you know at least 10 of them are going to be confused by it. Ok, total speeds. A, ok, that's going to be, [works]

I: Before doing, before doing this out, what will you expect, you had said before you expected B to be the answer, but now with the new

T: I don't know what I would expect. Looking at this, wow, looking at this,

I: What did you say, wow?

T: Looking at the, now I think they're going to be equal, interestingly enough. 2gh, ok, that's it's total speed. Well, now that I think about it it kind of doesn't make sense. Here, that's going to be v sub 0 plus, yeah, ha, total speeds are going to be equal.

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[1] *Concepts*: Tony applies the common sense that "you can't have negative time."

[2] *Concepts*: Tony affirms that the answer is common sense, to the point that he would not need to explain it for the course.

I: And that doesn't make sense.

T: Well, it might, now that I think about it. Yeah it does, because they both started at the same height, ok, so you can bring energy into it, these make sense so those total speeds are equal, yeah, because they started at the same height, and they both started with the same kinetic energy, so, as they fell, their kinetic energies, ok, as they fall, their potential energy is decreasing, cause they're getting lower, and so their kinetic energies have to increase the same amount, and since they both have to hit, since they both fell the same distance, they both have to hit the bottom, when they get to the bottom, they have to have the same kinetic energy, which means the same speed. And so yeah it does make sense, now that I think about it. If this had started off, not being thrown horizontally, just being dropped like it had before, it would be different, because, its kinetic energy started off as being 0, and the only kinetic energy it has when it hits is it, the amount it got by falling, whereas, being since it was thrown horizontally, it had an initial amount, and they both had the same initial amount, and they both gained the same amount by falling. Interesting. *1*

Second interview - 4/11

I: What are you doing in the course lately?

T: Oh, we ah, rolling motion and gravitation, since then, and we're just starting simple harmonic motion.

I: So is it all pretty good?

T: Yeah. Rolling motion's kind of tough. That's something I never had any kind of background in before, and the book didn't give a whole lot of examples, as opposed to normal, so the problems were kind of hard, but other than that, no problem.

I: That's the rotational stuff.

T: Yeah, that's kind of, that's part of rolling motion.

I: But the earlier stuff in rotation, that's all

T: That was fine, yup. Midterm Thursday over the whole bit.

I:" Oh, that's in two days, that's right, how are you preparing for it?

T: Hmm? Well, mostly going over my old class notes, and going over just problems, since he's, very good, very problem oriented tests, which is the kind I like anyway, um

I: What do you mean by problem oriented?

T: Um, well, none of this multiple choice type stuff.

I: Oh, I see.

T: There's a little at the beginning, but most of it is problems, that's the kind I like. Like last year the chemistry professor was like, his tests were, he'd have a question with like five possible answers, and they can be multiple, you can have like 2 or more answers per questions, and he says there are a hundred circles total, go, find them [laugh] hundred answers, you can have like five out of six on one question.

[A little clarification of this exam format.]

T: These are more problem oriented. I like these because, if you got it right, you know it, or at least you knew what was going on, whereas, those other ones, there's so many trick questions

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1 *Coherence*: Tony feels it is important to check the result with another line of argument, because he is surprised by it. *Concepts*: His surprise seems based on an intuitive expectation; he uses a conceptual argument based on conservation of energy to understand the result. *Independent*: Tony's effort to sort the matter out was self-motivated.

it's annoying. There was one that there was two answers, 3.65 times 10 to the second, and 365, I figured, he's not going to put the same number down twice, so [etc on this question, tricks...]

I: But on these exams, you pretty much know if you get it right.

T: Yeah, I mean there's always room for error, in, you know, adding numbers and stuff, but you always end up with some kind of final formula, and if all the units work out in the final formula, and you've probably seen it, the formula somewhere before, or something like it, it kind of just feels right. If it looks familiar, and the units come out right, it's a decent bet. But there's always room for error. But, ah, I feel far more confident. I tend to know where I make mistakes, and, so that's why I'm usually pretty good at predicting my test scores, I usually know where my mistakes are, and usually good at estimating how many I made. These kind of tests, I feel like I have a lot more power over my score. *1*

I: Um, alright, well, let's see. The first thing I want to do today, is just go through the book a little bit, and we'll just pick some pages somewhere, and, actually, what have you read up to?

[Tony reviews what parts of the book he has read. He has not read Chapter 14, but expects to be familiar with the material, simple harmonic motion, from his math course in differential equations.]

I: Um, ok, so let's see. This is as good a place as any, on rotation stuff. So why not, I don't know, just, starting here, um, tell me, you know, you're going through this, what are you thinking when you're reading it, what are you, what do you pay attention to or not pay attention to.

T: Um, hmm. I usually tend to, see it depends, if it's something I've seen before, I'll just kind of skim over it, and make sure there's nothing new that I haven't seen before.

I: That's why I didn't want to do the, otherwise I would have done chapter 14, but you said, the simple harmonic stuff, that sounds like that's what you'd be doing there. Is this something you've seen before?

T: Yeah, no this is something I had not seen before.

I: Ok, so

T: Basically I go through and I kind of, what usually happens is I will have seen it before in lecture, so I'll kind of know what to expect, anyway, and the derivations, he will have done in lecture, hopefully, and hopefully I understood, and if I didn't I would have gone back over my lecture notes and figured out the derivations he already, got it from, but, I've gotten to the, he does the derivations pretty fast, I've gotten to the point now where I usually understand them as he's going along. I get my thought processes up so I can see what he's doing while he's doing it. (ok) Basically the book for me is more *2*

[1] *Formulas*: Tony describes knowing whether an answer is correct in purely symbolic terms. Reconciled: This is in fact a useful strategy; that Tony uses it does not necessarily mean he thinks it is the only strategy; moreover, he may mean to include conceptual reasoning in "it kind of just feels right."

[2] *Coherence*: Tony mentions derivations in particular as important to 'go back and figure out.'

of a reinforcement of the lecture. I go through it and it kind of reminds me of what was done in the lecture, like I'll go through and I'll remember seeing this someplace, you know, in my notes or something, and I'll say ok I remember when he derived that. So I really don't actively sit down and, I just kind of, I don't know, I kind of read over it, and it just tends to absorb, or something, I have a decent memory when it comes to stuff that's written down on paper, so I just kind of read over it and a lot of it is just reinforces the lecture. 1

I: So can you simulate that in some way for what would happen on this passage here.

T: This passage here, ok I'd probably kind of, I'd skim this area here, because this is the part I will have seen before in lecture, I'll see the math, and I'll kind of skim this.

I: You'll skim the whole, you'll skim all of this stuff.

T: Right here, yeah, cause all this is doing here is just proving these equations, and I've already seen these proven. So I'll read it again to reinforce the lecture.

I: Ok, and you feel comfortable with it and so you don't

T: I'm comfortable with it, and I don't go on, it's the examples I pay a lot of attention to. Most of this, I will have seen before, and if I haven't seen it in lecture before, but if I haven't seen it in lecture before, I'll read over it pretty carefully, but usually I tend to read the lectures first and then the book later, cause I like the reinforcing bit. And the examples I pay close attention to, cause they always show you various ways of using the information that's given, you see in this part, when they're explaining what it is, they really won't tell you, you know, if you were given a problem like this you can use it in this way, they won't do that, that's what the examples are for. And that's the part I pay close attention to the examples here. And then I just, I see how this was used, I go over the example really carefully and make sure I understood how it's done, and if I don't, I'll make a note someplace, ask my TA or whatever, or I'll call somebody up who knows. But, ah, I'll find out what the example is, so I'll understand it

I: You'll call someone up

T: In chemistry I would call somebody up, in physics I've never

I: What would prompt you to call somebody up?

T: If I didn't understand what was going on. I'm not going to do that in physics at all. In chemistry once or twice I had a little bit of trouble, just in the beginning, because I didn't have any background.

I: So basically, something like this, this is stuff that you feel you already got from lecture, and so you don't

T: Yeah, it's just, for the me the math, it really clicks. I mean, they could probably take out most of the words and I would just go through the math and say ok, and I wouldn't worry about it. [unintelligible] The words gives it a really good, gives the math a physical meaning, is what I like about that, and that's something I'll read this for, is to get more physical understanding of the math. But usually I don't sit there, I don't read it all that intently. 2

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1 *By Authority*: Tony's description implies that learning involves storing ('absorbing') information. Reconciled: Based on his comments earlier in the same passage, where he described 'getting his thought processes up' so that he can 'understand as the professor goes along' and how he "would have gone back over [his] notes and figured out the derivations," Tony may have meant to distinguish learning from being reminded about what he had learned. This is supported as well by his comments in the subsequent discussion about already being comfortable with the material before reading the text.

2 *Concepts*: Tony describes the text being useful to give him a "physical understanding of the math."

I: Is it, have there been any times, so, you, you learn in lecture.

T: Yeah, I do, I learn, that's mainly

I: Does it ever happen that you don't get it in lecture and then you get it from reading the book?

T: Um, not this year, cause I've had a pretty decent physics background. Last year, a lot, last 1
year in lecture it would be right over my head, and I would have no idea what was going on, and I'd go to the book and read it. And then, usually the book would make sense at first, but then we would see it again later, and something, since we're always building on stuff I'd see it again later and how it was used, and then I would, when I was reviewing for a test or something I'd read it again, and then it would make sense. The chemistry takes a long, the physics I usually get on the first time.

I: So you have a good background in physics, you had a good high school course.

T: Um, I mentioned last time, we had a high school course, I've had a good, the math, the 2
math course I took in high school, the college math course is what gave me a decent physics, it's not so much physics background, it's just, I don't know, I've always taken to physics very easily. My background is just a high school course, we never really did anything, like moment of inertia I had no clue what it was until this class. (right) I knew how to compute them from math classes and stuff, but I didn't know what they were, until this class. (right) and I still to this day don't understand it, and he said in class, don't bother asking me about this because they've done whole books on the subject and I can't explain it, what it is is a gyroscope

I: He said this in lecture.

T: Yeah, he said, this is just a very basic introduction to gyroscopes, it's going to seem really strange, and I'm not going to test you on it, so don't worry about it, this just gives you and example of higher physics, (ok) and um, we were talking about angular momentum earlier, I was talking about angular momentum is conserved, so if you had a gyroscope spinning, it has an angular momentum along its axis, and if you [unintelligible] if you try turning, it's like, if I'm holding the axis here and if I turn it this way, what happens is since I'm applying a torque, since I'm trying to turn it this way, the torque is up, because of the, the vector is up anyway, and since the change in, um, angular momentum is equal to the torque, the angular momentum is this way, the change is this way, it actually goes up, but you're trying to turn it this way, so you push this way and it wants to go up, and it doesn't make any sense. I mean it makes sense in that, you know, that happens, but I can't for the life of me explain why. [laugh] And if it's spinning the other way, and you turn it, it will go down. And you sit there, and it's exactly right, cause I've done those with actual gyroscopes, and I've wondered about why.

I: So you say it makes sense that it would happen,

T: No, I mean mathematically I see the proof and stuff, but I have no clue why. Also, the 3
thing I have is um, in the proof, this is what I was talking to him about, I had a problem with the fact that, the fact that the vector is pointing along an axis is just like a convention of math, it just they say, oh, well, we'll make it a right hand rule, and it didn't make sense to me that that could dictate that it would go up instead of down, in a given direction, and [the

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[1] *Independent*: The paragraph implies that listening and hearing is not sufficient for understanding the lecture.

[2] *Concepts*: Tony distinguishes knowing "how to compute" moments of inertia from knowing "what they were."

[3] *Concepts*: The difficulty Tony describes here is entirely conceptual. He can follow the mathematical derivation, as is clear from his summary. However, he does not understand how that derivation is tied to the mechanism, why the derivation is based on more than mathematical conventions. The answer he got from the professor was that the phenomenon was used to define the conventions, but that still leaves him without an understanding of the mechanism for "why you pull this way and it goes up."

professor] explained that, well, it wasn't so much that the math dictated it, it's just that it went the other way, they would have just said minus that vector, they would have made it so the math fit, and so it just happened to work out that way, and so that's cool, but ah, it still doesn't make sense to my why, you pull this way and it goes up. He said there are some tops out nowadays who do things he doesn't
even understand, there are so many weird things you can do with angular momentum, gyroscopes and stuff. It still, that doesn't make sense to me, but I accept that for now, because he said there's

I: It's not on the

T: He showed me a book on gyroscope precession or something, and it was like that, and he said that's an introductory book. But ah, that didn't make sense to me, so I made sure I went to everybody and talked to the TA couldn't answer it, and the professor couldn't answer it, and that kind of stuff. *1*

I: Wait, you asked a question that they couldn't answer.

T: Well, not couldn't answer, but said, you know, it would take books and books to explain this to you, and stuff.

I: And did you ask a specific question, or did you just say this doesn't make sense.

T: Well, I was asking how they could say that a mathematical convention could dictate reality, because I mean, just because we said the right hand rule the vector goes up, that makes it go up? It should be the other way around, it should be because it goes up that make the right hand rule, and they said yeah, that is the way, they made the math to fit the reality not vice versa. And I was questioning that proof, and that was basically all I had problems with. And he said in class as to why is actually does all this stuff I really can't explain to you, that's a whole course in itself as to why it actually does that.

I: This stuff [the passage on moment of inertia], though, clicks.

T: Yeah, that makes

I: And this stuff not only clicked, it clicked the first time you saw it.

T: Hm, oh yeah. Or if it didn't, I knew it would, so I wouldn't worry about it later. Like in lecture, if I see something that I don't understand, then I will usually, it will click later, after I've digested it a little bit or I've read over a book and I say ok I see how that happens. That happened a lot last year, last semester in chemistry. *2*

I: So it, when it doesn't make sense in lecture you go to the book?

T: Hm? Yeah, and if that doesn't work I go ask TA or whatever, explain it to me, but I haven't had that problem in physics yet, except for that one gyroscope thing.

I: So that's been the only thing in physics that didn't click in lecture the first time.

T: Basically, mm-hm.

I: Well, could you just go through this and explain it, I mean, something

T: Sure, well basically this whole page is centered around proving this. All this is saying

I: Proving K = 1/2 I omega squared, ok.

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1 *Independent*: Tony describes an extended, self-motivated sense-making process. His difficulty is more specific than the professor's note that students should not expect to understand angular momentum: he was concerned with making sense of the relationship between the mathematical formalism and the physical phenomenon.

2 *Independent*: Tony mentions, vaguely, a process of 'digesting' the material.

T: And this, kind of that's, K isn't so much a physical, it's a quantity, but, K is kinetic energy first of all, and it's saying that the kinetic energy of something is the 1/2 I omega squared, I is the moment of inertia, which is the ah, lets see, if something is spinning around an axis, the more mass it has like out away from the axis, the more energy it has, the harder it is to stop, the harder it is to start up again, there's more mass outside, and that's basically, that's what moment of inertia is, the physical definition is um, each little tiny mass particle, times it's rate, times the distance from the rotating axis squared. (ok) So the more, if you have a cylinder with the same mass as a disk, the cylinder will have a higher moment of inertia, whereas the disk has some in nice and close. 1

I: Well, why should that be the definition of moment of inertia.

T: Actually, that's because it is really handy. [laugh] Actually, it gives that right here, because, I mean you're figuring out, it starts off, let's go through the proof here. The kinetic energy of a rotating body is each of its little tiny particles is going to have an energy of 1/2 it's tiny mass times the velocity squared, that's just the definition of kinetic energy, and then, that's the same thing, since v, it's velocity is going to be the same thing as its radius from the center of the axis times how fast the whole body is rotating around that axis. That's um, that's just a physical fact, it's um, if something is rotating, if the circle is rotating, the radius of the circle times how fast, it's angular velocity is going to be the speed of any given particle on it. So you substitute that into the equation and you get this.

I: Ok, so this is

T: That's a physical

I: When you say a physical fact, what do you mean, this is something that

T: Um,

I: It's in the book.

T: By physical fact I mean it's just something that, how to explain it. I mean it's like, hmm. I don't know, it's like those basic equations I was telling you about earlier, with the velocity is acceleration times time, (ok) that's just kind of, you have to have that as being kind of given, because acceleration is how fast you're getting faster, and time is how long you do it, so, you multiply them together you get velocity. 2

I: So, are you saying that it's obvious?

T: Yeah. It's not so much obvious that look at it and say oh yeah, but it's something that you have to kind of draw a picture of, and you kind of see what's going on. 3

I: This one, you draw a picture of v = r omega, and it just sort of

T: Yeah, it just sort of makes sense. It's like saying a cubes area is one of it's sides cubed, it's, you kind of simply have to look at it and say yeah, length times of width times height, they're all the same thing, cubed. It's kind of the same, the same

I: When some people, when somebody says to me, it's, that's a fact, um

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[1] *Concepts*: Tony explains moment of inertia in informal terms, as well as in formal terms.

[2] *Pieces*: Tony's description of "physical fact" sounds like a description of an independent piece. Reconciled: in the discussion that follows, Tony elaborates on this description in a manner inconsistent with *Pieces*.

[3] *Concepts*: Tony's statement that "you kind of see what's going on" implies the involvement of intuitive knowledge.

T: Well, it's proven back in rotational motion. [1]

I: Somebody might say that the area of Rhode Island is, and state whatever the area of Rhode Island is, and that's a fact, and it's just a fact, so I guess that's not what you mean.

T: No no, no no. It's provable, it's back in the chapter. Basically it's like a provable statement, [2] you prove it with geometry, is what it comes down to. That's what is, it's a geometric relationship, ha ha, that's the phrase, a geometric relationship. Substitute that in here, and you end up with 1/2 this quantity m r squared times omega squared. (ok) Well, if you define the moment of inertia as each, see they're trying to come up with a relationship for the whole body, without having to relate to computing each individual little piece's kinetic energy. So if you have an expression here with 1/2, and then this is the m sub i, this is the little tiny body, and then the r squared sub i, and omega squared is for the whole thing again.

I: So, each of the separate pieces has the same omega?

T: Yeah.

I: And why is that?

T: That's the thing, a record turning, each piece doesn't really have it's own omega, omega is a property of the whole body, and each piece has a velocity that is its distance from the axis times omega. Omega is kind of, a record turns at 331/3 or whatever, actually, I'm not sure what 331/3 represents, but suspect it has something to do with angular velocity, basically it's how fast it's turning around. There are parts closer in to the record that are moving slower than parts outside on the record, but the whole records turning at 331/3.

I: So the inside of the record's maybe turning at 20 and the outside

T: No no, see, the whole record is turning at the same time, but the individual pieces have [3] different velocities. One way of thinking of angular momentum is the speed, the amount of angle it goes through in the same time. Like if my arm goes from, goes through 90 degrees in a given amount of time, this will be slower velocity wise,than the outside of my hand, but the whole thing is rotating at the same angular velocity.

I: Moves through the same angle, ok.

T: This is a property of the whole body, this is just a constant, we got from this. The only thing left is the m sub i r sub i squared. so if you take a quantity I, and call it this, and this is computable over the body, then you would have this, um, this definition of kinetic energy of a spinning body, and that's basically the whole idea of what they're saying there.

I: All right, so you're saying here, all this clicked during lecture, and you can sort of hop thorough here and remind yourself of that, but, fine. all right, enough of that. Oh, wait, there were two problems from the homework, there were two problems, from long ago, number 3 and number 8. If you remember how you did them.

[Move to problem #3, Ch. 11.]

T: Ok, well, 3 is basically a, more of a definition problem, it tests to make sure you know the definition. The definition of angular velocity is r cross, velocity, r cross p, it says an aircraft passes over you with ah, [pause, drawing?] has a speed of 90 km/hr, ok, oh, angular velocity, I was thinking of angular momentum here, get rid of that. Ok, angular velocity is, that's just

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1 *By Authority*: the statement implies that Tony considers this the basis for his understanding. Reconciled: in Tony's next statement, he clarifies his point to be that the formula can be proven, not that the formula was given in the book.

2 *Coherence*: It becomes part of Tony's explanation that the formula is not 'just a fact,' but can be proven. He then continues with the main task of explaining the text passage.

3 *Concepts*: Tony gives a conceptual argument, closely related to the 'geometrical proof' he referred to earlier, to explain $v = r\omega$.

what we were talking about a second ago. Let's see, velocity is r omega, ok, ok, 90, 100 km per hour, and it's 10,000 meters up, that's just in the definition, you figure out, w is velocity over radius, is, ok, the trick here is, 900 km, ok, it's also a unit type thing, this is 10 km, is 10, 000 meters.

I: Ok, but, well, here's the reason is that I pull this question out, is that it seems strange, omega, the way you were describing it, with your arm and the record is about things moving in circles, and this is an airplane moving in a straight line overhead.

T: Here they're talking about instantaneously, as if, if right here, if it were going in circles around you, how fast, how many an, what's the angular velocity it would have, and, yeah, since it's, it's instantaneous angular velocity. Basically what it comes down to is if it were going in circles, what would the angular velocity be, and it's still angular velocity with respect to you, it's just that its not going in a circle, but at that point, it still has a certain distance from you and it's still going a certain distance on its own. *1*

I: Ok, so it still has an angular velocity with respect to you.

T: Right, it's just that it's not going in a circle, it's, the angular velocity will change, as soon as it, it's different at each point, because it's going in a straight line, that's like when you sit there and you watch a train come, you'll see it come, and it's kind of sit there, and as it goes by, it's zoom by, so it's angular velocity, the faster you turn your head that's what angular velocity is, shoooo [turns his head to demonstrate], and then it'll start slowing down again, but at each point it has an instantaneous, at that point it has an angular velocity with respect to you, if it were going in a circle, but it isn't actually going in a circle. At that point, what is its angular velocity, and then its

I: You don't have to do the arithmetic.

T: Ok.

I: So, so omega is v over r and then you get the units right.

T: And then um, the next part is, you figure out, a few minutes later, you figure out how many more kilometers its gone, however many that is, (ok) and then you end up with something like this. This is kind of tough in that its angular velocity with respect to you here is, since this has to be a 90 degree angle, you have to take this portion,

I: Why does it have to be a ninety degree angle?

T: Well, if it were going in a circle its velocity would be this direction, so you take the portion of this velocity vector that is at 90 degree angles, this would be some angle theta you'd have to figure out from the numbers given, this would be some amount here you take 2

I: Which is the right angle here? This is the right angle?

T: Yeah, that's called a projection.

I: And this angle here

T: That's also 90. So basically what you're trying to do is you're trying to find this distance here, which you have to get from some geometrical, since you know this distance that it came over and stuff, you can figure out this, and then you figure out this from, once you figure out the theta here, you can figure out what this, the angle this is going to be, it's going to be v cosine theta, and then you get that times this distance r, actually divided by, v over r. And that's something I could see, a vectors have always really

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1 *Concepts*: Tony immediately explains the answer to my question with a conceptual argument, which he elaborates with an example.

2 *Concepts*: Tony gives a conceptual response, based on the meaning underlying the formula and not on the established procedure.

been a strong point of mine, but I could see people having real problems trying to figure [1] out why that is, why you take that projection, I don't know, it just makes sense to me because I've always been decent with vectors.

[Tony solves Ch. 11 prob 8 - straightforward, routine solution.]

I: Alright. Um, another problem. This is a merry-go-round, and two people on the merry-go-round, and let's have it turn that way, and they're playing catch with the ball. The question is, there are two parts to this question, the first part is, well, they're playing catch, and this guy is going to throw the ball to this guy. The first part of the question is, if you're this guy, how should you aim, ok, where should this guy aim to throw the ball. The second part is, if we're standing on a bridge or something up above this, and we're not turning, what do we see the ball do?

T: Yeah. Hmm. Ok, when. Ok, when he's actually throwing the ball, this is a very easy problem to overthink, either way

I: What do you mean by that, an easy problem to overthink?

T: Well there's two ways of looking at it. If you just assume that the ball is independent of the wheel, then he should aim over here, cause the guy will then turn and he'll catch it, but I think, what I'm thinking about it, while he's actually throwing it, he's going to be turning, and that's going to give the ball, I think he might be able to throw it just at the guy, I'm not sure. That can't be right. Cause when he's throwing it, at the same time, he's still, he's giving it like a, cause as he throws it he's turning, so the ball is launched, it's launched a little bit off to the left anyway, because, once it leaves his hand hand, does it keep that or not? Hmm. See that's what I mean by overthink. If you consider the ball so that it just all of a sudden have a velocity of it's own accord, then he should definitely throw it up this way, for the ball to catch it.

I: Then he should den=finitely throw it which way?

T: Over here, he should aim to this side so the boy will end up and catch it (Ok, now) Unless I'm

I: Is this, so, what would that look like from up above on the bridge?

T: Just like this.

I: Just like that.

T: And he would catch it, I'm thinking, see, that's the point, I'm not sure, if he actually throws is, his arm might be giving it a sideways momentum, because as he's throwing it, he means to throw it straight at the guy, as he throws, the whole wheel will have turned and give the ball a kind of a sideways, it might go that way anyway.

I: So as he, since the wheel is turning, like this, the ball would also after it leaves his hand sort of curve around toward here.

T: The ball itself, yeah, the ball itself can't actually curve, you're right. So it must, he must [2] have to aim it to the left of the guy. Because, once it leaves his hand, even if he has a sideways momentum it's going to have to go, the ball itself can't curve, I'm trying to question

I: I thought that's what you were saying.

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[1] *Independent*: Tony statement that he could "see people having real problems... why you take that projection" indicates a sense of an involvement in making sense of the material.

[2] *No misconception*: Throughout our discussion of the merry-go-round problem, Tony is confident that the ball will move in a straight line as seen from the bridge.

T: No no. As he's throwing it the ball is curving, because its in his hand. He's throwing it straight but he's turning, so the ball is going like that, but once it leaves his hand it has to go straight.

I: Why is that?

T: Cause the wheel itself isn't acting on the ball in any way.

I: But it was turning before he threw it.

T: Yeah, [unintelligible]

I: I mean, the wheel is going around in a circle, so shouldn't the ball also curve after he throws it too, because

T: Oh, once the ball is thrown it's not going to curve, that's definite, because, the only reason something curves is because something is pulling at it, and once it leaves his hand, there's no force pulling on the ball, except gravity, but ah, the reason it turns while he's turning is because the wheel is pulling on the boy and the boy is pulling on the ball, not pulling, but, see like if there was a little chunk of dirt right here, it would just fly off, if it weren't attached, because there's no more force of the wheel pulling it down. The only reason it goes in a circle is because there is a force pulling in on it. *1*

[Continued discussion of this problem. I ask Tony the related question of how the one person should throw the ball in order to catch it himself, and how that would look from the bridge. Tony again answers that the ball would move in a straight line, as seen from the bridge. I asks whether someone on the merry go round will see the ball move in a straight line.]

T: Actually he'll see it do really weird things. Cause after he throws it, he'll throw it at there, it will actually go there, it'll seem to curve off that way, cause he's also moving this way, so as the ball goes in a straight line, he sees it going like that. To him it'll seem to curve off to the left, I would think.

I: So why can it curve to him but not to us?

T: Because he's moving, in a, in in a different, he's in a moving reference frame. he's actually in an accelerated reference frame. (ok) In the same way he sees things on the horizon moving more than we would. **2**

I: Suppose I argue this, he's moving that way when he throws it, so if he were to throw it straight across, what the ball would do is go that way.

T: Yeah, that's what I was thinking too. Hmm.

I: So that would argue that he should throw it even much further to the left over here.

T: Yeah, I am trying to figure out, that is definitely true. And the ball would have an initial velocity down, hmm, hmm. [pause] But his hand would curve, but I'm not, I'm thinking it wouldn't curve that much, cause when you throw a ball it's fast, it isn't a whole lot of time. [pause] Maybe you're right, he may actually be, [unintelligible] [pause] Ok, I'm going to go with, I'm going to go, based on the assumption that, since he's going to throw it fast enough, that the wheel won't have any effect on the ball, I don't know, like his arm turning or whatever, in which case, if I go on that assumption

I: Then he should aim here?

T: No, no, he shouldn't aim there. I'm trying to think of [pause]. Yeah, I think you're right, I think maybe he should even aim maybe past, in kind of off, this way, so that the, I think he

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[1] *Concepts*: Tony's gives a conceptual explanation, rather than just an appeal to a rule.

[2] *Coherence*: Tony's explanation is coherent with his explanation for the stationary reference frame. This reinforces the coding above for *No misconception*.

has to aim a little bit this way, but not a lot, that's where he thinks the boy is going to catch it. But I'm trying to actually envision throwing a ball. Hmm. [pause] See that's something that for some reason doesn't click, that's why I have to think about it. See it makes sense yeah, cause he's moving this way the ball is going to have a velocity that way, but for some reason that doesn't seem right, yet, I look at it and it is right, so, yeah, he's going to have to aim it a little off to the side, and he's going to actually have to throw it harder than he expects, cause if he just aims for it to land right there, it's going to land back here, since he also has a, [unintelligible] of force, he's going to land in here someplace. Yeah, I think that's the call. And that's assuming he throws it fast enough that he's not throwing it like this, where he's turning. Although actually, does that have anything to do with the velocity? No, cause he's still, the velocity is still instantaneous. Ok, so yeah, that's what my guess is right there.

I: So you're happy with it.

T: Um, not totally, but, this is one of those problems where you can like think this way and think that way, and up and down, left and right, and ah, there's no way, that's the point, on this kind of problem I would never be totally happy with, but, on a test, what I would do is, I would say, ok this is my answer, if it's wrong, so be it, and if I think of something later on, I won't change my mind unless I think of something definite that I didn't think of before, like you pointed out with the velocity there.

I: So now this problem is one that you wouldn't know that you got right.

T: No, I wouldn't be able to

I: But you were saying on an exam

T: Cause there's no algebra and stuff involved to, I've never seen a problem like this before, that's the main reason, so it would not look familiar in any way. *1*

I: So if there's no algebra you don't know if you got it right.

T: Um, I wouldn't make that correlation, but algebra helps a lot. I mean I can have a non-algebraic one and still know I got it right, but this is more like a multiple choice question, he'd say, would it land, should he throw here, should he throw here

[end tape side]

I: Is there any way for you to be happier with your answer?

T: Um, hmm. I suppose if like, what I'm trying to think is to actually set up the algebra for this problem. If he's going to have, velocity, initial velocity of the ball is going to be, [unintelligible] plus, whatever he throws it, hm. Probably not. [laugh]

I: Ok. I have one more question to ask and it's just 3:00, so let's go on to that. This is a ring, or mass 5 kg, all around the ring, and it's rotating, its radius is 2 meters, and it's rotating so that any point on the rim is moving with speed, um, 3 m/sec. What's the kinetic energy?

T: Kinetic energy? Ok. Ok, moment of inertia of the ring is going to be, straight mr squared, [pause, works] kinetic energy is 1/2, ω squared, ok, ω, let's see, [works] 15, this better be in joules, [works] Oh, ok, I am missing a m/sec squared, ok, so [works]. 45 joules.

I: Ok. So, let me see. Somebody did this on a test, from another course, and got it wrong. The way that person tried to do it was write, was use 1/2 m v squared. **2**

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[1] *By Authority*: In saying that he would not be able to know whether he had gotten this problem right, because he had never seen one like it, Tony implies that he needs to be shown how to solve problems. *Formulas*: Tony draws a distinction between problems with algebra and problems without. These codings are unreconciled.

[2] I set up the question differently for Tony than for other subjects by telling him that the student who had used linear kinetic energy got the problem wrong.

T: You actually could do that, if you really wanted to, but you'd have to do the integral around the whole circle. 1

I: How would you do that? Now, the person didn't take an integral, so, how would you do that?

T: Ok, that's the reason that this moment of inertia thing was invented to begin with, was to get rid of having to do this. Each piece is going to have a kinetic energy of 1/2 of its little mass, dm, times its velocity squared, and it's velocity is going to be 1/2 r omega, dm, and you have to take it over, you have to do that integral, ok so the mass is going to be dm, is going to be density times that little tiny volume. The density of the ring is going to be its mass divided by its volume, see that's another thing is I have to just, you have to give like a thickness, or I just I could actually just create an arbitrary one, I don't think it would make a whole lot of difference. Think of a small circle, you have to do it that way. It would come out to be the integral of 1/2 r ω, ro [ρ] dv, then you have to figure out the volume of this ring. Since it's a ring you probably have to actually go, ds, call it arc length, or dl, and then that's a line integral. It's actually you could set up a, whew, harsh. Ok, if I want to go and do it this way, ok, with respect to x, ds = square root of dx squared plus dy squared, equals, you have to go through and figure out an actual equation for this ring, which would be, x squared plus y squared,

I: And do a line integral around it.

T: Yeah, and do a line integral around the whole way.

I: Well, this, what this person did was just said, oh, mass is 5 kg, and the speed is, and just used 1/2 mv squared

T: Yeah, 1/2 mv squared is like, the center of mass has to be moving at v squared. Here the center of mass is just standing still.

I: Ok. So that wouldn't work.

T: No. Definitely not. The whole thing doesn't have any moving kinetic energy it only has rotational kinetic energy. If it were also moving you could say 1/2 times the total mass times the velocity of the center of mass. 2

I: Ok. And here, um, this works, why does this work?

T: Um, I'm not really sure I set it up perfectly, because you have to go into all this really intense math, but all you're doing is you're taking a small amount of volume, figuring out

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[1] *Coherence, Concepts*: Tony explains, in conceptual terms, the connection between rotational kinetic energy and linear kinetic energy, which was the content of the passage he had explained earlier.

[2] *Pieces*: This indicates the separation between rotational and kinetic energy I was probing for with this problem. Reconciled: Tony is responding to my specific question of whether it would work to put the mass of the ring and the velocity of its rim into $\frac{1}{2}mv^2$. Tony's argument is not that one cannot use $\frac{1}{2}mv^2$ to calculate the energy, it is that one cannot just substitute mass and velocity in the manner I described, which is correct. (It would not work just to substitute in the values without the step of noticing that every point on the rim moves at the same speed and the sum of the mass elements is just the mass on the ring.)

Reading the whole episode, it seems clear what Tony was doing. He accepted that the student's solution was incorrect, based on my statement that the student got it wrong. (Tony also commented at the end that he had miscalculated when he tried to check this in his head.) This led him, not to separate rotational and kinetic energy arbitrarily, but to expect that there was something in the process of summing the kinetic energies he was overlooking.

what mass it has, and figuring out what velocity it has, and multiplying it together, and adding that up for everybody.

I: So, but you're adding up lots of linear velocities?

T: Yeah, no, um, energy isn't a vector, we're adding up a lot of vectors, a lot of energies.

I: You're adding up lots of

T: Energies.

I: Linear energies, but not rotational energies.

T: Yeah, well rotational energy, the only reason there's energy rotational is because each thing is moving, and at any given moment, instantaneously it's moving in a straight line.

I: So, what if I argued that each of these pieces instantaneously is moving in a straight line, the sum of all their masses is 5 kg, and they're all moving 3 m/s squared, I mean 3 m/s, excuse me, so 3 m/s squared, times 5, 1/2 should give all the energies.

T: Um, let's see. Becausssssse, hmm. [pause]

I: Would that be a legitimate way to solve the problem?

T: No, it definitely wouldn't. I'm trying to figure out why not.

I: And why do you say it wouldn't?

T: [long pause] I'm trying to figure out, I'm trying to, about the derivations now, [pause] Hmm. So why wouldn't the, [pause] Hmm, this is the beginning of the moment of inertia, why wouldn't [long pause] Hmm. If you sum them up, well this is annoying. Ok. Basically because, well, I can see it mathematically, is this is not the same thing as [pause] those aren't the same things.

I: Right.

T: And that's kind of what you did, you said 1/2, this is the sum of all the masses, and [1]
actually, ok, yeah, this is, that's it, you're saying that this is the same thing as this, which, mathematically you really can't do. Oh wait a minute. [sigh] [pause]

I: See, actually, that looks like it would be a good way to show that it does work, because, um, distributive law of multiplication over addition. I mean, why, why can I pull the 1/2 out but I can't pull the v squared out, v is the same for all the masses.

T: Well, it's the same in magnitude, I was just looking at that myself. It was the same in magnitude, but the actual vectors were different, but I'm trying to figure out a way of working that in here. Cause, the reason the whole thing isn't moving is because for each point, there's a point exactly opposite on the circle that's moving in the exactly opposite direction, so these vectors kind of cancel out. That's why the whole thing isn't moving, but we're not adding vectors here, we're just adding magnitudes. Hmm. [pause] Oh, sorry I'm chewing your pen.

I: That's all right, I do too. It's almost quarter past three, so

T: That's ok, coach can wait. I have a problem here.

I: [laugh]

[Short exchange about being late for practice.]

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[1] *Independent, Coherence*: Tony believes he knows the right answer to the question, but he does not have an explanation for it. He engages in an extended, self-motivated effort to resolve the difficulty, to the point that he makes himself late for crew practice (he had said on arriving to the interview that he needed to leave at 3:00 to be at practice on time).

T: I think you're right, it seems like, mathematically, [pause] All I can think of, is that, something to do with the, each of these m's is infinitesimal, all I can think is it has something to do with that.

I: Ok, um, but you, you have a, it should be wrong.

T: Yeah. Um

I: Now are you saying it should be wrong because I told you that this person got it marked wrong, or do you have your own reason.

T: No no, the reason it should be wrong is because the 1/2 mv squared, this is, in referring to any kind of body or system of particles even, the v squared refers to the center of mass, and here the velocity of center of mass is 0, it's not going anywhere. (Ok.) So this is the equation for translational kinetic energy, whereas the rotational one is definitely that, without, beyond the shadow of a doubt. The question is, why are they different. [pause]

I: Well, cause I was a little bit sneaky, because the person got it marked wrong, and went back and complained, and they marked it right.

T: They did?

I: After the student complained. It got the right answer. It came out to the right answer.

T: Did it? [works] [laugh] It does, doesn't it. Huh. That still doesn't answer why it isn't true in every case, except that, ok, that's why it works, because um, hmm, a part of it must have been that, cause you said that, because you said it was wrong, I assumed it was wrong, cause now I look at it, that clears away the blockage that it has to be wrong, so it's come up with the same answer. Here, yeah, you're right, each piece does have the same velocity, whereas, ok, the reason you need moment of inertia is normally you're not dealing with something that has everything the same distance from the center. Normally you have something, a cylinder, or something along those lines, or usually you have like a disk, in here, but here everything is moving at the exact same velocity. Hmm. By that same token, that should also work for cylinders too, rotating about their axis of symmetry. Wow. I like it. I shouldn't have looked at it, I looked at that to see if the answer came out right, and I looked at this thinking that was mass for some reason, and said no, it's not going to come right, and I just kind of glanced at it, I didn't, if I looked at there and saw they had come out to the same answer I would have started questioning, but I was intent on that. ***1***

[Brief continued discussion of problem]

Third (final) interview - 4/25

I: So how did the midterm go?

T: Fine, 92 out of 100. [Tony gets his exam.]

I: So, what I'd like to do is go through it and you tell me how you did each of the problems, and ah, what went right or wrong.

[Tony explains the first problems, which were straightforward. The next was the car and water problem: a car rolls on level, frictionless tracks, filling with water from vertically falling rain, and draining water from a hole in the bottom. The question asked whether the speed of the car increases, decreases, or remains the same.]

T: Um the second one, I remembered this one from class, that's how I did this one. If you have the thing coming along here, if this were plugged up, the rain would be falling in so that its momentum would not be, its momentum would stay the same, because the raindrops were falling in perpendicular to the car, they have no momentum relative to this of their own to

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1 *Independent, Coherence*: That the answers comes out the same leaves Tony, briefly, with another conflict to resolve, which is why it does not always work out this way.

bring in, so, it would be, the momentum would stay the same but the mass would be going up, so it would slow down. This way, what happens is, that's slowing down, plus there's a drain in the bottom, so some of the water drops are flowing out, and taking momentum with them, so here not only is the mass, here the mass is probably staying the same, but the momentum is decreasing, so the velocity is going to decrease.

I: Why would the rain falling out take momentum with it?

T: Because it's part of the car before it drops out, and then as it leaves, it kind of falls, these raindrops here are hitting straight down, so they're moving perpendicular to the path. These aren't falling straight down, typically these would be going, the only reason that that goes back is because of air resistance, if there weren't any air it would be going down at the same rate the whole boat's going down.

I: It would go down at the same rate the whole boat is going down.

T: The hole, the hole above it is going down.

I: The hole above it

T: If this was the hole, the water falling out, it would fall like this, because it, the water is moving at this speed before it falls, it's going to keep moving at the same speed.

I: So it would keep moving forward.

T: Right.

I: And that would mean that it's carrying momentum with it?

T: Yeah, not like carrying, that's kind of the wrong phrase, but before it falls it's part of the 1
system of particles, and after it falls it's still going to be part of the system of particles, until it hits the ground. (ok) Basically I just remember that from class, I remember it, he said ok, cause a lot of people have problems, it took me a long time to figure it out myself, when he said it in class, I said what. I thought about it later and, didn't think about all that just now when, that's what it was in class.

I: Oh, so when you solved it, you didn't even think about it, you just remembered it.

T: Right. When he first said it in class I had to think about it for a long time.

I: Did he do exactly that in class?

T: Close. He did one where rain was falling, and one where the rain had stopped and it was moving along and it was draining out, the two have the same effect to decrease, so

I: Ok. If the rain weren't falling and it were just draining out then it would slow down.

T: It would still slow down. 2

I: And why is that?

T: Cause of the momentum thing.

I: So if there weren't anything falling into the car, but rain was draining out, then it would still slow down. (mm) So what you remember from lecture is that he did one where it was

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1 *Independent*: Tony says that when he first heard it in class, he "said what," ie he did not understand, and that it 'took him a long time to figure out himself.' In this situation, it is clear Tony's use of the phrase "remember that from class" does not indicate he feels hearing in class is sufficient for learning. For the test, he just remembered, but when he first heard it, he "had to think about it for a long time."

2 I note a misconception, because it comes up again later in the interview. Tony misunderstood in lecture: the car would not slow down.

plugged up and then another one where it was not plugged, and in both cases it slowed down so it would have to slow down there.

T: Right. Right.

I: Ok.

T: Ok, this one here, oh this is just a mathematical proof here. I went through and just fuddled around basically. Um, ok, I knew how to, I knew the derivation up to there, T squared equals that, and from there on in, I knew it was going to have to be, I saw that this had a good resemblance, that formula for v, to this, so I just substituted that in there, and I figured that would work, and it ends up, it doesn't end up working. And this is just substituting in to the um, *1*

I: So this had a resemblance to another formula.

T: Yeah. So I derived the other formula, and then plugged

I: And what was the other formula?

T: The formula to T squared.

I: Oh, I see, so v = 4 pi r cubed over 3, you remembered the 4 pi r cubed kind of thing, (right) from somewhere else, I see, ok.

T: Ok, here, for the first one, everybody got marked points off for this, I just said the sphere, cause I knew that it had a lesser moment of inertia and all this kind of thing, if they had equal masses, but the guy wanted to know why. It didn't say, it says which hits the bottom first, I put sphere, he said well why, takes of 5 points for it. I still have problems with that, but I'm not going to worry about it. And then ah, this one here, we
did the derivation in class, but I honestly didn't remember it, so I went through and I did my own derivation. I still haven't actually gone through and figured out what I did wrong. I thought it looked really good, but, I kind of knew it was wrong at the time, because I had seen the formula before and that didn't look familiar at all. Um, so I kind of flubbed that one up. *2*

I: So this, the question is, it takes a ball 10 seconds to get to the bottom, how long does it take the cylinder, ok, can you kind of go through and tell how you were doing it?

T: What I did, ok, I set up a force diagram and figured out the frictional force and the torque due to that, then um, hmm, maybe this would be what I did wrong.

I: So let's see, you have F s equals mu s mg.

T: Yeah, that's the frictional force, equals the coefficient of friction

I: F s means static force.

T: Yeah, static friction.

I: But the ball's moving.

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[1] *Formulas*: Tony's solution involves pure symbol manipulation. Reconciled: in saying "this is just a mathematical proof," Tony may mean this is not really a physics problem.

[2] *Independent*, *Coherence*: Tony did not remember the derivation, so he did his own.

T: The ball's moving, but at any given instant, the point touching the ground is not. If the ball were slipping then it would be kinetic friction, but as the ball rolls along its point stays stopped relative to the ground, so it's static friction. 1

I: Does that make, does that make sense? The point stays stopped

T: Hmm? Yeah. Mm-hm. It's like, at that instant point of time, the whole ball's kind of revolving around the point on the ground, actually. (Ok.) Um, yeah, it makes sense, ah, I don't know if the rest of the class did, I had no problem with it, I like it.

I: Ok. Um, this is ah

T: That's a torque.

I: Torque sub f, what's the f for?

T: The friction.

I: Oh, ok, equals r mu s mg, is the radius times the friction force equals I alpha.

T: I think that's where I made my mistake. I think, well

I: What's where you made a mistake?

T: I thought this was where I made my mistake, somewhere in here, but looking at it it doesn't look wrong, cause here, yeah, that's right, it does equal, the torques got to equal moment of inertia times acceleration, and there's no other torques. Hmm. So anyway, I got the acceleration there, and then I related the actual acceleration to um

I: Ok, so you have v equals r omega,. and then you write a = r alpha.

T: Right, just took the derivative of both sides, that was just to make sure for myself that was the right formula. 2

I: And how do you know v = r omega is the right formula?

T: Um, I just know it. First of all I had it on my sheet if I had to check it, but, mean we knew that, that's one of those basic formulas that you just kind of know. If you don't, I could have going through and kind of derived, but

I: You just kind of know.

T: Yeah, um, that's what I meant last time, there's just certain basic equations that you learn, and, you don't, I mean, you've seen the derivations, once or twice, and you just don't bother deriving them after that, cause you know that they are right. 3

I: Ok.

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[1] *Concepts*: Tony gives a conceptual explanation for a calculation procedure. It is not necessary for the coding, but it is worth noting that it is common for students to have misconceptions regarding the motion of the point of contact of a rolling ball or wheel. The idea was discussed in the lecture and in the text – Tony is not inventing it. However, although it is usually counter-intuitive, Tony does not appeal to authority but to a conceptual argument to justify it. Shortly later in our discussion of this problem, Tony shows that he understands the argument well enough to debug his solution.

[2] *Coherence*: Tony checks his memory with a derivation.

[3] *Coherence*, *Independent*: Implicit in this statement is that at some time it is important to 'see the derivations,' and that, if he felt it necessary, he could 'derive every equation.' Tony's statement about not deriving "every equation on every test" again supports reconciliations of indications I made by arguing that Tony's use literal memory does not necessarily imply a belief that knowing in physics means remembering.

T: If that makes any sense. If I went through and derived every equation on every test, it would take forever.

I: Ok, so now we got a equals mr squared

T: Mr squared yeah, so substituting the alpha in there, and evidently that's wrong, because, that's not what it should be. And actually, [unintelligible]

I: Ok, so this is wrong, $a = mr^2 \mu_s g / I$. [reading what Tony has written]

T: What I did from there is I said ok, here's the equation of the movement of the center of mass of the ball, um, ah, that's what my problem is, I think, maybe not. [pause] Maybe because, it's kind of rotating, ok, that's my problem, um, I just told you that at the instantaneous point the ball rotates around a point, I didn't do that. I have it rotating around the center here. So this should be, the torque should be this part of the gravity here, and then, that'd end up with a different, that would involve the angle here, and a bunch of other stuff. And then, ah, that would get, yeah, I [unintelligible] that's what I did wrong. I had this ball rotating around the center, when it's not right here, it's rotating around the point on the ramp. Cause I'm taking this as a torque, I should have taken the gravity of the torque, and [unintelligible] did the same thing, this would be equal to I alpha, and so on and so forth, and this I would have been changed, it would actually I plus m r squared, cause it's not at the center of mass. ***1***

I: So

T: I'm just talking to myself, I just realized what I did wrong. (ok) So then from there I went ok, here's the equation of the ball, evidently that's right, cause it, gave me a big circle, and I went from there, and figured out what the, the relation to the times would be, using, using these equations, which turns out to be wrong, but um, I went from there, and if this had been right, then the whole thing would have been right, but I made a mistake on how I did that, how I derived that. But I just remember the formula offhand, I would have, I would have remembered it, but that's the one, I had a cheat sheet of formulas, and I somehow missed that one on my cheat sheet. So I went ok, time to derive. Usually I do that, usually I do that reasonably well, I go through and I can rederive, if I forget an equation I can rederive it on a test, but, not this one, I just didn't pay attention or something. ***2***

I: So this, so, could you have done it the way you did it, and you just made a mistake, or did you, you said, you said, I guess I'm not sure, what did you do wrong?

T: Ok, um, when a ball's rolling, it's instantaneously rolling around the point that's stationary on the ramp, and the whole ball is kind of rolling around it just at that moment, and then it rolls over the next point, and so on and so forth, so actually point of the ball kind of goes like this, [gesture] stops for a second, stops for a second. And, when I was setting up my force diagram here, I was assuming it was rotating around the center, I forgot that it was instantaneously rotating around that point there, so then I set up my torques with, to figure out how these forces would make it rotate around the center, but I should have figured out how these forces would make it rotate around the point on the ramp there. ***3***

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[1] *Coherence*: Tony resolves the conflict, looking to understand what was wrong with his solution rather than simply dismissing the method. *Concepts*: He explains the mistake in conceptual terms, closely tied to his conceptualization of the calculations: "I just told you... the ball rotates around a point, I didn't do that... I had the ball rotating around the center."

[2] *Independent*: Tony is 'just talking to himself,' an indication that he is not only trying to explain the difficulty to me, but he is interested in resolving the conflict for himself as well. *Coherence*: Tony generally expects he can rederive equations on a test.

[3] *Concepts*: In answer to my question of what he did wrong in deriving an equation, Tony answers in purely conceptual terms.

I: Now, guess, I mean a standard method for this is to find, is to separate rotational motion from translational motion.

T: Right.

I: Is that what you're talking about.

T: Yeah, I'm trying to figure out rotational motion now, and the translational motion comes in when I start plugging these, r i, a equals r m alpha,

I: So are you that it would be incorrect for me to think of it as rotating and moving?

T: No, not at all, but my

I: I mean rotating on it's own axis.

T: No, it wouldn't be wrong, I'm just not sure, maybe you have to find a different, well, hmm, it wouldn't be, the way you said it, no it's not wrong to think of a circle as rotating on it's own axis and moving at the same time, but you have to add those two motions together, you got to do that, that could be a problem. [pause] Hmm. Well, I think, that wouldn't be wrong, to think of it that way, but I can't see how to apply that to the problem. I definitely see how to apply the other way to the problem.

I: And the other way to the problem would be to

T: Think of it as rotating around that point instantaneously.

I: Ok.

T: Cause, I remember, I vaguely remember what the formula is for a here, and looking at it [1]
that way would give me, the formula.

I: Ok, well let's ah, [flips pages]. Ok, how should I do this now? Um, let's do this, I'm going to give you a problem. I take my watch or a pen or whatever, and I throw it up in the air, ok? (mm-hm) And catch it. (mm-hm) Just after it leaves my hand, but while it's still moving up, (mm-hm) what are the forces on it?

T: Just after it leaves your hand?

I: Just after it leaves my hand but while it's still on the way up.

T: Huh, just gravity. Unless you're thinking of, by saying just after your hand you mean the force of your hand is still affecting it, but, just gravity.

I: Unless I'm saying

T: Gravity from everywhere, gravity from

I: You said, unless I'm sa, you said something about the force of my hand still affecting it

T: Well, if you mean, if you mean like, the instantaneous like where it's [gesture]

I: Oh, no no no, it's no longer touching

T: Like where it's halfway

I: my hand, it's no longer touching my hand

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[1] *Weak Coherence*: Tony solved the problem the way he did, because that was the way it was solved in the course, and he does not seem concerned to understand how to solve it as I suggested. This is an unreconciled coding.

T: Ok, just gravity [1]

I: but it's on the way up.

T: There's gravitational forces of various kinds, I guess. Mostly the earth. Well, let's see, hm, hm, hm. [pause] Just the earth, not just the earth's gravitation, but, earth's gravitation, sun's gravitation's pulling it along at the same time as the earth, (ok) and so on and so forth.

I: Ok, well, now, someone else will say, that, I push it, and I'm putting a force in it, and the force is making it go up, and after it leaves my hand, the force, there's a force up, making it go up, and

T: Mm-mm.

I: No. Why?

T: Um, while it's going up, it's still decelerating, so I mean, there's a force point down, it's causing it to decelerate, and then eventually stop, and then come down the other way. Like if you're in a bus that's slowing down suddenly, you still feel the force backward, cause you lurch forward, even though you're still going forward. [2]

I: Does the gravitational force stay constant throughout the whole, while it goes up, stops at the top and comes back down.

T: Well, if you consider it constant with this small space here. If you were to like shoot it way the heck up it would not be constant. (ok)

I: Um. We did, did we do the merry-go-round question? (yeah) And we did the ring problem. (mm-hm) Ok, so now I think only one more that we should do. [comments on finding paper, pen] This is a, two cliffs, or something, and this is a bridge, across. There's a 98 pound juggler, (mm-hm) with 3 one pound balls, (mm-hm). Do you know what's coming?

T: I can, no, no definitely.

I: Ok. These are one pound balls. [writing] Each one weighs a pound. But the bridge can only hold, exactly, one hundred pounds, it's a government made bridge, so that if this juggler just puts all three balls in his hands and walks across the bridge, it's going to collapse, because he and the balls together weigh 101 pounds. But he's a good juggler, and so he thinks is ok, well, what I'll do is I'm going to juggle, and I'll juggle so that I'll always keep two balls in the air, (mm-hm) so I'll be able to make it across the bridge. The question is does that work, does he make it across the bridge?

T: I seriously don't think so. Um, [pause] No, it doesn't. Because the, the weight, the um, if you can kind of consider the man and each of the three balls as like a system of particles, and it's um, they're exerting forces on each other, but you can, you always can ignore the forces on each other, because every time, say he throws up a ball, he is at the same time, the ball is at the same time pushing down on his hand. So, you're saying he keeps one in his hand at all times. (yeah) I don't think it makes a difference. Um, hmm, I don't think it makes any difference.

I: Well, so if he keeps 2 in his hand then what happens.

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[1] *No misconception*: Here and after my question to challenge, Tony responds "just gravity."

[2] *Concepts*: Tony explains conceptually that motion in one direction does not mean a force in that direction. His argument is an analogy to a very different kind of situation where the point seems more intuitive.

T: Yeah, well I was thinking about it in a different way, though, you see the whole, hmm. 1
Normally I would say, I mean, cause like a system of particles you can treat the center of, the gravity as being at the center of mass, which instead of being his normal center of mass would be up some, cause the balls' being there, but, he, when I think about it though, cause the force that's keeping him up, namely the bridge is only acting on one of the particles, so maybe that screws things up. I'm part overthinking it, however, but what the heck. Now I have to prove it one way or the other. Hmm. [pause]

I: Now what are you drawing?

T: Well, normally, ok a system of particles, you can just treat gravity as acting on the center of mass, and go from there, which will give this whole system a weight of 101 pounds, but, now that I look at it, [pause] ok, this would be an external force here, normal force, [mumbles]. Ok, ah, I don't think so, doesn't seem

I: You don't think so for this or for making it across?

T: For making it across. But, hmm, see, you know, if I get a problem like right or wrong, if I 2
get it right anyway, I can like justify it physically, this one I can't seem to do.

I: I'm not sure what you mean.

T: Well, normally if I'm not sure of a problem, I go and I think about theory, and I go ok, ok, now I see how the theory applies and I can apply that theory to the problem and say ok that makes sense, and the whole bit. This I, here, the theory is, since, ok, you can consider the man and the 3 ball's are exerting forces on each other, every time the man throws a ball, I mean the 3 ball's actually, the [inner/inter?] forces between the 3 balls are 0, from except for minor gravitational stuff, and the man is the, they're each exerting a force on the man, as they come down, he exerts a force on them, as they go up, and as
he catches, and as they come down he's exerting a force to stop them and they're exerting the
same force on him pushing down, as he throws it up, the same thing, he throws it up and they 3
exert the same force on his hand, and all that stuff, and all these forces, they interact and they kind of cancel out, so you create, because the force of him pushing up on the ball is the same as it pushing down on his hand, since his hand isn't, he isn't moving, the ball does

I: So those forces cancel out?

T: Yes, because they're equal and opposite.

T: So why does the ball go up?

T: Hmm? Because the forces cancel out, but they're acting on two different bodies, one is acting on his hand one's acting on the ball.

I: So what do you mean by cancel out?

T: Well, if you look at the man in himself, then you would see forces, but I'm looking at the whole system, (ok) and in terms of the whole system, the forces cancel out, which leaves, only external forces to the system will affect the system's center of mass, which in this case would be 101 pounds, but I wasn't sure, because the bridge will only be affecting one of the things,

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1 *Coherence, Concepts, Independent*: Tony identifies two different ways to think about the problem, in terms of the center of mass and in terms of an individual ball, and says he has "to prove it one way or the other." Tony had an answer based on center of mass considerations, but he thought it was important to check it against another way of thinking about the problem. In this way, he generated a conflict and then set himself the task of resolving it.

2 *Concepts*: It is important to "justify it physically," by which Tony seems to mean mechanistically.

3 *No misconception, Concepts*: Tony expects there is a reaction force on the juggler's hand. When I challenge, he supports this with conceptual arguments, distinguishing that the forces "cancel-out" in regard to the "whole system" but not in regard to "the man in himself."

but now looking at it, it still counts as an external force, the bridge, on the one man, it [typically?] also exerts on the other balls too, because it's pushing on him, and he's pushing on the balls, and so on and so forth. So, I'm going to have to say no, I'm going to have to go with theory, but now I still can't really, I'm trying to figure out, if the ball comes, I'm trying to think of it in terms of momentum. If he has 90 pounds, how many, the balls must be giving him more than 90, see it shouldn't depend on how high the ball goes, but in any case I would have to say no, because the gravity is acting on the center of force, mg, which is 101 pounds, and the normal force can only support 100 pounds, so I would have to say no.

I: Ok. In describing that you said I'm going to have to go with theory.

T: Yeah, I can't seem to, I can't justify it physically in my own mind, without, there's only, use equations on how high he's throwing the balls, on something that shouldn't matter, what it is it shouldn't matter at all. I can see, I can definitely see that as each ball comes down, it imparts momentum to him, which gives him an extra weight, but then, I don't want to go through and figure out, it shouldn't make a difference, how high or anything, and the way I'm thinking of that that does make a difference, so I'm just going to *1*

I: Ok, so give me an es, how confident are you in your answer.

T: Mm. [pause] This is the way I would leave it on a test, I'm confident in that respect, but I wouldn't be sure I got it right until after the test.

I: Ok. Is there anyway to get yourself to be more confident?

T: If I could like, justify this to myself physically, but I don't see anyway of doing that without going through and figuring out 20 thousand little equations and stuff.

I: So, ok, you, this is, I'm confused by what you mean by theory and what you mean by physically, cause, this is theory, but then

T: Yeah, this is just, I mean, center of mass, force, and that's that, but here I'm not taking into account any of the forces in between the things, I'm trying to figure out where these ball's will impart that force, obviously they would impart a force to him when he catches them and throws them up, but I'm not sure if that would ever be

I: Ok, so that's, but when you say about it physically

T: Yeah, that's I'm thinking of him actually throwing them up and catching them and stuff, I'm trying to figure out how his force is going to be over 100 pounds, to support him, but then also he has to support the 3 balls, I would think. Hmm. So I'm not totally confident about the answer, but that's the one I would stick with on the test. It makes sense to me, I understand the at the balls would be imparting momentum to him and he throws them and catches them and stuff, I'm just trying to figure out would that momentum be more than 2 pounds worth, and there's no way to figure that out without going through tons of, and it would depend on the mass of the balls, and how high he throws them and all that stuff, stuff that I really couldn't, stuff that shouldn't matter, and I would have to describe as constants, and there would be tons of things, and eventually you'd have to watch all the constants drop out somehow, that's what I would stick with on a test right there.

I: Ok. Um, let's see what we have left. About half an hour left. All right. Well, I'm going to turn this to a more general kind of conversation. Um, the question is, how much do you use,

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1 *Independent*: Tony adds here the phrases "justify it physically in my own mind" and "justify this to myself physically," indicating a sense of his own involvement in generating a physical justification. *Concepts*: Tony speaks of the possibility of using equations as a tool to help him 'justify the problem physically,' that is, to help him understand the mechanism. Because of his informal expectation that "it shouldn't make a difference, how high" the balls are thrown, he is reluctant to do that work.

just from the things that you've been saying, how much do you use, um, common sense kind of

T: I tend to use it as a check. 1

I: Huh.

T: I tend to go through, I do everything like theoretically, and what not, sometimes I use common sense to derive the theory, just to help me remember it, and then, but it always doubles as a check. If the answer is done, it makes sense, I know I've gotten it right. Usually I know when I've gotten a problem right, if I've gotten it wrong, I usually at least didn't know I got it right. There's, like, most of the problems on that test I knew I had right, and it was just like

I: Well, actually I want to look at that a little bit. On this one, when you were telling me how you did it, you said

T: I knew I had this one right.

I: You said something like, I can't remember you're exact words, you said something like you couldn't just do it, you had to do equations.

T: Yeah, it's not something I can look at and say, if I had a th, I suppose a really expert physicist type would look at that and scoff and circle his answers, but I need to, just to double check myself. This one here I knew pretty much instinctively. This one, I didn't know simply right off the bat, but as soon as I drew this here, as soon as I remember that energy is dependent on distance, I said yeah, I knew that. They were both ind of, I just had to kind of remind myself, they were both kind of obvious. (ok) Actually, after I wrote the equations down, this one was more obvious than that one.

I: Really.

T: Yeah, just a little bit, because, the energy mean, the whole I idea, this one you had to figure out, ok, since it's been going half the time, that means half the acceleration, therefore half the speed, and all that stuff, half the time of the acceleration anyway, therefore half the speed, therefore half the momentum, whereas here you went, oh, half the distance half the energy, once less step, but I was sure of both of those once I was done.

I: Now why should it be half the distance half the energy?

T: Hmm? Um, cause there we spent a long time in class proving that gravitational potential energy is based solely on height and not how you get there, like, if you have no friction, and you have a ball, and you let it go, it would roll down the hill and up the other side, exactly the height it started off with, cause here you have a certain amount of potential energy, and as it rolls down it coverts that into kinetic energy, and as it goes back up again, it's converting it all back into potential energy again, and ends right there, but the energy level is still constant, throughout the whole thing, and it basically it only depends on how high it is, not where it's going.

I: But now there's an example where, I would think someone's sort of common sense would say it should be, you know, if you take a longer way to get up there then it must take more energy. Wouldn't that be sort of a common sense kind of way to look at it?

T: Um, yeah, but that, I usually tend to modify my common sense during the year, cause it 2
also makes sense to me why, I mean it makes the, why I see that and I go Oh yeah, then it also makes sense to me why it takes the same amount.

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[1] *Apparent Concepts*: Tony speaks of common sense as "sometimes" useful "just to help me remember it," thus assigning it an ancillary, limited role. This is unreconciled.

[2] *Independent*: Tony thinks of learning as involving modification of his common sense.

I: So when you say you modify your common sense, what do you mean by that?

T: Um, I mean, common, it's not, it doesn't make much sense to, bring in one thing after you've seen examples of the other and stuff. I just, you know, if I think one thing and it's proved me wrong, and the other thing does make sense, which in this case it does, it makes sense that it would go up all the way, then I have no problem

I: If the other thing makes sense.

T: Yeah, like with the gyroscope thing, it doesn't make sense to me right off the bat, so I have to go question it, and, yell and scream and stuff. *1*

I: So now, how do you tell when it makes sense to you?

T: I don't know, just based on like past experiences with stuff I've seen or read or whatever, it just fits in, that's all I can describe it is it just fits in with everything else.

I: Ok. I realize I'm asking ah, kind of ah, it's difficult to

T: Word

I: Put into words maybe, I don't know

T: That one I was sure of [does a lot?] in class, basically.

I: And this was the other one, you said this one, and you said you just remembered.

T: Mm-hm.

I: Is that using common sense?

T: No. He did it in class, with almost the same drawing on the board.

I: So

T: So.

I: Who wouldn't use it,

T: Yeah

I: Ok, I would actually, if you could just explain then, a little bit more about why this works out the way it does. I guess because you said on the one where it was on a drain, I understand what you were saying with, you had it, the water's coming in so it's increasing in mass, um, if this were plugged up, so the water doesn't add any momentum, but it adds mass, and so if the momentum is supposed to stay the same, the mass goes up

T: Mass goes up, velocity goes down.

I: Velocity goes down. Now one way to look at that is I'm just using an equation, I'm just using p = mv, (right) right, and so I'm not really using common sense, I'm just looking at this equation, p = mv, if m gets bigger, v gets smaller, (right) is that, would you call that, would you agree with that, is that, you're not using common sense you're just using an equation.

T: Yeah, definitely. *2*

I: Ok. Is this a case where you need to, make it, you said you

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[1] *Independent*: "yell and scream and stuff" further indicates a belief that learning involves a personal process of reconciliation.

[2] *Formulas*: Tony agrees with my argument that using $p = mv$ in this context is not using common sense, just using an equation. The coding is based on my assumption, for one, that the equation expresses accessible conceptual content. For another, the application in this particular instance should also be intuitive. This is unreconciled.

T: Here I would look at the equation, you see, it's like I usually say, I go, common, not common sense, but I'm, just past experience would say ok use the momentum equation, mv, mass is gong up, velocity is going down, so velocity has to be going down, and then as a check I would say, well, it's getting heavier, so it's also getting slower, and that's kind of what went through my mind. I use the theory first and I just check it with the common sense. [1]

I: But in this case it's not getting heavier, because the water is draining out.

T: Right.

I: So now where's your common sense?

T: Ok, this took me a long time to figure out. Ok, um, thing that you say, like, the tank and um, instead of thinking it as a tank and the water, think of it as like if, two things are tied together and they're thrown, and, um, without, say there was like a timer on one of them so that it would all of a sudden, no, that's impossible to do, um, hm hm hm. Just think of it as like, as two particles going along, with some momentum mv, (ok) and all of a sudden they separate and one drops, and the other one keeps going, well, the total momentum of the whole system has to stay the same, [gesturing throughout this explanation], (right) when this one drops, it's going to actually drop this way, (right) but it has some momentum, ok it should be 2mv, say they're equal, (ok) um, this is going to have some momentum mv, and it will drop off, and this will have some momentum mv and keep going. So the 2mv you start off with, when this thing drops, it stops, when these rain pellets hit the ground they're going to stop, but it actually doesn't make, does it make a difference or not, um, yeah it does actually, [mumbles] [2]

I: You say this took a long time to figure out.

T: Yeah, I still have a little problem with it.

I: When you saw this in lecture, you spent a long time afterwards trying to figure it out.

T: Yeah, I still don't completely understand it, to the point that it makes sense, quote unquote, but ah, I understand the theory behind it, I mean because when this thing stops, the total momentum of the system has to stay the same, so it's going to have half the speed as it did when they were both going along.

I: But, when this one stops, so when the one that, I see, this is analogous to the rain, when this one hits the ground

T: Right, that's the point, that can't happen

I: Then this other one slows down.

T: No, see, that's the point, that's the part I have a problem with.

I: But the whole momentum of the system has to stay the same.

T: Mm-hm.

I: What's the system?

T: Hmm? The system here is the cart and the rain inside it.

I: Is the ground part of the system?

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[1] *Apparent Concepts*: Tony describes only a rough comparison between a result and a general expectation. As well, he states this as an example of a general practice. This is unreconciled.

[2] *Concepts*: Tony constructs an analogous situation, abstracting out what he considers the essence of the problem: two connected masses separating. This reasoning is informal and mechanistic.

T: No. That's the part that bugs me. There's no way I can, [unintelligible] force it to slow [1]
down. So therefore I'm thinking about something wrong here, so thinking about it, it should slow down right as they separate, now how can I prove they do that?

I: And it should slow down, why should it slow down as they separate?

T: That's what they're doing here.

I: Ok.

T: [long pause] When I look at, I can't, it doesn't make sense at all, because there's no reason that these two things attached here are going to go slower than these then they are separated, that doesn't make any sense. Then again, in that case it's still moving forward, but so would that. [long pause] Hmm. Well, I mean, I see it in terms of this one here, you have like, um you have like, you know, say five particles, each moving along at a velocity v, and you have, well, that's doesn't make sense either, because if you have, like here is v 6, and [unintelligible] one of them drops off, and momentum is 5 mv, but the mass is 5/6 of what it was before. Wait a minute. The mass of this is 5/6, ok, you have 6 mv, and one of them drops off, the total momentum is going to be 5 mv, I don't know why that would make it go slower. [mumbles] [long pause] Well, all I can think of here, is looking at it this way, I have 6 particles, and one of them drops off and starts to
fall. Um, the momentum beforehand is 6 m times v sub 0, whatever, and then the momentum afterwards of the total system is going to be 5 m v sub 1, the velocity these are still going, plus the mass it dropped off, times v sub 2, and then since v sub 2, it is going faster because it's dropping, I, I, that may have a major part of the problem actually, because

I: It's going faster because it's dropping.

T: Yeah, if it's going along this way, and all of a sudden it'll keep going this way, and it's also adding on a longer and longer (ok) downward vector so the total velocity is going to be much faster, and I'm starting to think that has a, that has a big part of the problem, I'm not saying this, because it is going faster it's making this slow down, but as soon as it separates it's taking this little bit of momentum with it here, and you can kind of ignore that in terms of the car, and what that goes a little bit of speed at, and that's the only way I can think of it, cause in looking at it, looking at it at a total system point of view, this is right here, the total momentum beforehand has to equal the total momentum after. (ok) And, ah, I want to take that back, no it doesn't, cause there's a force on one of them. Damn. He never explained this problem very well in class, he never did any math behind it, that's what's bugging me, because I can't, I mean he never did any math behind it basically, he just said ok, well the droplets kind of take momentum with them, and I was kind of saying well what, and I still, I have to see, easier tot see a mathematical thing so I can try to make it make sense. [Sigh] This is wrong, because there's a force there.

[Continued discussion of the problem. The audio-tape stops here, missing about 5 minutes of conversation. Tony and I try to remember what was missed. Tony had compared the situation to that of a ballerina twirling and holding bricks in her outstretched arms: if she drops the bricks, should she slow down.]

T: If I assume this problem is truth, she should slow down, but if I ignore that that's truth, I [2]
don't know what she would do. I would, my first instinct would that she would stay the same, but that doesn't make any sense either, because, it kind of makes sense if you have like 2 heavy things and you're spinning around you drop them, you would slow down, because you have the lack of those two big things to drive you along so to speak,

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[1] *Concepts*: Tony considers a mechanism for what causes the car to slow down essential to his understanding. *Coherence*: He expects that the whole-system explanation and his sense of mechanism should agree. *Independent*: Tony engages in an extended, self-directed effort to make sense of the situation.

[2] *Concepts*, *Coherence*: Tony expects the different examples should all give the same results. He is looking for a situation that should be analogous but in which the outcome is intuitively clear.

I: And how would they be driving you along.

T: Well, just with momentum. If you have something big and you're twirling around it's hard to stop, but if you let go of it, it seems like all of a sudden you would slow down. I mean, instinctively the ballerina makes more sense to me than this, I could easily see a ballerina with two weights spinning around, and then drop them I could see her slow down.

I: And this is the one that doesn't make sense to you.

T: And this is, yeah, I couldn't explain in the ballerina's case either, it just kind of, I don't know.

I: Ok. We went through some other stuff too, but we're not going to get it back, so let's go on to, um, there's a question, there are a couple of things I'm interested about, about how people work, in this course, and one question is how much do they use their common sense, and sort of two sorts of extremes are, one person might say I don't use common sense at all, I combine equations, and another sort of extreme might be, everything's in this, everything's common sense, this equation's common sense. Where would you be?

T: Somewhere to the second part, towards it, cause mostly, I mean, if, normally when I do a problem, I will stick with equations, but each equation that we've, I've derived from what I consider common sense. Each equation to some extent make common, makes sense. *1*

I: This is back to those basic set of equations.

T: Yeah.

I: Is a basic equation one that is

T: Common sense

I: sort of common sense, or modified common sense, as you said before.

T: Basically, that's true, those really, basically I just consider common sense, and the more advanced ones that I can't physically picture, then I just accept that as truth and use them, and then, I mean you can usually look at an equation, and it may not, maybe not physically picture, but you can look at parts of it and say, ok, if I were to raise this, this would happen, and if I were to lower this this would happen, but usually with me, um, you know, except for, like $p = mv$, that's a common sense type equation, I would just, I would go through, I just do, I'm totally theory, and then at the end it has to make sense, I use it as a check, more or less. Like the problem here. But usually if I can't figure out, is this problem right or not, what's the answer to this problem here?

I: This one?

T: Yeah.

I: Um, at the very end let's do that.

T: Ok.

I: Um, next, do you have more to say about that?

T: Yeah, like here I couldn't decide between, like common sense I couldn't picture it, but I have the theory there, so I would go with the theory.

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[1] *Concepts, Independent*: In answer to my direct question, Tony indicates he believes physics is mostly common sense. He elaborates on this to say that, while he "normally... will stick to equations" in solving problems, "each equation... I've derived from what I consider common sense." His statement that *he* derived each equation from common sense indicates *Independent*. This excerpt also supports earlier reconciliations in which I argued that Tony may make use of the formalism as a tool, and still believe it represents conceptual knowledge.

I: Oh, on the juggler, you didn't have a common sense answer, you

T: No,

I: couldn't picture it.

T: That's the problem I had with it, I couldn't use my check so to speak, I couldn't, I had the theory there, but I *1*

I: And you were talking about, making a physical picture, is that what you meant?

T: No, I meant doing it a different way, like another mathematical way, like you go through and prove the thing and decide and, this is the simple way, this is a lot more pure theory than it would be if I went through and figured out all the momentum of the ball hitting the hand, and all this stuff.

I: But that wouldn't be common sense either.

T: Hmm? More so then, I mean I would be able to relate to it a little bit more firmly, because it's more basic, so to speak. But um, here I had the theory done, but I couldn't, I can't check it with my common sense because it doesn't click, so to speak, so I'm unsure about it.

I: Would you say with the more, finding momentum, each one, that's more at a level where you can see how it happens?

T: Yeah.

I: So that, it's more basic, so that's (yeah) closer to common sense because you can see each (right) little piece of it, (right) in a way. And here it isn't really talking about what's happening, (yeah) it's talking about the end result. (right) Ok. Um, that's my

T: Interpretation of what I was trying to say.

I: Interpretation of what you were trying to say. Is that, does that seem right?

T: Yeah, that's exactly right, it's like I said before, a little bit more basic.

[I ask how much Tony accepts what he is told In response, Tony discusses incompetent TAs. Closing comments.]

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[1] *Concepts*: This is essentially the same indication of *Concepts* that came up earlier in the discussion of the juggler problem, Tony's expectation of a mechanistic understanding. It is worth noting in addition Tony's use of the word "check," which he here elaborates to mean finding a mechanism, and not just an apparent association.

Subject: Daniel

<u>First interview - 2/24[1]</u>

[General background, Daniel comments that he "did not like chemistry at all."]

I: You didn't like chemistry at all, what was

D: It was, chemistry, that chemistry 1A course was too deep for me, it's more, I'm beginning **2** to ask myself why I'm learning this thing because, towards the end of the year, we're learning about, like what angle the benzene ring bonds at, (uh-huh) and, I just, at the moment, I was so fed up with chemistry labs and I began too ask stupid questions like, why, who cares, I mean, when am I going to use this later on in life, while in physics if I think about being a civil engineer, there is just more light at the end of the tunnel than I'm going to use what angle the benzene ring bonds at. (right) That's why I like physics more. And, chemistry was just so much harder to grasp for me because, you can't see, you can't picture the atoms going around the molecules, while in physics you can sort of see, ok, if you throw the ball up and it goes down, I can sort of picture that, yeah, this one's going to go faster as the other one goes down, you know that one where he says, throws two balls up, I can see that, I can picture that in my mind, while in chemistry I can picture nothing in my mind, because you mix acids together, all you see is um, mixtures, fluids mixing together while in physics you can actually see things happening, (huh) and you can, I can picture things happening, that's why physics is just so much easier for me.

[Details about Daniel's background and other courses. He was taking a self-paced Calculus course (the same one as Tony), "getting a borderline A" and "not satisfied with that." Discussion about the structure of the self-paced course, convenience. Several minutes of discussion on Daniel's never having graduated from high school: he left as a junior, because he was "bored," getting very high grades without much effort, and Berkeley does not require high school graduation. Back to his other courses, he is also taking economics, which keeps his "options open."]

D: And, I could have taken physics 8A, but, I want to take physics 7A, [laugh] because, I mean, um, I've been working with this one girl and helping her with her physics 8A homework, and it's just so much easier the way you approach 7A problems and 8A problems. Everytime I see 7A problems, I think more complexly, and then, I started doing 8A problems, and I still think in a very advanced, more advanced level, and I don't get anywhere, while in **3** 8A, basically, all I think about is basic things, you stay down in basics, like v = v oh plus a t [$v = v_0 + at$], and that will get you somewhere, and x max will not get you much. [laugh]

I: And what won't get you much?

D: The formula for x max and z max's, (ok) in 8A it just doesn't get you very far, I found out, it's, it seems like, I expect the problems to be hard, while, when I finally think about it and look at it it's simple.

I: Huh, well that's really interesting.

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[1] There were several logistical problems with this interview. I had been planning to have Daniel go through his lecture notes and solve some homework problems. However, I did not have a copy of the textbook (suddenly repossessed by the physics department and unavailable otherwise), and Daniel neglected to bring any of his course materials. As a result, most of what happened in the interview was impromptu.

[2] *Apparent Concepts*: Daniel explains a role for informal knowledge in apparent, direct experiences that correspond literally to the content. He does not see relevance to constructed 'pictures,' such as are used in chemistry to conceptualize reactions.

[3] *Formulas*: Daniel describes the difference between the two courses as a difference in the formulas that apply.

D: And I don't know how to, every time, every time I help her I write this big thing about, first thing, think basic, because, that works, and if they, [unintelligible] yeah, that's really simple, there's this big difference, so sometimes I really wonder if I should have taken 8A, but after I studied the midterms for midterms, preparing for midterms, I was like, no, this is not that bad at all, because I understand this stuff and I'm learning much more than she is. (right) So that's why I think taking 7A is not a big mistake.

I: Huh. But, that, thinking basic doesn't work in 7A, it works in 8A.

D: It doesn't get you for, very far in 7A, because the problems usually are so much more advanced then the 8A problems.

I: Ok, ok, wow. So this is good, you help out somebody in 8A. Is this, is this a formal situation, or just a friend

D: Just a friend.

I: Yeah. Um, so you've already partly answered it, but how's, how's 7A going, how do you like the course, the lecture, the

D: I don't like the lab, but I never really liked labs that much.

I: Why don't you like the lab?

D: The first lab was really just a bad experience for me, it's, it's frustrating, especially with the um, kinematics lab with the friction problem, (mm-hm) I mean,

I: What was the friction problem?

D: It's where you slide a block down the ramp, (uh-huh) and taking into account the friction, and you time it, distance, velocity, (yeah) taking into account friction, and you, at the end you calculate the mu k or you calculate the g.

I: So the idea is to find mu k or to find g

D: Or g. And it's just, ah, the inconsistency you come up with problems, that, ah, you cover 20 cm in 5 seconds, while you cover 10 cm in 10 seconds, (uh-huh) with the same angle and the same procedure. It's just so inconsistent, that, my data curve was just [gesture], (huh) and, I don't deal with those very well.

I: That's inconsistent, 5 cm in 5 sec?

D: No, 20 cm in 5 sec, 10 cm in 10 sec.

I: Oh, oh, I see, and how did that happen?

D: It's just that, when you slide the block down the ramp, (uh-huh) in the 20 cm it encounters less friction from, other, like just from one side, while I think in the 10 cm one, maybe it drift off to the side and like starts friction with two sides, [unintelligible] forces on two sides. It's just so inconsistent that I don't see, if the methods are this poor, (right) why should I do it, it's a waste of time. Because the datas I'm getting is zero, I just want them to let me do problems and I get so much more out of it, (right) then starting to, trying to plot a data curve for something that's, obviously has no use, or I don't see the use of. (huh)

I: And is this something, that, do you have ideas for improving it, or do you just, or would you just as soon scrap lab entirely, or

D: I, well, in some cases the labs are helpful, because, one thing I found about labs is that, **1**
when you relate labs to lecture closely, and do problems that are closely related to each other

[1] *Concepts*: Daniel says that labs are useful to help one understand the procedures based on experience. Reconciled: The statement is also consistent with *Apparent Concepts*: labs provide literal connection to the formalism.

like finding, well, not simple things like finding velocity and finding distance, but doing similar problems in lab than what you're doing in homework problems, (right) you see things more, and you're doing it, you're actually doing the procedure, you're like, oh yeah, in order to find this, I have to do this, this, this, and this, and when you take the midterms, or test in high schools, I can picture that in my mind and it helps me understand the procedure will work, so I don't have to memorize anything, I just, think back to the labs, and saying, yeah, this is what you're supposed to do.

[Brief discussion on high school labs, then several minutes discussion about teaching assistants. Daniel complains about their being disorganized, late to sections, unprepared, and comments on their sloppy attire. Discussion moves to lecturers, at similar level.]

D: And some of our lectures are usually, put me to sleep, after an hour and a half lectures, they usually put me to sleep, but, he's done a very well job of keeping me awake, you know, just like, staring at him, trying to concentrate. And it's hard to concentrate for an hour and a half, but so far he's doing a good job (uh-huh) making me concentrate for an hour and a half. (All right) Well, at least so far the course hasn't been as scary as others described it to me.

I: Oh really.

D: Because, oh physics [unintelligible] that was the hardest physics ever, and I, (uh-huh) So far it's not that bad [unintelligible]

I: Ok. How do you like the book?

D: [pause] So far, I don't have that much complaint with it. I don't, so far I don't have any complaint that I can remember. Sometimes it's kind of vague, but, it's not something that [unintelligible] for asking the TA. It's not that bad. I can't really say, because there's nothing to compare it to. [unintelligible] physics 8A book, maybe the content is good, but [etc on the way the pages are set up - layout criticisms] also, from a 7A point of view it needs more depth.

I: All right, well let's see, ordinarily, I forgot to tell you, but, if you could bring lecture notes, and we go through, and flip through, you tell me what happened in lecture and what you got out of it, but, you took a midterm yesterday, how did that go?

H: When I took the midterm I thought it went well, I really prepared for this midterm, because, from my experiences last semester being cocky out of high school and everything, (yeah) midterms was simply a living nightmare for me, especially I prepared a lot earlier.

I: How did you prepare, what did you do?

D: I read ahead of time, I read like, last week, (mm-hm) like early, mid last week I start reading all the, flipping through pages from chapter one, chapter two, and I flipped over the pages over and over and over again, until the weekend, (mm-hm) until I convinced myself that, yeah, I understand all this. And then I began to do problems, and I just did most of the even problems at the end of each chapter, (mm-hm) until I think I have the grasp for it, even to understand the concept about, because so far what we've been doing is just basically breaking down, um, breaking down, you know, like with forces problems, you break it down into the free body diagram. (mm-hm) Free body diagram, once you get the diagram, everything is pretty much, everything pretty much makes sense to me, (mm-hm), that's what I've been practicing is doing free body **1** diagrams and everything, just doing lots and lots of problems, and really concentrating on it. So I took the midterm, and I thought it went pretty well, because, I finish early, with checking and everything, (hm), and, it's not after I get the, um, answer sheet back that I realize, like, wow, he did some of these things differently, [laugh] than I did.

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[1] *Coherence*: Daniel describes free-body diagrams as a core idea. Reconciled: It is consistent with *Weak Coherence*, because free-body diagrams were stressed as central repeatedly in the course.

[Discussion of Daniel's score on the midterm, his estimate of what his grade will be. We move to a homework problem Daniel had solved in an earlier assignment: A car starts out moving on a horizontal ramp over a row of parked cars. The ramp is 6 ft high. How fast should the car be going to jump 80 ft?]

D: Well, first of all you find, um, distance it um, the time it will take to drop the 6 feet, (mm- 1
hm) so, first just time it takes to drop 6 feet, and use that time, plug it into the x max equation which is, v oh squared, sine two theta over g. 2 v oh squared. No. Well, something like that. And, basically, wait, you can't do that, can you? [pause] Oh, when you find t here, you can find the velocity at the end, and then the initial velocity you can use the cosine, um trig functions, it's going to go here Somehow, I'm lost on this one. They give you 6 feet, they give you 80 feet, and find the time it takes to drop from here to here, and then, well, [laugh] if you, this is so simple, if you find the time from here to here, this is 80 feet, then you can find how, um, the velocity, um the x component of velocity, right? (ok) And if you know the x component of velocity, and, you know this angle right here, then you can find the initial velocity.

I: Ok. Want to do it?

D: [laugh] Ok. I'll give it a shot. 8 feet, I mean 6 feet, is this distance. One half a t squared. So 2
12 equals a t squared. t is square root of 12 point 2 [12.2]. distance is 80 feet, equals velocity times time, um, and so velocity is 80 over square root of 12 point 2, and that's velocity in the x direction. So, if v x equals v oh cosine theta, theta is equal to tangent s, arctangent, 80, then, [unintelligible] v oh is v x over. cosine, 6 over 80. So that's how I think I did it.

I: And you were solving there for the velocity, of launch.

D: Initial.

I: Ok. And this was the false start, or, you never used this.

D: I never used that.

I: Um, all right. Where does this come from? 6 equals 1/2 a t squared.

D: Distance equals 1/2 a t squared. That's the distance formula, isn't it? 3

I: Um, yeah, I guess so. Why is that the distance formula?

D: It was derived in the lecture, for one thing, and, I've had that ever since seventh grade and it just sticks to my mind. [laugh]

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[1] *Formulas*: Daniel describes pure symbol manipulation.

[2] *Formulas*: Daniel has solved for the time in the air by finding the time it takes to drop 6 feet:

$$12\text{ft} = \frac{1}{2}\,\text{a}t^2.$$

In this, he has made an error of dimensions, using 12ft and $a = 9.8\text{m/s}^2$, but the conceptual structure is correct. He goes on to find the velocity of launch:

$$v_0 = 80\text{ft}/\sqrt{12.2}\ \text{sec, and then}$$

$$v_x = v_0\cos\theta,\ \text{where}\ \theta = \text{arctangent}\ \frac{6}{80}.$$

Daniel's error here is a further indication of pure symbol manipulation. The problem statement says that the car has only a horizontal component of velocity initially, but Daniel describes using the horizontal component to find the total. He uses the spacial angle, between the horizontal and the spot where the car must land, as the angle between the initial total velocity and its horizontal component. Thus he is clearly some distance from having an informal understanding of his solution, because, as he has it, the initial velocity points directly from the starting point to the finishing point – right through the parked cars.

[3] *Weak Coherence, Formulas*: To Daniel, $d = (1/2)at^2$ is something to memorize There is a derivation, but he is not concerned with it. He knows the formula because he remembers it.

I: Seventh grade? Really? (yeah) You had 1/2 a t squared in 7th grade?

D: I think so. I've grown up with [laugh] 1/2 a t squared, I can't get it out of my mind.

I: All right. How about this, where does this come from?

D: Um, I just learned that this semester. (uh-huh) That's from the book. 1

I: Ok. And the book just provided it, or did they figure it out, or

D: The book just provided it. I think they proved it too, but, I'm not sure.

I: All right. Another problem. Let's see. They had to take my book, so I don't have my book either. Um, so I'm remembering these problems, there's one, that was, ah, I think it was, a pulley and a block here, and a block here, is this right, and now, I don't remember what their masses were. Let's suppose, why don't we just make this one, or was it on a slant?

D: Both of them are

I: Oh, you got asked both of these.

D: They are both [unintelligible].

I: Ok, well, let's do this one. 4 kilograms and 2 kilograms, and find the acceleration of this block. [The two masses are connected by a cord; one is suspended, and the other rests on a surface. The cord passes through a frictionless pulley.]

D: This has always been a confusing problem for me. First thing I do is obviously F equals ma, 4, 9.8, approximately 40, so the force going down is ten, so the force going this way is also 10. And the ten Newtons of force will have to accelerate 6 kilograms of mass, so

I: So, this was 40, I'm confused, you said, F equals ma, 4 times 10 is 40, oh, so all right, the force here is also 40, ok [D switched the 10 to 40]

D: And since this is accelerating 6 kg of mass, then that's about, just a second, then, you take, uh, I guess I don't have to account for friction in this problem, huh?

I: No.

D: That's how I did it.

I: So you have here 40, is this, is this a force of 40 acting on 2 kg?

D: No, on all.

I: On all. Ok.

D: Because, it's pulling, let's isolate this part. (ok) If you want to look at it this, (ok) this is the only force that is going to be acting on the whole system, is just gravity pulling this part, (ok) gravity pulling this part will generate a force of 40 Newtons, but it's not only pulling this part, it's also pulling this part. So, you have to have 2 kg of mass, that's why it's pulling 6 kg 2

I: Ok. Great. Another one. This one is from the questions in one of the sections.... The question was this. When you're standing on the earth, your feet exert a force against the earth, so why doesn't the earth accelerate away from you?

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[1] *Weak Coherence*: It is reasonable to Daniel that the book would just provide the formula.

[2] *Concepts*: Daniel gives a conceptual account for his calculation. Reconciled: It is consistent with *Apparent Concepts*, because the correspondence to conceptual knowledge could be superficial.

D: Because the force you exert on the earth is so insignificant compared to the mass of the earth, that, even thought the ma, um, there's gonna be an action reaction, you exert a force on the earth, (mm-hm) the earth exert a force on you, since the earth's so big that, the little force it exert on you is enough to keep you, enough to make you walking, well, the, however, tens of Newtons you exert, the earth would, budge the 6 times 10 to the 24 kg mass of the earth. It wouldn't, um, it's just um, F equals ma, um, this is um, for instance you're exerting x amount of force, 100 Newtons of force to the earth, well, 100 Newtons exerted on you would accelerate you somewhere, because you probably weight more or less 100 kg, that will accelerate you 20 meters per second. (mm-hm) But if you plug in a humongous number here, then the a is very small. That's why it doesn't accelerate from you. [1]

I: But it has a, what if I said, but it has a very small acceleration, it accelerates, but it's just very small. Is that what your answer is?

D: Mm hm [Yes].

I: So I'm standing on the earth and I apply a force to the earth, and it has an acceleration, it's just a very small acceleration away from me.

D: I would say so. If I really think about it, the number is going to be ten to the negative something, (uh-huh) and there's going to be other forces cancelling the force I am applying to it.

I: Oh, there are?

D: I would

I: What

D: Well, um, if I am walking, I am applying a force up this way, (mm-hm) sure somebody else is going to be walking a different direction to me applying a force that way. [2]

I: I see. On the other side of the earth

D: [laugh] Tell me if I'm wrong!

I: Oh, I should say, I didn't, this is something I left out. Part of the idea for me, is to just find out what all of the students think, and it's very strange to ah, not to tutor, but I have to be careful not to tutor, because i don't want to find out, I want to find out what you're getting from the course and only from the course. So

D: This is not what I'm getting from the course by the way.

I: Oh really.

D: Halfway. It's just that, this is one of things where physics is so much easier than chemistry because, physics you're dealing with, um, physical things that you can see, (mm-hm), so, the only thing that I'm applying from the lecture is F equals ma and the action reaction thing, (right), while, my theory about I'm sure somebody else is going to be giving an opposite force [3]

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[1] *Concepts*: Again, Daniel gives a conceptual explanation. Reconciled: It involves concepts that could easily be apparent. In fact, Daniel's response shows a common misconception. His account neglects the attraction of the earth toward himself. He says that the earth does accelerate slowly away from him.

[2] I note a misconception: Daniel speaks of motion associated with force.

[3] *Pieces, Apparent Concepts*: Daniel is aware of different levels of knowledge, but he describes them as separate pieces with separate sources, only conveniently related. There is what he knows "from looking out the window," which he applies as apparent informal knowledge, and there are the formulas he knows from the course. *Independent*: Daniel speaks of relying on his own sense of the situation. Reconciled: What Daniel says may reflect only his confidence in his familiarity with certain content, rather than his conception of learning.

going that way is just something I [unintelligible], like, yeah, I'm going that way, gonna exerting a force that way, picture somebody else going to be walking that way and exerts a force this way and it will cancel. [laugh]

I: Right. Ok, so that's, um, I lost what, you said there's, this is not something from the course

D: Not fully. (uh-huh) My explanation for this thing is not fully from the course. I mean, I took, I applied some of the things.

I: And now where is the rest of it from?

D: From looking out the window and thinking about it [laugh] .

I: Ok, ok. So now let's see, I have another one. [pause] Oh, I don't know what, all right, let me think for a moment. [pause] Well, let's do this, um, [pause] so let me give you, [pause] terrible. Well, all right, another question. Somebody, a friend of yours is thinking of taking the course, and has never taken physics before, asks you what's physics, what do you do in [1] that course. What's your answer?

D: What do I do in that course? (yeah)

I: What's, what's the course about?

D: Well, that, that's not usually, [laugh] first thing I would say is, um, are you taking 7A, or you should take 8A, that's what I'd first tell him. [laugh]

I: Why is that?

D: Because even though I didn't learn anything from my high school,uh I got some background in physics at least, I have some idea what physics is like, and I did learn some really, really basic things, [unintelligible] I just realized that. If I really put my effort into high school physics, 7A would have been so much easier, because I would have gotten such a big head start. So if somebody asked me, so far, what I learned in the course, I would say everything I learned in a whole year of high school, and a lot more. (mm-hm) Um, [pause] such a strange question, ok. Um, it's like I'm rattling out the syllabus to you, ok, we're learning vectors, vectors addition, vector multiplication, dot product, friction, forces, um [pause], enhanced friction forces [laugh].

I: Enhanced friction forces?

D: Oh, well, it's like blocks, [unintelligible] it was like, not merely just sliding a block down the ramp.

I: Oh, oh, oh, blocks on blocks.

D: Yeah, it's like, blocks on blocks and other things like that, pulley problems, and just, and multiple pulley problems, which is even harder. (right) One pulley here and another pulley, and it, here (right) work problems, work energy. What's interesting would be, conservation of energy, (mm-hm) cause there is just so much things that you can, that you have to think about while, so far [unintelligible] conservation of energy. Um, kinematics, motions in one directions, motions in three dimensions.

I: Ok, well that's a pretty, a, that's a pretty complete answer. All right, I've thought of something else to do. Circular motion, and something's moving in a circle, it's going around in a circle that way. (mm-hm) What's the acceleration of the, what's the acceleration of the object?

D: v squared over r.

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[1] I note an interview error. Because I was improvising, I asked a question I had not meant to use, because it seems likely to create a context fairly distant from the course.

I: Ok, where's that come from, and which way is that, which direction does it point?

D: Which direction does it point? (yeah) Good question. This is [unintelligible]

I: This is what?

D: Just the problem I need to answer [laugh], cause I don't know, I'm not sure about the 1
answer. Ok, I remember the ferris wheel problem, this is, um, Um, at the top, when you're going, at this point, in a circle, (mm-hm) [unintelligible] in the ferris wheel, the force of gravity is going to be pointing that way, while, I think the acceleration is pointing that way, so you're actually at top here, experiencing less weight, I'm not sure. But in the bottom I know for sure that

I: Here

D: It always diff, the acceleration always points away, from the circle.

I: It always points away from the circle.

D: [pause] [unintelligible phrase] like, comes away, [unintelligible] think about it, this is going 2
to confuse me more and more. This is one of those things that always confuses me in this, centrifugal and centripetal force, (huh) cause I always forget which one goes which direction. But I think it points away, the acceleration does. So,

I: What's v squared over r?

D: That's the centripetal acceleration is v squared over r. Something I remember, from the 3
book.

I: Ok. v squared over r, could you figure that out, or is that just from the book.

D: Acceleration is v squared over r. Can I derive that. [pause] No.

I: Ok. So let me. let me ask, you have, ah, centripetal acceleration is this way,

[tape ends]

I: from the course, or is this, when I asked you, which one was it, this one, (uh-huh) and you said, well, this isn't really from the course, this is ah,I get half of this from looking out the window, (mm-hm) kind of thing, with this, is this from the course, is this looking out the window.

D: This is from the course, that's from the course. For sure. Cause, I, I try to, um, look at, if I 4
look at this problem, (uh-huh) I know that, over here, the acceleration has got to be pointing down. I learned that from the course. If I forget, I can always remember going down the roller coaster and feeling the um, e, the extra gees when I'm down there. So that, and so, well, I don't know, you decide but that's how I remember it, (ok) But up here, um, I'm confused because, I know that,

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[1] *Independent*: Daniel speaks of the question as one he needs to answer, and he begins to try to do so. Reconciled: Daniel might mean simply that he needs to know the answer, not that he needs to construct it. *Concepts*: Daniel makes a conceptual argument. Reconciled: The use of informal knowledge is consistent with *Apparent Concepts*.

[2] *Pieces*: The phrase "I always forget" implies that the distinction is something to know by memory. There is also the misconception here that force/acceleration in circular motion is outward.

[3] *Weak Coherence*: Daniel describes $a = v^2/r$ as a formula one should know, but it is not important to know why it is so. That he supplied the "can I derive that" makes this an indication of *Weak Coherence* rather than of *Pieces*.

[4] *Apparent Concepts*: Daniel says that experiential knowledge serves to help remember the formal knowledge. As well, the application is apparent and a misconception.

if I go on a roller coaster, and then at the peak, I know that my body's sort of pushing up a little bit instead of down, so I'm feeling less g, but somehow in the course I have, um, I remember doing, well, trying to understand one of the problems with the um, pilot, (mm-hm) the one that's going on a loop, (mm-hm) and, at the top of the loop, his um body presses against the chair, so, that, for an [in?]experienced pilot it seems like the sky is the ground and the ground is the sky, it's like the whole thing's reversed because, his body's pressing up against the chair, (mm-hm) and, that way, yeah, so it is pointing upward, because his body's pressing up against the chair, and that's the acceleration force up there. And, if you go on a roller coaster, at the dip it seems to be going up too, so that's why I think it's going upward. 1

I: Ok. What if this is not, ah, so you, you have gravity pointing down, what if this is not something moving in this kind of a circle but in this kind of a circle, so a horizontal circle.

D: Ok, horizontal circle? (yeah) Velocity's always pointing tangent to the, um, circle,

I: Ok, and you know that

D: Well, from the course and from, well, if you ask me a question and then you try to explain it to me, ok what happens if the string gets cut off over here, (right) then, well, I can picture that it's just going to go straight, that's

I: A lot of people picture that it's going to go out. A lot of people, when you ask them that question, and the string breaks, say it's going to go outward, in some direction.

D: That's why I think it also [laugh] half, part of, partially physics. I learned that partially from physics and partially from experience, (uh-huh, ok) and I applied some of my physics to it. Um, centripetal acceleration, and centrifugal acceleration, I always get those two mixed up. I know that one of them points toward the circle and one of them points away from the circle. And, I think centripetal points inwards. Somehow, that's from, what somebody's, when we're studying physics together, someone told me, I think, centripetal goes in, but I grew up thinking centrifugal goes in, so I'm confused about that one. (ok) [unintelligible] 2

I: Ok. That's fair. I think we need to stop.

<u>Second interview - 3/3</u>

[Greetings, friendly chat, then turn to going over the midterm. The first two problems Daniel considered easy: they had been done in lecture, so "it was just a matter of remembering," and, moreover, they were "very easy to visualize...very physical." One of these he misread. The third problem concerned a film, played backwards, of a rock falling. It asked whether the acceleration was up or down, and Daniel found this question ambiguous. We move on to the next question.]

I: The force between two point masses, m and M, a distance r apart is F equals G M m over r squared. So let's see, was this full credit, so this one was all right. Was it easy?

D: Yeah.

I: How'd you do it.

D: First I was trying to solve this equation, F = G mM over r squared, except that, um, at this part, the radius of earth's orbit, that could be r squared, but I wasn't really sure at the time. But then, I read this part, then about pi times 10 to the 7 seconds in the year, and when I first 3

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[1] *Coherence*: Daniel seems to be looking for coherence between his experience and what he remembers from the course. Reconciled: The excerpt may also be interpreted as Daniel's trying to choose between two pieces.

[2] *Concepts*: The statement can be interpreted as expressing an expectation of integration between the formal knowledge from the course and experiential knowledge. Reconciled: It can also be interpreted as expressing the difficulty of remembering the correct answer.

[3] *Formulas*: Daniel describes reasoning based on the literal content of the formulas.

read it, I know there's something wrong about putting this in here because, I felt that um, they wouldn't just put this in here for nothing, [laugh] because so far, all the midterms I've taken in the university, they've never put information down for nothing, just to throw me off, I've never had that. So, I was thinking about it for a long time, and I couldn't get to it, and suddenly it just came to me that it must imply something about velocity here, so since we're studying centripetal acceleration, I put this equation in, and, it just came to me.

I: So what was the first thing you thought? Originally you thought this was

D: I thought this was a value for r, or something, to give me a value for r.

I: The radius of the earth's nearly circular orbit around the sun, is 1.5 times 10 to the. meters. Ok.

D: I was just trying to solve for um

I: So that was, you used that as r, or you didn't use that as r?

D: Mm-hm.

I: You did use that as r. Ok. And so what were you going to do before you saw this and said oh.

D: I was going to solve this, except I have two unknowns. I was expecting them to give me the mass of the earth, cause that would be so much easier.

I: I see. Ok. But then you had this, and so you figured it out that way.

D: Well, then I thought of this, and everything just works out.

I: Centripetal force. All right, moving along. A girl throws a ball upwards at an angle of 16 meters from horizontal, the ball passes through a window 32 feet above the point where she releases it. Was this one easy? Looks like you got full credit.

D: Mm-hm. It's just a matter of plugging in equations. They give you a cheat sheet, and 1
basically I have this formula, the maximum height formula, which is

I: They gave it to you, or you

D: Well, we were allowed to use one sheet of notes. (right) And I had that in my notes.

I: I see. You don't have that sheet with you do you?

D: No.

I: Oh, that's too bad. So they gave you one sheet of notes, so you had this a maximum height formula in your notes, and so you just plugged in.

D: Yeah, and I also had the x max formula, so that was pretty easy.

I: Oh, so you had this one and you had this one and those were the two questions they asked, is that right?

D: Mm-hm. Basically with those two formulas I had all the information I needed to solve the question.

I: So let's see, how did that work, you had the maximum height, you used the maximum height formula there, and, oh I see, they gave you maximum height, (yeah) and you just solved for v. And then down here, now you had all this stuff with the x max.

D: Mm-hm. And since it's at the peak, it's only x max, and that was divided by 2.

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[1] *Formulas*: Daniel says that "with those two formulas I had all the information I needed." This conveys a sense that the formulas are the content.

I: I see. x max is for the whole, so that one is easy. So this one you lost, oh no, no exclamation point, how many points was this total?

D: 15.

I: And why was d crossed out?

D: We don't have to do d.

I: Oh, he cancelled that, at the exam, ok. So a 25 kg child slides down a slide, friction slows her descent, so acceleration is g over 4, ok, so now part a if she starts from rest what's her speed after 10 meters. You got that.

D: Mm-hm.

I: And that was easy.

D: Mm-hm.

I: And how did you do it, you had a v final

D: That's v final equals v initial plus at, but we don't know t we know a, um, um, distance is [1]
1/2 a t squared, that's a formula, we learned, I solved for t squared and t here, because I know distance, which is 10 meters, I know acceleration, which is 2.45, so I solved for t, t is 2.86 seconds, then I plugged it back into this equation, because initial velocity is zero, it's just a, we know a, and we just solve for t, so a times t is 45. It's easy.

I: Ok. and then the work done by gravity.

D: I used um, the formula was in the book, it's given, so it's just a matter of plugging in mg and v.

I: Ok, was that on your sheet?

D: Mm, yeah, that was on my sheet.

I: And, W equals delta K, not always, what does this mean?

D: Um, I thought I would be smart and [laugh] do it tow ways, and give them two ways of doing this problem, and obviously one of them was wrong and I got points taken off for that one.

I: Now you have the same answer for these two.

D: Mm-hm.

I: Ok, so, you were going to solve it in two ways, and one of them was wrong. (mm-hm) Which one was wrong?

D: This one was wrong.

I: Ok, this one was ok, and this one was wrong?

D: Mm-hm. This one, this says something about not always, but [the professor] said [2]
something about using the work energy theorem which is this, so I put it that way.

I: So you said work is change in kinetic energy, and they wrote not always, pointing to that.

D: Yeah [laugh].

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[1] *Formulas*: Daniel describes pure symbol manipulation.

[2] *By Authority*: Daniel's justification for using the work energy theorem seems based on the professor's literal comments.

I: What did that, does this mean anything?

D: Wooo, maybe. I think they're just trying to tell me this is not always the case, but, in this problem you can do it this way,

I: You can do it this way.

D: Mm hm, in this special case because [the professor] had specifically mentioned that you can do it with the work energy theorem.

[A problem from the first midterm: "*A 3-kg block rests on top of a 6-kg block, which rests on a frictionless surface. The maximum sideways force that can be applied to the lower block so that the two blocks move together without slippage between them is 27 N. a) What is the coefficient of static friction between the two blocks? b) What is the maximum sideways force that can be applied tot he upper block so that the two move together without slippage?*" Daniel got 10 out of 20; the grader did not specify what was wrong.]

I: They just put 10. So do you know what was wrong?

D: Um, not over here. Well, I have the solution set, it's just that the way they thought of the problem, like I told you, was different than the way I thought of solving the problem. Um, I sill haven't had time to look at it yet, why that is so, but you do the first problem such that, the top
block's 3 kg, the bottom block is 6 kg, and this is frictionless, and you apply 27 N force to the lower block, so the two blocks move together by a slippage as, 27 N, (ok) um, so when a force of 27 N is exerted on the bottom block, it is being accelerated at, equation F equals m a, but if you push the bottom block you also have to move the to-p block, so m is not just 6 it's 6 times 3, so you're accelerating the whole system by an acceleration of 3.0 m/sec squared. So [pause] if the top block moves together with the bottom block, then the top block also accelerates at 3.0 m/sec squared, so the force applied to it is 3.o m/sec squared times 3 kg which is 9 Newtons, so the friction is 9 Newtons, the frictional force, [pause] because it's opposite to the, um, force exerted by the push. So, the force due to friction is 9 N, and that's equal to mu k mg, which is mu k N, and solving for mu k I got 0.306, which is the exact answer they got. **1**

I: So part A was right, so the 10 points you got was for part A.

D: Yeah. Part B. So, if the maximum frictional force is 9 N, they don't slip, if the top and bottom don't slip, they must have equal acceleration between them.

I: [pause] Mm-hm.

D: So I said that, well, force is mass times acceleration, so force must be also 9 Newtons because there's no slip, they don't slip. I think that's where I went wrong.

I: Why?

D: Because here I'm taking into account both blocks, and I'm not giving the equal amount of force for this 9 Newtons for both blocks, I think I'm just saying that this is for the top one, while this I'm doing for both top and bottom.

I: So you got 9 N is 9 kg times acceleration.

D: I think the right answer is 13.5.

I: Where'd you get that?

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[1] *Concepts*: Daniel gives a conceptual explanation of the solution to part A. Reconciled: It involves intuitive knowledge Daniel finds apparent. This interpretation is supported by his subsequent explanation.

D: Um, I got that, 13.5, except that I scratched off that theory because I thought it was 1
wrong and I thought this was more reasonable.... 13.5 Newtons I thought was way too much. Because I thought that if, frictional force is 9 Newtons, and you push the top one 13.5 Newtons, then it will slip. It's more reasonable if I use 3 Newtons while the frictional force, the maximum frictional force to hold these two together is 9 Newtons, they won't slip. So I scratched off 13.5, because I thought it was unreasonable (right) but obviously I was wrong [laugh]. Because, the way I figured this problem is that, you exert a force on the bottom one, and you also have to take into account the mass for the whole system. So, if um, I exert a force on the top one, 9 Newtons, then I'll also have to be accelerating 9 kg of mass.

I: [pause] So, that, you exert a force, so what happened here, you had F = ma, 9 Newtons equals 9 kg times a, so 9 N was the force you were going to apply to the top block, (mm-hm) so you put 9 kg

D: No. 9 Newtons was the force I was going to apply to the whole system. 3 N was the one I was going to apply to the top.

I: Ok. Ok. And how did you choose 9 N is the force to apply to the whole system?

D: [pause] I think over here [pause]

I: Now there was, I see there was 9 N here, f friction is 9 N.

D: Yeah, well I was thinking of the problem so I was sort of like in a hurry, so I just figured, well, I think that's how I did it, so I think I'm going to use 9 N in here also, that's my way of thinking during final exams.... All I remember is I did get 13.5 N and I erased the whole thing.

I: So why don't you try to get it, and see what, ah

D: I think the way I got 13.5 N was very simple, very basic thinking, so basic that I thought it 2
was wrong [laugh] because it took me like 4 minutes to get 13.5 N, and I thought that must be wrong. Um, first of all with the 13.5 N, I also got different numbers over here.

I: On part A you got different numbers?

D: Numbers, and then I got 13.5 on part B, and I said that couldn't be right, cause I remember 3
mu k at that time was point 4 [.4] something, and, first I started with a .9 mu k, and I was like going on to the next problem, except that I like, wow, that's really a lot of friction.... I've never seen that high of a number before. And thinking about it, that's a lot, [laugh] for two blocks together to have a .9 static friction is like glued together almost, so I think there must be something wrong with my answer there. Um, getting .9 was easy, I think it's just a matter of dividing 27 N by 6 or something like that, and it's just not thinking of any, not taking any of this into account. I forget how I got .4

I: But .4 seems like a reasonable

D: It seems like a very reasonable answer.

I: So how did you decide to do it again after that?

---

1 *Apparent Concepts*: Daniel describes what I call 'shopping' for a method. He tries a solution technique until one gives an answer he thinks is reasonable, without trying to understand the underlying physical mechanism.

2 *By Authority*: In addition to his sense of the reasonableness of the result, Daniel's choice of technique is informed by a guess of how hard the problem must be. This shows a lack of emphasis on his ability to make sense of the material for himself.

3 *Apparent Concepts*: Further indication of 'shopping.'

D: I can't really remember that one, except that I just think that maybe I did it wrong and I decided to do it again. This way just seems so, I just started writing out how I did the problem, explaining it, [laugh] it just make it so much more, rational to do it this way, and I'm convinced that this was the right answer, after finally writing all this thing out. [1]

[Daniel tries to come up with the answer of 13.5]

D: F equals ma, I'm applying 9 N force here, the mass I'm applying it to is 6 kg, I'll get an acceleration of 1.5. So, F equals ma again, for the whole system, don't know F, [unintelligible] 9 kg of mass, it's going to turn out to be 13.5 N. That's how I got it, why is it so

I: So this is how you did it

D: How I got 13.5.

I: So you said F = ma is 9 N equals 6 kg

D: Oh, yeah, that's how I got it, because I was saying with that 9 Newtons force, I'm accelerating the bottom one by 1.5, and I'm also accelerating the whole system by 1.5, except that, yeah, that's the first way I thought, yeah, that's how I got 13.5, so then I thought again, I'm pushing the top one, [pause] [laugh] yeah, that's right, I think, why did I think this way?

I: This is right?

D: Well, that's the right answer. Um, they don't slip, they must have equal acceleration, so F equals ma, [pause] except that here when I'm exerting, when I'm calculating accelerating, I'm only taking into account the top one not the bottom one, why is that. [pause] Ok, so you have to do this problem backwards, from this, maybe.

I: Do part A backwards.

D: [pause] The thing that makes me decide about how, why I want to do it this way, it's [2]
because if I want to apply a top, um, if the maximum force that I am applying, I'm only applying a force to the top block, then there's no reason for me to calculate 9 kg from this, because why take into account the bottom one, because I'm only exerting a force on the top one, and I only want to know, if those two equals, um, something like 9 N, I'm just saying that, well, I'm pushing the top one, and the maximum I can, um, accelerate the system is Newtons, because that's a frictional force, so the acceleration has to be [one?] meters per second squared, over here, so the first time I did it, it was 1.5 meters per second squared, and then I changed this part too, because, I basically modelled this problem after this problem.

I: Ok, so you were happy enough with this one that you decided to

D: To do it the same way, to model it this way. Since it's not right, then I don't know, how, how that is right, or how that is true.

I: Ok, so you got 13.5 this way, when this was .4 up here, so since you redid this you redid this, and got this, and now you know that this is the right answer, are you, do you think that this is why this is the right answer?

D: I'm not sure, to tell you the truth, because it just doesn't make sense for me to do it that way, and so I'm not so happy with this answer.

[Transition chat. Daniel received no credit for the next problem.]

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[1] *Concepts*: Daniel spoke of the method as being more "rational," as opposed merely to the result. This indication is unreconciled.

[2] *Coherence*: Daniel is clearly disturbed by the incoherence he perceives in that the conceptualization for the first part of the problem was not successful for the second part. This is an unreconciled indication of *Coherence*.

I: A chain is held on a frictionless table with one half of it's length hanging over the edge, you told me about that this one was tough. How much work does it take to slowly pull the chain onto the table? Ok, so what did you try, what's happening?

D: Um, the way they did it was with integration, and that's the way I sort of did it too, except, 1
I think that I looked at the problem differently than they did, for some reason, because the way I looked at it is there's a chain, and half of it's hanging on top, I mean hanging down, and you want to pull it all the way, so that all of the chain is going to be on the table. And, if work is force times distance, then if I pull this whole chain all the way to the table, then I also have to work in order to pull this over here.

[More discussion of that problem. Based on his reasoning, about work, Daniel thinks the problem statement was unclear and wants to complain. We move to talking the lecture from the previous day.]

I: So if you could start from the beginning of the lecture, and just sort of talk about what went on in the lecture.

D: Oh, great [laugh].

I: And, what made sense to you, what didn't make sense to you, and any comments about the lecture, and so on. Whatever.

D: I think I never really had much problems with his lectures, I mean, basically, everything made sense, when I'm sitting there and looking at it, because, um, it just came to me, everything, except that the only problem with it is, ok, so if you tell me all those things it's going to make sense to me and I'm going to understand everything, but, if you give me a problem, how do I know where I'm going to start. That's a different thing.

I: So, explain that?

D: It's like, the problem with most students I see is that, there's a tutor sitting by them, 2
explaining to them how to do the problem, most of the time they're going to catch what the tutor's going through, what the tutor's doing, most of the time, except that their problem is, once the tutor works it out they go, oh yeah, that's really easy and they go on to the next problem and go, ok, where do I start. I think it's just a matter of starting and, getting all of the
right equations into place, and that's the hard part, because when he's doing example 3
problems in lectures, of course they're going to get it because we don't have to think oh, let's start from here or from here, because he's doing that thinking and we're just following and saying, oh, yeah, let's start from there, [unintelligible] of course, I mean why would he start in the wrong place, [laugh] if he thought of the problems, or he knows how to do it. So, yeah, all his lectures basically made sense, because it's just, he's just deriving equations and saying blah blah, and if you read the book, that's what the book says anyways, so if you read the, especially if you read the book ahead of time, everything will make sense to you, most of the things. So, following his lecture is not a problem for me, at all. Getting, doing the homework problem [laugh] is another thing, it's just two different things. So, what do you want me to do, you just want me to explain to you what's going on?

I: Yeah, tell me, what would be a good place, this is, what was here, this is ah, drops hitting the wall, the bullets sticking, rocket motion [all flipping pages], I don't know, why don't you just, this was the thing exploding?

D: Mm-hm.

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1 I note a misconception. Daniel thinks it takes work to move the chain horizontally on the table, perhaps related to an idea that motion is caused by force.

2 *Independent*: Daniel said one cannot understand simply by listening in lecture. Unreconciled.

3 *Apparent Concepts*: Daniel speaks of the content as formulas and of the difficulty he has with problems as "a matter of starting and, getting all of the right equations into place."

I: Tell me what that example was about, tell me what it shows, what you got out of it.

D: Um, what I got out of it. What [the professor] was saying is that, you shoot a particle, well, a particle goes in a parabolic trajectory into the air, and it went into the air, it explodes, and what he's saying is that even though the particles will go off in a bunch of directions, the center of mass of that particle will still follow a straight parabolic trajectory path. And all of that is going to sum up through some, distance of the, [pause] um, yeah I think some of it is going to sum up to some distance, I'm not really sure about this one. Ok, here is this, when a particle explodes, [gives detailed description of values, manipulations.]

I: Ok. So this, this is all based on the idea that, if it explodes that the center of mass follows (mm-hm) in the same path. Now, why would that be?

D: I never really thought of that, I sort of accepted that concept. [pause] I've never really thought that. [1]

I: Ok. [pause] Ok, impulse. What's impulse?

D: The definition of impulse, change in momentum times change in, no, that's not impulse. Ah, I don't know the definition of impulse, to tell you the truth. If it's just a matter of deriving all these things, then I haven't spent my time looking at

I: Well this was just yesterday, so

D: I don't think I want to tell [laugh] you and make a fool out of myself.

I: Ok, that's fine. So then there was this, and then rocket motion, did this one make sense, the rocket motion?

D: Um, which part?

I: Um

D: The very last part about the, um, where it starts, involving the L n and stuff, that's sort of cloudy grounds for me.

I: Ok, how about up here, where he's got m b v dt plus [unintelligible]

D: All this is sort of not yet clear to me, I haven't really attempted to look at it yet, but the concept of um

[tape side ends]

D: The concept of in outer space, where, the only way a rocket can go forward is to, to shoot off all this hot gas, in the opposite direction, does make sense. Um, why that makes sense, um, [pause] the way I, for me that just, if I picture it in my mind, I can't think of any other way to explain it. I just accept that. I mean, of course it makes sense [laugh] to me, I don't know why, if somebody asked me that, yes, it does make sense. The only way I can think of explaining it is by using Newton's Laws, the three Newton's laws, where, if you exert a force this way, then the object also has to be going that way with the same force. So that part really makes sense to me, and I have no problem with that. This part, I haven't really looked at. So I don't know what to tell you about this part. [2]

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[1] *By Authority*: Daniel believes it is reasonable to accept the fact without thinking about it. Alternatively, it could be taken as an indication of *Apparent Concepts*, if one interprets Daniel's comment to mean that the result is intuitively plausible and therefore does not need further consideration. My choice in coding is based on my assumption that the result would not have immediate intuitive appeal.

[2] *Apparent Concepts*: Daniel says he accepts the idea because he can picture it literally, but he "can't think of any other way to explain it." The phrases "the only way a rocket can go forward" and "the object also has to be going that way with the same force" are indications of a force-causes-motion misconception.

I: Ok, that's fine. All right, so, this was about, he's talked now about work, and energy, and momentum, and impulse, you've covered in the course, and you've read, (mm-hm) does that all seem like it makes sense, is that, there are details missing, or do you generally have the idea, or where are you will all that?

D: What do you mean if there are details missing?

I: Um, well for example on this one, you said this part you don't understand, but this other part you do, (mm-hm) so is this a detail, or is this I don't understand

D: I don't understand this, I haven't really looked at it, if that's what you mean, I haven't really studied, and tried to understand it. Um, I can't really say, how much detail the course should go into, because I just don't know enough about teaching to be able to make that judgement, so basically everything that's going to be taught to me I'm going to accept as important, because, I don't know any better. **1**

I: But, just the way, for work and energy and impulse and momentum, so you fell like you basically understand what's going on.

D: Basically. In general. If we were to go to specific problems that's going to go into depth, then I'm not sure if I'm going to be able to really grasp everything that's going on, or going to happen, but in general I would say I basically know what's going on. **2**

I: Ok, so I'm going to just ask you a question about, tell me roughly, or whatever, whatever level you want to tell me, what's the difference between energy and momentum?

D: The difference between energy and momentum? [laugh] Wow, that's really specific [unintelligible] Momentum has something to do, in my mind, the connotation of momentum is that [gesture – sound of hitting something]. In order to have momentum, then I would say I have to start with energy, because momentum is, the way I picture it in my mind, is something, moving, it's really closely related to inertia, is that, something's moving, at some velocity, then that particle has some momentum, while energy is what makes that thing have momentum, I would say. Because if that thing has no energy, then it wouldn't have any momentum to move. Momentum, the connotation of momentum, always have to [say about?] some particle, and that particle has to move somewhere, and if it hits something, then it's going to exert a momentum to that other object, and it's going to give either all or some of it's energy to the other object, the energy is always going to be conserved, the total energy, of the system is always going to be conserved, and the energy is of course going to be like heat, or like friction, or something like that, but total energy is always conserved if a collision occurred. Um, momentum, ok, if there's a particle sitting here, and it's just standing still, there's a particle going straight at it and then, um, hitting it, and then that particle now is stationary, initially begins to move, I would say that, um, the particle that hit the resting particle gives the resting particle momentum such that it moves, and, it's, then if I say, then momentum is closely related to energy also. So I'm going around in circles. Without getting anywhere. **3**

I: So how do you know if something is conservation of energy or conservation of momentum?

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[1] *By Authority*: Daniel does not feel he is in a position to determine what are central points and what peripheral.

[2] *Apparent Concepts*: Implicit in Daniel's statement is that understanding generally (qualitatively) is separate from being able to "really grasp the details" (quantitatively).

[3] *Concepts*: Daniel interprets the question entirely conceptually. He does not feel able to respond to it, although he is quite familiar with the formal differences. He also seems to feel that this is a question he ought to be able to answer. Reconciled: Daniel appears to be looking for an apparent connection by which to understand the differences. That is, the kind of conceptual association he seems to expect, if he is going to have one, involves a literal association with something directly perceivable.

D: How would I know? (yeah) [pause] In this collision, I think, I [unintelligible] tell you. Do you have a specific problem, maybe I could say [laugh] ?

I: U, could you come up with a specific problem, um, just this is a situation where I'd use conservation of energy, this is one where I'd use conservation of momentum.

D: Ok, conservation of energy shouldn't be that hard. Um, [pause] Ok, for instance, if you're [1] on a roller coaster, and going at constant velocity, then you have constant force, constant force doesn't necessarily imply constant energy though, kinetic energy, well, ok, yeah. Conservation of energy, when you're going down a roller-coaster that's going through a loop, at the bottom of the loop, obviously the roller coaster is going to go faster than it is at the top of the loop, so the kinetic energy at the bottom of the loop is going to be greater than the kinetic energy at the top of the loop. But, then you say, if there's a law of conservation of energy, then the kinetic energy at the bottom of the loop is greater than the kinetic energy at the top of the loop, then where does all the other energy go, and it's going to be potential energy, because it's going to be at the higher spot, and that's conservation of energy....

<u>Third interview - 3/13</u>

I: Let's go through the lecture then.

D: [Finds notes] Seventh, was that the last lecture?

I: Thursday, the ninth would be the last lecture.

D: Well, this is starting from here.

I: Ok so what do we got, this is ah

D: We got, go through momentum, and some collision stuff.

I: What's this one, what is this? I was not at this lecture, so

D: He's talking about momentum, and how things like rolling forward, and whether it gains momentum or not if there's like water flowing into it.

[There are two examples, both involving a cart rolling on level, frictionless tracks. In one case, the cart fills with water from rain falling vertically: the cart slows down. In the other case, the cart drains water from a hole in the bottom: the cart continues with constant speed.]

I: So what is, what happens?

D: Um, well in this case, since the um, momentum stays constant, the speed of the cart slows down because there's no mass, and kinetic energy is given by this equation.

I: so what happens to kinetic energy, does it go up or down or stay the same?

D: Um, if momentum constant and the mass is increasing, then it should be decreasing.

I: It should be decreasing. How come?

D: If I look at this equation [$K = p^2/2m$], p is always going to be constant, and [v?] is always going to be constant also. And the mass will be increasing, because there is going to be more water as time goes by.

I: Ok. Why is momentum constant?

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[1] The phrase "going at a constant velocity, then you have a constant force" is another indication of 'force-causes-motion.'

D: [pause] Because, there is no external force going this way, it's just vertical, this is the y axis, and I would imagine, well, if you think about it, the mass is going to be increasing, obviously, because there is going to be more water, in this cart, and obviously, if I think about it, I can sort of picture it, well, if there's going to be more mass, then the speed is going to be less, and momentum is just mass times velocity, and if the velocity is decreasing, most likely proportional to the rate at which the other one's increasing, then I guess it should be constant. [1]

I: So you're saying if the mass is increasing the velocity should be decreasing just because

D: Mm-hm.

I: Just because.

D: Yeah, just, well, because it will take more work, I mean the same amount of work has, in order for the velocity to stay constant you need more work to move the heavier mass. [2]

I: Ok. [pause] Ok, so if it's picking up mass as it gets heavier, then you'd need something

D: More.

I: You'd need more.

D: To keep it

I: Ok, so you figure, so it must slow down, since it might as well be proportional, so

D: Well also, if [the professor] says it's going to be constant, and it's going to slow down, I not going to argue with him [laugh], I'm just going to think about it yeah it makes sense. [laugh] [3]

I: [laugh] Ok.

D: Ok, next one is that, if there's a hole in the cart, and the water's dripping out, um, then the momentum will get smaller and smaller, because, um, the mass is going to be, decreasing, but the velocity will stay the same.

I: Now that doesn't seem to

D: Yeah, that doesn't seem to match with my ideas. [pause] So. [pause] What should I say? [unintelligible] gets more, gets less momentum and velocity stays the same, as the water drips out. [pause] Well, I don't know what to say, so, you got me here. [laugh]

I: I got you. [laugh] Ok, um, well, all right. So what do you do in a situation like this?

D: What, does it bother me, (yeah) yes. Well, [unintelligible - sounds like "for the last two"], that's the easiest way, [laugh], or ask somebody else who is smarter than I am, [laugh], who knows more about physics, that's what I'd do. But, yeah, that seems strange. During the lecture it seemed to make sense. It has something to do, I guess it has something to do with, the force of which, this thing is, I don't know, it just seemed to make sense from lecture. Oh well. [4]

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[1] *Apparent Concepts*: Daniel's explanation is that he can picture it happening, not that he can account for the effect by a mechanism. Note as well the circularity in Daniel's reasoning based on the formula: v decreases because p is constant; p is constant because v decreases.

[2] *Apparent Concepts*: This is another instance of Daniel's use of 'force-causes-motion.'

[3] *By Authority*, *Apparent Concepts*: Daniel describes his understanding the point as based, for one, on what the professor said and, for another, on he an apparent interpretation.

[4] *By Authority*: When confronted with the difficulty, Daniel believes his only recourse is to consult "somebody else who is smarter." *Weak Coherence*: Daniel commented that "it seemed to make sense" during the lecture. These two problems were presented as consecutive examples in lecture, along with mechanistic explanations. By what Daniel says here, in listening in lecture he did not compare the two examples in any way. To him, at least without the probing in the interview, it was sufficient to know the two results, not to look for coherence between them.

I: Well, what about your argument that [the professor] says this is true, so you're not going to argue with him here either.

D: Yes, that's true, I'm not going to argue with him there, but, yeah, exactly, I'm not going to argue with him there, [laugh] but, I don't know how he got that.

I: Uh huh. This one you know how he got.

D: Well, no, if I don't know how to get this one, I don't know how to get this one either, 1
because obviously, this whole statement has disproved what I said about why I got this one.

I: So if you don't understand this one then you're not real confident about your understanding about this one either.

D: Mm-hm. Of course not.

I: Ok. Well that makes sense. So do you want to try to figure it out or do you want to just go on, I'll give you a choice.

D: Well, I don't think I'm going to come up with it in the next fifteen minutes, so it's just going to waste your time.

I: No, it doesn't waste my time. Well, ok, then lets go on. Ah, what's, this is F, F bar, grains, mass, is this ah

D: This is a scale, supposedly.

I: This is a scale here, and this is stuff falling in?

D: Mm-hm.

I: Grains falling in.

D: My notes aren't that clear at all. [laugh]

I: Grains colon mass m. What's that?

D: That's the amount of grains, mu grains fall per second.

I: Mu grains fall per second. Ok.

D: So the cumulative grains is just mu m g, the amount

I: So what is this, is this a problem, is this an example?

D: It's, I think it's an example problem.

I: And, it's what are you trying to do with it?

D: We're just trying to find the reading of the scale, with [unintelligible], so I guess in this case, it's going to be, I guess it's just going to be like, a universal thing, where you have this general case of stuff, and grains are going to fall out, then the whole scale reading's going to be this. (Ok) [unintelligible] function of time. What's given is the mass of the grains and the number of grains that fall per second.

I: Ok. And so how does it work out?

D: Um, I think all this is simply kinematics, or whatever, the um, formula is given previous 2
chapters, and kinetic energy will equal potential energy, and solving for v squared, it's just

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[1] *Coherence*: Daniel expresses the belief that the two examples should cohere. That they do not, he takes as an indication that he does not understand either of them. Unreconciled.

[2] *Formulas, By Authority*: Daniel explains the content as a formula, which, he says, is given.

going to give this, 2 gh, and then, um, so, um, momentum, p, mv is going to equal this, so that's just the change in momentum per grain.

I: Ok, and why is that an interesting, why do you want change in momentum per grain?

D: Why do you have change in momentum per grain? (yeah) I haven't a clue. Um, because, delta p over delta t equals something, it's similar to something, I don't know. I think it's in the book.

I: Ok.

D: Mm, well, let's see. Here it looks like it's um, force, the change in um, momentum over time, seems to be equal to force, so, if we know the force, then we know what the total scale reading is for this, if this is, that's the way scales work.

I: Ok, so then this is the answer. [pause] Ok, and what. This, what's this, just about

D: About, um, this is about potential energy, and kinetic energy, about how, um, and, also he wrote the equations, about this train [unintelligible] off the hill, it's going to roll down, and it's going to undergo an [unintelligible] and they want to know if this is going to barely reach the top of the hill, they want to know, um

I: How high should the hill be.

D: Mm-hm.

I: Ok, does that make sense?

D: Yeah, this is, goes through a bunch of equations, that is basically in the book. That was one of his easier ones. [laugh] 1

I: Um, let's see, what's happening here. [pause] What is this a graph of?

D: Impulse? [laugh] I'm not sure. [laugh]

I: So you were saying, physics is getting hard, lately,

D: Yes, this is what

I: This is the hard stuff.

D: Because it's just a matter, bunch of equations where, you sort of just sit there and take notes in class, and wonder what he's trying to get to you. He does go at a fairly fast pace in his lecture, and [pause] the general concepts is not that hard to grasp, but, when he gets to the specifics and the nitpicky details of the whole problem, you sort of have to sit there and think about it. For instance, that cart problem, he just whizzed by that whole thing, and it's just, you're trying to draw this thing as fast as possible, and when you like listening to him, he said it in such a way that you just accept that, because it's um, as if it makes pure sense, but when we look back at the notes, it's one of those things that, you say what. 2

I: Huh. So that, your talking back about this cart stuff.

D: Yeah, that's what I'm talking about.

I: Does he ever say stuff in lecture that he says it and it doesn't seem, it doesn't make pure sense right then? Do you ever go what, right in lecture?

D: Um, sometimes, but I can't really remember, he doesn't do that very often, except when he get to like the details about all the equations, deriving all the formulas, then all he does is sit

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1 *Formulas, By Authority*: Daniel described the content as equations from the book.

2 *Apparent Concepts*: Daniel describes the content as formulas, with 'general concepts that are easy to grasp.'

there and say, I guess, he must know more than I do, so he must, he must be doing this for some reason, [laugh] so you're just like copying things down, and someone will say, oh, yeah, and then you got this equation, and you say oh yeah, I've got that in the book somewhere. [laugh]

I: Ok. So the general, is the general stuff not hard to grasp with this too?

D: It's the general stuff is not that hard to grasp, I mean the general stuff is much more different than the homework assignments that he gave out, or that the professors gave out.

I: Ok, so give me an example, tell me, give me a fell for what you mean by general stuff.

D: Um, general stuff is like, the um, inelastic collisions, when the thing's going to hit at the [1] bottom, then there is going to be change in velocity, because with the same amount of momentum you have to move more mass, so the velocity is going to slow down, and stuff like that, and, just, basic general things like that, that's not hard to grasp at all, it's just a matter of straightforward formulas, plugging into numbers and formulas, or [unintelligible phrase] one of the answers does. When it starts, when for instance, homework problems, says something about, prove the angle, and
like, and elastic collision, when two billiard balls hit each other, find the angle of the param, in terms of like, the parameter of the diameter and stuff like that, and he doesn't go over that in [2] lecture, so it is not, easy for, most of the, all of the people I've talked with [laugh] they just go, I don't know how to do this, and I heard one of the um, like my friend, and she knows people who got 90-something on the midterm, so, ok, that's a blow to my ego, fine, but anyways, she was saying how, this supposedly genius just sat there, for 20 minutes and not be able to do this problem, so here I am going, oh great, he doesn't know to do it, I'm supposed to know how to do it. And then I went to the physics tutor at my dorm, and he said I have no idea what he's trying to say here, [unintelligible phrase] [pause] contradictory to what I've learned so far.

I: What, this is your physics tutor talking? (mm-hm) Saying it's totally contradictory

D: To the concepts I've used.

I: And what is this, this is a problem you got on your homework? (mm-hm) What was the problem?

D: I don't have the book with me, it's number 31, but, a ball glued to the, it's a billiard ball glued to the table, and there's going to be a ball going along, and it's going to hit, and it's going to be deflected, they want to find the angle at which it is deflected, and then the second part of the problem is they want to find, the change of momentum, the magnitude of
the change of momentum, and they also said that the collision was elastic, so if the collision is elastic, that means the momentum before has to equal the momentum afterwards, but since [3] that ball is glued firmly to the table, it will have no momentum after the collision, thus there shouldn't be any change in momentum, in the collision.

[Further discussion of problem - neither he nor his tutor understood it. Discussion of the tutor, who is assigned to Daniel's dormitory: how he conducts office hours, etc.]

I: Ok. So how did we get on to this, you were telling me about, ah

D: Specifics and generals.

I: Specifics and generals, that's right. Um, so I got a feeling you're more interested for yourself in generals.

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[1] *Apparent Concepts*: The content is described as "straightforward formulas" with apparent interpretations.

[2] *By Authority*: Daniel's criticism is that the lecturer does not give enough information.

[3] *Formulas*: Daniel's understanding of the law of conservation of momentum is as a rule: "if the collision is elastic, that means the momentum before has to equal the momentum afterwards."

D: [pause] Not necessarily.

I: No.

D: Because that means I would be, simply just another physics student, like, if I'm only interested in generals, then I wouldn't be taking 7A, I'd be taking 8A.

I: I see,

D: Right?

I: Ok.

D: But it's not a matter of I'm not interested in it, it's, I don't think he's giving us a fair chance to be able to work out the problems.

I: Really?

D: He's just not giving, I don't think he's giving enough infos or inputs on how to, um, go [1] about the problem, because, none of his, well, I've looked through my notes, and there's not much correlation between what he says in lecture, and how to approach this problem. It's kind of like there's a missing gap between them, and we're supposed to fill that in by ourselves? I don't know if it's fair or not, but

I: So is this a gap between the kinds of things he does in lecture and the homework?

D: Mm-hm.

I: Cause he's done, I mean this, some of these look like pretty complicated problems, from lecture.

D: Yeah, some of it is.

I: So, I, are you saying he doesn't do any problems that are this hard during lecture?

D: Um, well, he does hard problems sometimes, not often, but most of his problems are basically general, or, kind of easy, but, the hard problems that he does in lecture and the hard problems that the professors assign are different, because [the other professors] are also using the same homework assignments.

I: Right.

D: So, I don't know what [they] do, maybe they give inputs to how to do problems, but [my professor] doesn't mention anything about homework problems at all.

I: During lecture.

D: Rarely. I mean, during the beginning he said, in the first two weeks, but he hasn't mentioned any things like that.

I: Ok, discussion sections, you're not ah

D: I'm not happy with it, I didn't go to my discussion section last time, I went to another one, and, I guess I'm not going to go to mine again today, I'm just going to go to one tomorrow, but, I basically decided that I'm not going to go to that discussion section any more.

[15 minutes of discussion of teaching assistant, Daniel is not happy with them, etc.]

I: Ok. [pause] All right, let's go back to this. so this is elastic scattering, this is stuff that, ah, this is all from last lecture, this is all from Thursday.

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[1] *By Authority*: Daniel feels the lectures do not provide enough information, rather than that filling the "gap" is part of what he needs to do to learn.

D: This is Thursday's lecture.

I: And this is all hard, generally.

D: This part, with, how to convert, velocities from the center of mass frame to lab frame, oh, he just whizzed by it [laugh], and, um so did my tutor, [laugh] he just goes, ok this is how to do it in the center of mass frame, then you convert to the lab frame, then you convert it back to the center of mass frame, then you plug this in, then you go back to lab frame, and I'm sitting there saying which one is which, and what's the difference? He didn't go into this at all, I'm, lost.

I: So you're still lost. [flips page]

D: This is not bad, the one with how kinetic energy, in a, partially, inelastic collisions, about how it only loses energy with some proportion e, or some constant e, that, I think it's called coefficient of restitution. That, he explained that very clearly.

I: See now this would seem to me like to be a very hard problem, with this, you know, infinite series summing, and all this stuff.

D: Well, I've been doing this in math [laugh] so,

I: Oh, so you're used to the math.

D: I'm used to the math, this part, well, yeah, I mean it seemed like it's going to be complicated, and if I really look into it it's going to be complicated, but, the concept that all you're supposed to do, all he's doing here is finding K1 K2 K3 K4 and finding, quote unquote, series for that, I can't deal with that. I may not be able to get to this point, in like, [snaps finger] 5 or 10 minutes, but at least I know how to get somewhere close to that, the same answer, except in a more complicated form, not the simple form, [unintelligible algebra].

I: Ok. So this one was ok. What's next, um, how about this.

D: Um, mv equals m v f cosine theta 2, um, this has something to do with, oh yeah, I have one question of this is that, first of all I forgot what goes on in this, but I'm assuming that there's a particle here and it strikes a particle here, and the two scatter apart, except my question, what I thought from what I read in the book was that it's supposed to split in a, 90 degree angle, instead of just this, so, that's my question on this problem, of what he's doing. [laugh]

I: I'm not sure, what's your question?

D: Um, like in the billiard ball problem, if there's an elastic collision, and this ball hits this ball, this ball is also supposed to go that way, if it's not glued to the table, and the angle formed between them is supposed to be 90 degrees.

I: Ok, and this, between them is not 90 degrees.

D: Yeah, that's the thing.

I: Ok. [pause]

D: And the same here. Hmm, what is this.

I: So this is the same thing.

D: Mm-hm. Except this

I: Oh, this is a neutron.

D: This is the neutron bomb.

I: What was this, happening here, this circle? [pause] Oh, here's a right angle, is this what you were talking about?

D: Um, maybe. [laugh] Let's see, what are we doing here. Ok, so this is an elastic collision where the masses are equal, and big m is initially at rest, so, initial kinetic energy equals final kinetic energy of first mass and final kinetic energy of second mass, and here, if you take the 1/2 m's out, because there all equal, you get this formula were v initial squared equals v final squared plus v final squared, big v final squared. And, special case where masses are equal, and elastic collision scattering is that, if you draw a circle with the diameter of the initial, um, then, in each case the velocity finals of both of them will stay on the circle, it won't go past the circle, it will just go to, it will be arcs of the circle, and that's what it's saying here, angle formed 90 degrees, I think, over here, this is angle theta, which, in which the thing splits part. And from there, with trig functions, you should be able to figure out both of them.

I: So this was conservation of energy. (mm-hm) here, cause it's elastic, and this is conservation of momentum (mm-hm). Um, what is the difference between energy and momentum?

D: Oh you asked me that last time too.

I: I know.

D: You got me there. You still got me [laugh]

I: Ok. So that's something that hasn't really cleared up.

D: I know it's in my notes somewhere, I wrote it down somewhere, I don't know where, or I read it somewhere. It's written somewhere [laugh], at least I got to that point, I remember seeing that sometime, but, no, I haven't cleared that up, I haven't really fully understood that.

I: Ok. [flips pages more] What's this. Now we're getting into rotational stuff. (mm-hm) So tell me what's going on in here.

D: Rotational stuff, I think should be easier than collisions, because it's just a matter of **1**
transferring all the speed, velocity, acceleration that we learned in linear, into rotation, rotational frame, and all you're doing is changing the symbols, and, instead of v you use ω, or something like that. All the formulas look the same, it's just a matter of using different notations, they are the same concept, and I remember in high school I didn't have much problem with this either, once I figured out what the symbols are for [laugh].

I: Well, that's what he's got in here. All right, well here's, here's a new question, to start on. Let's see, at some point, I'm going to have a book, and we'll be able to do problems out of the book. If someone were to ask you, is physics more like math or more like chemistry

D: Math.

I: Math.

D: Simple.

I: Simple answer. Why?

D: Chemistry I'm dealing with acid base titrations, and all those things. All I remember from chemistry is that, yeah, there's a lot of math, but, the math I'm doing now in physics, I'm all the way up to the math I'm doing in math, instead of math I did in chemistry. Um, the concepts of math in chemistry deals more with, chemicals, [laugh] that's not saying much, but, you're dealing with more minute things, instead of physical things that I can see and imagine. For instance, um, I would calculate the pH in chemistry, I can't see that, I can't
see what's reacting, except with an indicator I can see colors changing, but, I don't see angles, I **2**

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1 *Formulas*: Daniel describes the content as formulas: "it's just a matter of using different notations." *Coherence*: Daniel describes the rotational formulas as the same as the linear formulas. Reconciled: They were introduced precisely this way in lecture.

2 *Apparent Concepts*: As earlier, Daniel describes a role for informal knowledge to literally "recreate" the event and not to images one constructs, such as of a chemical reaction.

don't see anything but the changing of colors, while, collisions, ok, so, a car hits something and it's elastic, yeah, of course, I can find examples that I've seen in life, where something to hit is going to scatter, it's going to be elastic, and like that, and it's just more easy to deal with in way, because, you can actually think, or, recreate the process in your mind, while in chemistry, you can't recreate what happens in your mind in acid base titration.

[Daniel explains why physics is like math" "I am using a lot of the math concepts for physics..."]

I: So, ok. You're taking calculus this year, right? (yeah) Um, so, all right, what will I do with that. So, with physics, you can see it and recreate it more, but does that help you, say, with kinetic energy and momentum?

D: [pause] Um, in some ways maybe. It's, [pause] can you be more specific, it seems to me, because, I can think of, I sort of can think of general cases where I think I can relate it, but I'm not really sure. Try giving me a specific problem, and how I can portray that in my mind, like how [unintelligible]

I: Ok, you want me to give you a problem.

D: Ok. Sure.

I: Ok. Um, [pause] A, ah, [pause] this is a ball on a string, and here's the pivot. I'm going to let the ball of the string drop, so that if I would just let it drop, it would swing back and forth like this, except what I'm going to do is over here, I'm going to put in a post, that's some distance below this pivot. The length of the sting is l, and this post is some distance h below it. And the question is, how, how big does h have to be, in order that the ball will swing completely around the post.

D: Ok. Um, so you want me to find h, in this case. (mm-hm) [The professor] did the same problem in lecture. So what the string's going to do, is gonna be here, and it's going to be caught, and it's just going to be a smaller loop. [pause, sketching?] That's, and you want to find, h, such that this goes around like that. Can I look at my notes?

I: Sure.

D: With this, you can't really recreate it in your mind, I can't, except that, ok, I see what's going to happen, and I see that, oh yeah it's going to go like that, except that, I haven't seen much examples to know, to remember, at the point it hits this, will the ball actually go faster or slower at the same rate or anything. **1**

I: Say that again?

D: Once it gets caught in the post, and starts making that loop, I can't remember whether the ball, the speed of the ball will actually go faster, decrease, or stay the same after that.

I: Faster than it was moving then, say, just before it hit.

D: Or before, [pause] if there's no pole here, it's going to be faster than

I: Ok. Well, you know cause when I let it go it isn't moving, right? (uh-huh) So you know it's moving faster than when I let it go, so when you say whether it's going to be going faster or slower you mean

D: Well, whether it's going to accelerate from this point, after it hits the pole or not, that's

I: I see. Now, so that's something to remember.

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1 *Apparent Concepts*: What Daniel thinks of as possible to "recreate," or the role of recreating, is limited. *Pieces*: Daniel says he does not "remember" whether the ball will go faster or slower: it is a piece he does not have.

D: That's something to, what do you mean?

I: Well, you said, you said you can't remember whether it would speed up or slow down.

D: Yeah, I can't remember, or stay the same.

I: And that's not something you can really picture either.

D: That's not something I can really picture either, no. [flips pages] I know he did it somewhere, where is it?

I: So the problem that I picked is one that he did in lecture?

D: Yeah, I remember it being in the notes, I remember reading about it, studying it. That's one of the hard problems that he did in lecture, I remember reading it, and thinking that too, but, I can't find it in my notes. Ok, anyways. Um, so, if there's going to be a length there, the energy down here has to be 1/2 m, mgl, no, just mgl, mgl equals 1/2 m v squared, so, v squared, v is equal to l [unintelligible] in this case, it's going to loop around here, so this distance here has to be h minus, l minus h, so, for it to completely loop around, it has to have enough energy to get up here. So the minimum energy to get up here, is the distance of 2l minus 2h, has to, that's potential energy, mg, [mumble work, algebra]

[Daniel has $2gl - 2hg = l$.]

D: This is the potential energy at the top, if I did it correctly, and this is the potential energy

[tape side ends] [Daniel solves to get $l = \frac{2hg}{2g - 1}$.]

D: One. [pause] Except that g is a constant, g is a number, so, something's wrong here.

I: What's wrong? You said g is what?

D: g is a constant, so I guess this is just a number. Ok.

I: I'm not sure what you're saying.

D: I'm saying that my length has to be, this is what I got for my length, the length has to be that fraction of h, (ok) so this doesn't look right to me.

I: What doesn't look right about it?

D: Ok, the way I approach this problem is, um, I know potential energy, from your given thing, and therefore I will know your kinetic energy over here, because of conservation of energy, and if this is going to be l, so the potential energy is going to be mgl, that's equal to 1/2 m v squared, over here, so this will cancel, m's will cancel, oh, that's what I did wrong, I forgot the g's. Um, so, 2gl is v squared. [pause] [unintelligible]

I: Ok.

D: So if 2gl is v squared, and that's kinetic energy, kinetic energy at the bottom needs to be 1/2 m g l, or just mgl, is kinetic energy at the bottom. So, ok, if distance here is l minus h, therefore, distance to get up from here to here is 2 times l minus h, so the potential energy at the top has to be mg times 2h, l minus h. That will equal, um, that has to be the same as energy at the bottom here, since it's going to loop around. So, if you set those two equal to each other, and cancel out and solve for h in terms of l, that should be your answer. (Ok) [works] Except that it should come out clean.

I: It should come out clean?

D: It shouldn't come out like this.

I: Oh, it shouldn't be, that looks too messy.

D: That looks wrong. [laugh] I just don't like the way that turns out. Do you want me to finish this?

I: Um, sure.

D: mgl, no, mgl, this is [mumble work] h equals l over 2. [Daniel solves $mg\,(2l - 2h) = \frac{1}{2}mv^2 = mgl$ to give $h = l/2$.]

I: So you have h = l/2. (mm-hm) So, that means it's halfway in there, and so

D: Well, I'm assuming that, um, if h is l over 2, right here, I'm assuming that it's going to go 1
up here, it's going to up here, you know here you will have, um, zero.

I: It will stop.

D: It will, yeah, it will have zero velocity, so I'm also assuming that it will drop this way, instead of back down, or, straight down. [Daniel sketches the options: the pendulum bob reaches the top of its circle, and from there either drops straight down or continues along the circular path.]

I: What will it do? You're assuming it'll what?

D: I'm assuming that, it's going to turn around here, it has zero velocity here, and you're saying that you want it to loop around like that, (yeah) well I'm still assuming that, after this point it's going to do that, instead of fall straight down. [in a confident tone]

I: You're assuming it will do that. (Mm-hm.) Um, is there any way to show that it will do that?

D: [pause] I think so, yeah. Something to do with [unintelligible] velocity. [pause, page 2
flipping] Yeah, the velocity at the top, of the circle, has to equal, [page flipping] that's the first time I saw that person [Daniel names a teaching assistant]

I: Oh, this was a problem he did in discussion section.

D: Yeah, that specific case was like on the second homework assignment or something, and, I 3
forget how to do that. I['m not] really good with remembering how to do problems.

I: So are you starting a new problem, or are you continuing on this one?

D: Um, now I'm questioning whether that's, um, all I know is that, this is, if it's h over 2, then it will get to the top, but, I don't know if it will go straight down, or if it will fall down, or go in the circle path. Um, what I'm trying to find is whether this is the critical velocity such that it will not fall down at this point. I'm not sure, I think I'm doing this right, but

I: You think you're doing it right [pause] But you're not sure, you're uncomfortable.

D: Mm-hm.

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1 *Pieces, Formulas*: Daniel feels free to 'assume' that when the pendulum bob reaches the top of its arc, where he has found that its velocity must be zero, it will continue along the arc rather than fall straight down. There is a direct contradiction here between the result v = 0 and Daniel's sense that the bob could continue. The result does trouble him, but it is clear he has confidence in the method. In order to maintain the result, he modifies his understanding of velocity, at least locally, to be a formal quantity without evident conceptual value.

2 *By Authority, Formulas*: Recourse is to check with authority; Daniel looks for a rule by which to answer.

3 *Pieces*: Daniel implies that being able to solve problems means remembering how.

I: Ok. So, go, tell me something again in this problem about how you use, you were talking about how you like physics, because you can visualize things. So, and you started to tell me how you can't visualize this because,

D: Well, I can visualize this, because I can draw this whole thing. Take for comparison acid- [1]
base titration, what am I going to, there's acid here, I'm going to pour base, just going to put base here, that doesn't give me much, where over here I can draw it, and yeah, ok at this point, this is what's going to happen, at this point this is the energy, at this point that's what's going to happen, and I can even draw, I can confuse myself saying, well, is it going to fall down over here, stuff like that, in that case it's easier than acid base titration, or a battery [laugh].

I: Well, sometimes you can draw little pictures of little plus signs and minus signs moving in different directions for batteries.

D: That doesn't help me much with numbers though [laugh] .

I: So this helps you with numbers?

D: Mm-hm.

I: And, in what way does it help you with numbers, just that

D: This doesn't help me with numbers, it helps me with variables in this case.

I: Ok, keeping track of

D: l's h, and all the other things. And if you, well, the connotation of number I have is that, um, 1's, 2's, and 3's, well, no, yeah, in this case it hasn't yet, but um, once I get to this level, this point, then I can start plugging in numbers and will get my answer.

I: So You say it will come up here, gonna fall, and then it stops right there.

D: It will have zero velocity, I didn't say it was gonna stop. [2]

I: Ok, it will have zero velocity, you didn't say it was gonna stop.

D: I'm saying all the kinetic energy will be converted to potential energy.

I: Is it moving at this point? You didn't say it's going to stop, so now say, is it going to stop or not?

D: [pause] Is it going to stop or not. [pause] Good question. This is one of those things where if you asked me a year ago I would say no, it would go on, but now that I learned more about physics, I start to confuse myself with all the principles I've learned. [laugh]

I: That's interesting. So if I asked you a year ago, that was, you were still in the middle of your high school physics course

D: Well, I'm assuming that I don't know any physics at all, [laugh], I'd say, no yeah

I: Ok, if I asked you before you ever took physics, you would say it would keep going.

D: [pause] Mm-hm.

I: And now that you've taken physics.

D: Um, could you read the question back.

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[1] *Apparent Concepts*: Daniel describes a limited role for informal knowledge: a visualization helps one keep track of calculations.

[2] *Pieces, Formulas*: Daniel explicitly distinguishes having "zero velocity" from stopping, thus abandoning a conceptual interpretation of the quantity.

I: Ok. So you know, you know that the velocity's zero, (mm-hm), but you're not sure if it's going to stop.

D: Um, [pause] I'm not sure if it's going to stop. See I'm always confusing myself, I'm thinking about too many things at once, because, it seems to me that momentum will carry it this way, or inertia, will carry it that way, but I'm also thinking about critical velocity of whether it will have enough speed at this point to make it go around or not.

I: Right.

D: And [unintelligible] would work out equations to prove that.

I: What would, what would the critical velocity depend on?

D: [pause] Height. The height at which this was dropped.

I: Ok, and you say you need more calculations to find that out. Do you know what they would involve?

D: I think all the information is given, I just have to remember which, concepts to use. **1**

I: Ok. this is a pretty hard problem. I did not give you an easy problem. But I wanted to come up with some kind of problem so I could find out, when you're talking about visualizing, is the difference from chemistry to see exactly what you mean, so I think I got that. All right, well, let's see. If it's going in a circle, what would you use up here, [pause] [page flipping]

D: What was the question?

I: I guess I'm giving you a chance to find it in your notes.

D: Ok I won't. [laugh]

I: That;'s fine, either way. If this is falling here, and then it comes up to the top, [pause] I guess what's confusing me is when you say, you know the velocity is zero, but you're not sure it's going to stop. What's velocity?

D: Speed with a direction of where it's going, that's the definition.

I: Ok, so if velocity is zero

D: That's zero speed.

I: So doesn't that mean it's stopped.

D: [pause] Yes, [laugh] if you say it that way, yes. But, something's bothering me about that. **2** Because, if it's stopped, then it's going to fall straight down, that means I did the problem wrong, by the way you specified, because you said that you wanted the thing to go around, (right) So, if you, if somebody says something, ok, there's this projectile in the air, and suddenly it has zero velocity, then yes it's going to fall straight down, because there's not going to be any components going in the x-direction and all that's acting on it is going to be gravity.

I: Ok, so if gravity's acting on it here, (mm-hm) ok so you're not, are you, so know, so that's, this is a trouble now, you got velocity equals zero, which means it should stop, but then it would have to fall straight down, which would mean you got the problem wrong. (mm-hm) Ok, um, if gravity's acting on it, suppose that you got this right, if gravity was acting on it, can you say anything about the relationship between how fast it should be going and what gravity is?

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1 *Pieces*: Daniel describes the difficulty as one of "remembering which concepts to use."

2 *Apparent Concepts*: Daniel has a strong intuition that if the bob stops, it should fall straight down, but this is not sufficient for him to reevaluate his calculation technique.

D: Square root of 2gh, or square root of gh, is the velocity at the bottom of the circle? That's just on top of my head, remembering doing the previous problem. So

I: What do you know about, how, when objects move in circles?

D: What do I know about

I: Yeah, what kind of forces need to be on them?

D: M v squared r over 2. Is that right, I don't really remember. Acceleration equals v squared over r.

I: Ok, that's

D: That's centrifugal [petal?] acceleration. Well, nothing's coming to my mind. That's all I can say.

[end of interview]

<u>Fourth interview - 3/30</u>

I: So how are you on this stuff lately, rotation stuff and torque and everything.

D: I am lost, [laugh] I am really lost. Especially with spring break coming up, my spring break plan really blew up, I thought I was going to set up this spring break where nobody's going to be home, so I'd be really bored and I'd be working.

I: Yeah, right.

D: I know, yeah right. [etc on decadent break]

I: So did you get this problem set in?

D: I got the problem set in, barely, the last one.

I: That was chapter 11, right?

D: Mm-hm. I went to the discussion section, and that really helped a lot, the one before mine, he was really helpful.

I: So how could you, you got it in but you're lost. Or is that, did you get

D: Um, I, see, if, the thing is that the other TA gave us too much hints, so, I know how to do, I'm not sure if I know how to do the problems, I didn't set it up, all I have to do is evaluate the integrals

I: Yeah, that's what you said.

D: So, I understand the problems, if they get set up in some way and all I have to do is finish it, but

I: But this guy didn't give you too many hints.

D: No, this guy gave me too much.

I: Oh, so this guy who was really helpful got you to set up the problems without knowing really what you were doing.

D: [pause] No, I understand how he got that far, but if I have to start from scratch, I don't **1**
know if I'd be able to do it or not. One of the problems he even set up the whole thing, and all I have to do is plug in some numbers, [laugh] It's pretty bad. I mean, it helps to get homework

[1] *Independent*: Daniel expresses the need to work it through himself. Unreconciled.

points, but, it just doesn't get me thinking. And I haven't been thinking of these things all over vacation [laugh], so my mind is empty, blank.

I: So is this one of the blank chapters?

D: Yeah, that and collisions. 10 and 11 has been blank. And 12 has been blank so far too, so I am, in a [beach box?] I've got to get my butt in gear really soon.

I: Ok, well, let's stay away from 11 and 12. All right, I'm going to give you some other stuff then. Here's a problem. [gets out pen, paper] Um, two balls are thrown, with a speed [writing all this] v zero, from a height, a cliff or something, a height h. One is thrown horizontally, and one straight down. Ok? So question A is, which hits the ground first, can you read my handwriting? (yeah) And which hits with greater speed?

D: Um, the answer to A and B is the one that's thrown down.

I: The answer to both is the one that's thrown down?

D: Mm-hm (Ok.) Um, assuming that you're throwing them straight down with velocity v oh.

I: And why?

D: Why? (mm-hm) Because if you're throwing straight, the one that you're throwing horizontally has, um, a velocity in the x component of v oh, but, um, the initial velocity in the y direction is zero, while if you're throwing them straight down, only gravity's acting on it, but you're also giving an initial velocity v oh, so it's starting out faster than the one that you're throwing horizontally first, and it keeps on picking up speed, adding speed due to gravity, so it will hit with greater speed. **1**

I: Um, if this were a homework problem, would that be what you'd do, is that enough?

D: That's [unintelligible], yeah.

I: Ok, great. So would you write out a little paragraph to explain, or something like that?

D: Mm, yeah, and diagrams.

I: Ok, don't write out the paragraph, cause that would take, ah, but, cause what you can say in words is fine, but show me what you'd use besides those words to say it.

D: This is ball 1, and you're throwing it horizontally, v_x is v_0, v_y is zero, and ball 2, thrown straight down, v_x is zero, v_y is v_0. So, at the moment of impact, x is just 0 plus $\frac{1}{2}gt^2$, all right, no, 0 + gt, while for y is v_0 plus gt, so obviously, unless v_0 is negative, this is going to have a greater value than this.

I: Ok, so ball 1 is just 0 + gt, and, ok. Um, so, which part are you answering, part A or part B.

D: This is part B. And I'm just assuming that, if this one is going to have, if this one has greater velocity, of course it's going to hit the ground first. I thought that was kind of obvious.

I: Ok, in fact it's obvious enough that there's no need to do this kind of stuff.

D: [unintelligible]

I: Um, so, this one, then, will take less time, than this one. (Mm-hm) Now is it possible that the time, since you had t in here, is it possible that this time will be enough smaller than this one that this one will come out equal or greater?

---

[1] *Concepts*: Daniel gives a conceptual explanation. Reconciled: It can be seen as apparent.

D: [pause] Unless the value for v_0 is negative, there is no possible way that 2 will hit the ground after 1.

I: No, no, but if 2 hits the ground so quickly, this is, what you're showing here, what is, this was to answer part B, right?

D: Oh, oh. Yeah, you brought up a good point. So it is possible, if it's um, yeah, I guess it [1] [pause] that's a tricky question. [pause] I wouldn't imagine that, there is a possibility you could find one specific example where the velocity would be less than the velocity of one, and I guess you can just plug in some numbers to see, it is possible to do that, to check about that possibility. That's my answer.

I: Ok, so, can I get you to work that through?

D: Sure. [laugh] Ok, let's see, let's take a thousand meters, and, you want, so you want to find an example where, um, velocity of two is less than velocity of one, I assume.

I: Well, the question for part B is which one is going to hit the ground with greater speed, and,

D: Ok, since you brought that point up, you have to give me an answer for h, a definite value for h, in order for me to determine this.

I: So you think it might depend on the height. For some heights 2 will hit with greater, and for some heights 2 will hit with less speed.

D: It depends on the height and the velocity initially. To answer this part of the problem [2] specifically. Some specific cases where there might be, there's not just one answer.

I: U, ok, so your answer to part B would be, then, this wasn't out of this book this is out of another book, so your answer to part B would be, it depends on the specific values of h

D: And v_0.

I: And v_0. Ok. So that's, that would be something you'd say on a homework? That's interesting just because, ah

D: Um, I would plug in some numbers first, to check if there is any possibility.

I: Ok, well, go ahead and pick numbers, and see what you got.

D: [pause] So, I want t_2 to be less than t_1, [pause] I want this to be pretty small, then. Ok, 0.01, [inaudible] 0.8 [hitting calculator] [long pause] I'm confused now. [long pause] [hitting calculator, writing]

I: You picked 500 to be

D: Velocity initial.

I: So you have v oh plus 1/2 a t squared. [3]

D: v_0, actually it's v_0 plus, at? Yeah, at. [pause] Can I open the book, can I use the book?

---

[1] *Apparent Concepts*: Daniel feels his only option is to try different inputs to the calculation – he does not think to reason about the mechanism underlying the calculation.

[2] *Pieces*: Daniel assumes that there are different answers for different situations.

[3] *Pieces*: Daniel checks to see whether the formula is $x = x_0 + v_0 + (1/2)at^2$, or $x = x_0 + v_0\boldsymbol{t} + (1/2)at^2$, without trying to determine the answer from the self-consistency of each equation. *Formulas*: I assume that the difference between v_0 and $v_0\boldsymbol{t}$ should be conceptually accessible: the question is whether distance is given by velocity or by velocity times time.

I: Sure. What are you going to check?

D: Formula. [flips pages] [unintelligible sentence]

I: [laugh] So this may serve the purpose of pulling your mind back to physics?

D: Yup. And this was v_0t, so it's less than that.

I: So you were checking this term, v_0t.

[Daniel tries another set of initial values.]

D: Ok, so that wouldn't cut it out, it won't work out, so I'll have to do some greater distance, I guess?

I: So what makes you decide that won't work out?

D: Because, that means, um, if the height of the cliff is only 5 meters, this velocity is going to be 500 something, while, over here obviously, um, you're dropping something from only 5 meters it's never going to reach 500.

I: It's never going to make it up to 500.

D: It's never going to make it up to, it's [never?] going to make it up to 20. Um [long pause] Come to think of it, I don't think there is going to be, any, [unintelligible], because v_0 is always going to be a big factor, and, even though this falling's going to be, um, your argument to me was, what if the value of t-prime is so small such that there might be a possibility of vt being much greater than this, but, I think because of the velocity initial, um, the value for t-prime depends on velocity initial, and it's inversely proportional, so, even though this seems small compared to this, um, the
final result will be, I think it's always going to be vt is going to be greater. Just because I feel that's right. (Ok.) So, back to the original question. Yes, I'm not going to think of any more physics, and yes I think v t is right. **1**

I: Yes you what?

D: I'm not going to think of any more [laugh] deep physics, and

I: I'm not going to think of any more deep physics [laugh].

D: That's what I do, though, I mean, you ask me a question like this, and I'll come up with the answer right away, and, then I start thinking of what if what if what if, and I get really confused, and I don't come up with an answer, so I end up sticking with my original, my very first answer.

I: Ok. Well let me ask you a couple more questions, and then I'm going to, ah, be annoying. Um, so, you have, ah, the velocity for ball 2 is $v_0 + gt$. Where'd you get that, how do you know this formula? **2**

D: Um, book.

I: From the book?

D: Mm-hm. I think so [laugh].

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[1] *Apparent Concepts*: Daniel's informal reasoning consists only of figuring out what he 'feels is right.' He does not think to try to prove his guess, either to himself or to me.

[2] *Formulas*: Daniel does not show a conceptual understanding of the formula $v = v_0 + gt$, which I assume should have a clear conceptual interpretation. *By Authority*: He bases his understanding on his memory of it as "a given formula." *Weak Coherence*: While he acknowledges there is a 'derivation,' he does not consider it essential to his own understanding.

I: Ok. Um, if you had to teach this to somebody, how would you do it?

D: If I had to teach that to somebody?

I: Yeah.

D: The only thing I would say is that that's a given formula, I mean I'm not a deriving person, so, if, that's why I'm not going to be a teacher [laugh] or a professor, but that's what I'd say, like, that's the given formula, you got to memorize that, otherwise you're going to be screwed.

I: Ok. You're not a deriving person.

D: I'm not a deriving person.

I: Ok, but you're sure you got it right?

D: No. [laugh] Something like that. I'm not exactly sure. [looks in book] Yeah, that's right, v_{0z}, gt.

I: Ok. Um, and this one you didn't have right, at first., well, you had v_0, and then you put in the t, what made you think to put in the t?

D: Fake, mem [laugh] memory. Something didn't look right. 1

I: Ok. Could, let's see, well, so you're at a, you're at a, you're convinced that the second one is going to have a greater velocity, when it hits, but I get the feeling you don't think you've proven it?

D: I haven't proven anything. 2

[Daniel works for a while, using the quadratic formula, comes up with two values for the time for one ball (the + or – in the quadratic formula) and one value for the other ball, but he doesn't know how to compare them.]

D: So the comparison would be slightly hard.

I: But you have plus or minus here.

D: That's another problem. [laugh] Um, one of them has to be ruled out, in most of the problems I've done, except in this case I don't know which one.

I: Are you sure that one of them has to be ruled out here?

D: Mmm. Yeah, I'm sure one has to be ruled out here [laugh].

I: Why?

D: Because if I work this thing out probably I cannot have, I cannot possibly get two answers, one of the answers has to be, um, impossible, because I'm taking the quadratic of something and usually you've got one answer that's not right. 3

I: Ok, so one of them has to be wrong, and now you're sort of

---

1 *Weak Coherence*: When I ask how he knew to put in the *t*, Daniel says he did it from memory, and starts to call that "fake." I interpret this to indicate he distinguishes remembering from some 'genuine' understanding.

2 *Weak Coherence*: Daniel is aware that the reasoning he has done does not constitute a proof, but he does not seem to consider it essential that he construct one.

3 *Apparent Concepts*: Daniel justifies his having to choose either plus or minus in the quadratic based on a fact that "usually you've got one answer that is just not right," a computational rule. After he finds out that one answer is negative, he rules it out based on conceptual knowledge that time must be positive.

D: Um, I have to find out which one is wrong and which one is right. So, if I take, oh, yeah, if I go, negative v oh minus square root of v oh squared plus 2gh, um the value for this has to be positive, so I'm going get rid of v oh minus some positive number, and I'm get t equals negative and I can't do that.

I: Ok, why not?

D: How can t be negative?

[I gives Daniel a trick for comparing the two times: square both sides. Daniel finds that the one thrown straight down has a greater speed, but he has only solved for the vertical component. I asks for total velocity, and Daniel finds that the one thrown down "still has a higher velocity." I asks about one step, and Daniel finds an algebraic error.]

D: And they are equal. The velocity at the bottom is equal, when they hit.

I: Is that what you expected?

D: Not really. That was one of the choices I thought, but I was leaning towards this one having the greater velocity, but I wasn't, I didn't understand what you meant by this.

I: The total speed.

D: Yeah, so, that might have something to do with it.

I: Ok, so now you have an answer, are you happy with it?

D: [laugh] What's your definition of happy with it? ([laugh], Um) I'm satisfied with my answers, yes, I can't really call it happy [laugh].

I: Well, because, I, well, at first you'd expected this one to go faster, um, when it hit, but maybe that was because you were thinking about just it's vertical speed and not it's total speed, and then when I asked you for the total speed, I guess I didn't find out which one you expected, although you did say that you were leaning toward this one, um, so, it didn't come out quite to the answer that you expect, so, I don't know

D: What's my feelings towards that?

I: Yeah.

D: Of course I have to trust my answers that I calculated better than intuition, so, I'm more than willing to accept this as an answer, as a better answer. [laugh] **1**

I: All right, fine. Let's go on to another one. So, let me see. We did the two rocks, here's another problem. Ten minutes left, that's not going to be enough. Well, we'll give it a shot and see what happens. This is going to be a merry go round, when I'm done, and there are two people on it, playing catch, trying to play catch. And this person has a ball, and this person is going to try to throw the ball to that person.

D: Ok. Is he going to aim at the person, or something like that [laugh].

I: That's what my question is going to be, so should he aim right at the person?

D: Oh my gosh, [laugh] this is so confusing. [laugh] I've seen movies about this, and it's given me nightmares.

I: [laugh] Nightmares.

---

[1] *Apparent Concepts*: Daniel recognizes a conflict between "intuition" and his calculations, and he resolves that conflict by choosing to believe the calculation. He does not try to understand why his intuition failed or what intuitive implications there are in his choice.

D: So when the ball leaves the person's hand, it's always, if this person's going to throw that way, the ball naturally will have a component this way, because this person is spinning, so, what does that do? [pause] [laugh] This is one of those problems, where I'm going to say an answer and it's going to be wrong, and then I'm going to calculate it, and I'm going to come up with a different answer, and I'm going to like my different answer better, but let's give it a shot. If it's going to have to move this way, and this person is going to move that way

I: This person is going to be moving that way? Ok. What's this arrow, you're showing, oh, you drew the circle again over here, ok.

D: Different circle over here. [mumbles] Is that what the ball's going to do? Yeah, so, um, first answer is going to be, I'm going to throw it over here.

I: Ok, so he's going to aim

D: Away from the other person.

I: Ok. So let me ask, if I'm, now, I'm going to sit up above, and not be rotating on this merry go round, but just perched on some kind of a bridge, (mm-hm) up above it, and this person, let's say this person's been doing this for a while, so he's got it all figured out, and, let's so, however, he's going to throw it he gets it to this person, (mm-hm) what do I see happen? [pause] And more specifically, what do I see the ball do when I watch

D: The ball is going to go straight.

I: The ball's going to go straight.

D: Yeah, the ball is going to go straight, ok, it's going to be [unintelligible] I think the ball is going to be straight, but if you're on the merry go round and you're catching it, the ball is going to look like it's going to curve to you.

I: Ok, if you're on the merry go round and you catch it the balls looks like it's curved, but if I'm watching from above I'll see it go straight.

D: I think so. Um, I'm trying to remember pictures in high school movies, I think so. **1**

I: These movies that gave you nightmares.

D: Yeah.

I: I don't want to bring back traumatic experiences. Ok, now can you justify that, why that would be right?

D: Probably not [laugh]. Why that would be right. Let me try

I: Tell you what. Go ahead.

D: Go ahead. [laugh]

I: All right, let me ask you, let's do a different question for a moment, let's just have one person on the merry go round, so this is this guy standing here. This guy wants to play catch with himself, the merry go round is moving and he's going to throw the ball up, and catch it himself, (mm-hm) how should he throw the ball, should he throw it

D: Straight.

I: Should he throw it, so if you're standing on the merry go round, should he just throw it straight up and down like that, or would he throw it say, ahead of himself?

D: He should throw it straight.

---

[1] *Apparent Concepts*: The question of which is correct is a matter of remembering, aided by literal visualization.

I: Ok, and if I'm watching from above, what will I see the ball do?

D: You're going to see the ball curve, [pause] or, this is going to be, yeah, you're going to see the ball curve. [1]

I: Ok, can you sketch that, just to show me

D: Sketch which one?

I: Yeah, when I'm watching from above, when I'm watching from above, I'm going to see him move from here, to say, let's he catches it here

D: Mm-hm, the ball's going to do the same thing. [Daniel sketches the ball following an arc curved with the motion of the merry-go-round.]

I: The ball will do the same thing.

D: The ball will go over here.

I: Ok.

D: It's also going up, but I can't draw in two dimensions on this.

I: Well, if I'm watching from above, I mean

D: Ok, the ball's going to go like that.

I: So if I'm watching from above, not considering the changing in height, I'll see the ball curve with him?

D: Mm-hm.

I: So he'll see the ball just stay with himself the whole time.

D: Mm-hm.

I: But here I'll see it go straight?

D: Mmm [pause] think. I can't think. [laugh]

I: This is how I make my living is asking annoying questions.

D: [laugh] I can't remember which one goes curved and which one goes straight. [2]

I: One will be curved and one will be straight.

D: Yeah, I know that for sure.

I: Now one meaning this case where he's just throwing it straight up, or

D: No.

I: Between these two cases or

D: I'm talking about between the person who is on the merry go round, references frame, and one who is not, they are going to see, the movement's going to be different, but [pause] which one's which, see, like now, I can just draw a conclusion, oh, yeah, I can change, I can, if it's going to look straight here, of course it's going to look straight here, but [laugh] I can't justify that, so, [laugh] I can't really say that.

I: Do you think that's right?

---

[1] *Misconception*: Daniel shows the misconception that a ball thrown from the merry-go-round might continue to curve, as seen by a non-rotating observer.

[2] *Pieces*: Daniel thinks of the matter as a fact he does not remember.

D: I knew it once.

I: Without justification, which is right, which do you think is right?

D: Without justification, which

I: Yeah, you don't have to justify it, but just tell me which one you think is the right answer?

D: Well it's going to be one or the other, right, and it would be a guess one is wrong, and, well, one I don't know, so I'm sure with both cases it's 50-50.

I: Ok. So the justification isn't only for me, you would like to have justification for yourself.

D: Mm-hm.

I: Ok. We need to stop. I have one more. Just a few minutes left. [Paper shuffling] This is a question from economics, just to branch out into a new field. Um, the national debt is D. (mm-hm) And we suppose that at some give year, the national debt is Daniel initial, and supposed that I know the national debt changes at a rate R (mm-hm) dollars per year.

D: Mm-hm, positive value or negative value?

I: Oh, the debt's increasing [laugh], so let's say R is positive, the debt's going up. So the debt's going up at a rate R per year, and T years goes by, can you tell me what the final debt would be.

D: [long pause] So it's RT

[end tape side, next side starts with 15 seconds of silence]

[Daniel has written $D_f = D_i + RTD_i$, crossed out the second D_i, then $v_f = v_0 + at$.]

I: Ok, so you wrote this down pretty quickly, and, why did you write v f equals v i plus at?

D: Um, trying to draw parallelism. Um, at first I was confused about, this was the first thing that came to mind, except I don't know if I am supposed to multiply this by Di also or not. (Uh-huh) Um, so, I sort of thought of it, as a different problem, as a velocity problem, and, that's how my thought process went. For going, confirming this conclusion, or making myself feel confident with this answer again, if I am supposed to, if I was to multiply this by Di.

I: Ok. Um, why would this, why this?

D: Um, because I imagined that will, [pause] ok, so we're not thinking about interest, so I'm [1] not going to worry about interest. [laugh] Um, the initial debt, I just thought about as something like initial velocity, just because both of them are initials, and if you say that debt is going to increase at rate R, or [unintelligible] T, well, in this case, velocity is also going to increase with rate a, and the time is just t. So if velocity final is given by this initial, I'm, I'm mainly, the way I thought of it is that, well, both of them are increasing with rate R, except that this is increasing at rate R and this is increasing at rate a, both of them take time t, increase of this given time, so is this, so this one must, the formula should be basically the same thing. Or should be quite close. So that's how I drew that conclusion. These sort of problems are like, not national debt, but stuff like compound interest, like how much money will I have at the end of blah years, basically it's just using different variables, so, it's from a while back. Um, that's how else I would use things with mathematical values, but since I'm taking physics now, these formulas are really sticking to my mind, so, [unintelligible] and that's how I think of parallel between these two problems is by using this.

[1] *Apparent Concepts*: Daniel associates the formulas based on a correspondence between quantities. He does not seem aware of the conceptual structure of the expressions. (The economics question seemed to make Daniel think of formulas from that course, rather than of his common sense.)

I: Ok, um, are you confident about this?

D: 75 percent.

I: 75 percent.

D: Like, I'm still confused whether or not, [pause] yeah, I'm confident, but wait, let me think about it some more. Ok the debt, this is going to be increasing, and this is going to be compounded [pause]

I: Compounded.

D: [unintelligible]

I: What do you mean by compounded?

D: Like, compounded interest.

I: Ok.

D: I don't know, I'm just thinking of too many things at once.   I'm 75 to 80 percent sure that that's the answer.

I: We're moving up. [laugh] 75 to 80. And this is right because of this? One reason I gave you an economics problem was to get out of physics, so that's why it's ah, you came back to use the physics to justify it, which is interesting. [pause] So

D: Are you asking me a question? [laugh]

[Discussion of the question, Daniel explains that he was confused: "it's too much closely related to money, financial banking stuff, and just the, concept of flat interest and compound interest is really getting me confused about this."]

I: Is there, would there be an area where you wouldn't, where it would be something that you wouldn't have to look up in the book, but you could figure it out.

D: I can like derive it, you mean?

I: Or whatever, or know

D: Or just know it?

I: Derive it or just know it.

D: Well, that's not pretty hard to memorize, for me, I mean, if I start really thinking about it I probably could get that, working like this [unintelligible] still I could probably still memorize that formula, but **1**

I: Why?

D: Because it is just used so often, and I can always just think about it if I'm not sure, I can always just imagine myself driving, and then derive it from driving. **2**

I: Ok, how would you derive it from driving?

D: Ok, for instance, I would start from um, stop, so my initial velocity is going to be zero, and then I'm going to try to figure out how long it takes me to get to um, or, a rock dropping, I drop something, this pen, um, I would have um, [pause] just pick some distance, like, some values, and calculate values for a, cause I know area for a, I probably could just come up with that

---

1 *Pieces*: It is implicit in Daniel's statement that knowing means remembering.

2 *Apparent Concepts*: It should be possible to "derive it from driving," but Daniel is not able to to so.

formula [laugh], I don't know how, but I can think about it and, [pause] because, the only, 1
um, this formula, distance is 1/2 a t^2, (mm-hm) that I know by heart, I've never forgotten that formula, and, the one that I'm always not sure is v = at, that's I'm kind of shaky.

I: This is the one you're shaky on

D: Um, yeah, 5 percent of the time I'm going to forget that formula or be unsure of that formula, but, if I know this, I'm pretty much going to remember that this is not going to have a square in it, this is, the t is only going to be to the power of 1, so if I start thinking about it I can probably come out with this.

I: Using d = 1/2 at^2.

D: As a reference, not

I: Now why is this one so easy to remember?

D: Oh, just form, I guess, first of all there is this 2 and 2 here, something that just sticks to my mind, I can't forget about it.

I: Um, could you figure, if somehow you had never taken physics, could you figure

D: Derive that, no. I could not have derived that.

I: Or this one, if you'd never taken physics?

D: Derive it mathematically, no.

I: Or come up with it in some way.

D: Probably.

I: So you could probably come up with either of these?

D: No, I could come up with this. If I totally forgot about this, I could not have figured it out. 2

[end of interview]

Fifth (final) interview - 4/27

I: Ok, so let's just pretty quickly go through this. So let's see, you got the second one wrong, full credit on the first, and third, fourth pretty much, so basically, you missed two problems, the second and the fourth and got everything else right.

D: Yeah, but everybody did the same thing.

[Everybody got around a 69. We move to the first problem, a multiple choice question, in which a rock is dropped off a bridge: when the rock had fallen for half the time it would take to reach the water, would it have half the kinetic energy it would have on hitting the water, half the momentum, both, or neither. There was a second part to the question, offering the same choices for when the rock had fallen half the distance.]

I: So just, same stuff, how'd you do it.

D: Um, plug in numbers, basically, as you can see.

I: Oh, so you just picked values.

D: Yeah, arbitrary values.

---

[1] *Pieces, Formulas*: Daniel says he knows the formulas by literal memorization.

[2] *Pieces, By Authority*: Knowing is remembering: if Daniel forgets, he has no recourse but to find out from an authority.

I: And plugged it in. Ok. So that was it, was it hard?

D: Um, no. 10 points in 7 minutes.

[The second question involved a car rolling along level frictionless tracks, filling with water that drains from a hole in the bottom: does the speed increase, decrease, or stay constant?]

I: All right, this one, [reading Daniel's solution on his exam] momentum stays the same, momentum has to be conserved, so if water pours in the same rate it goes out, and m is constant, so v has to stay the same.

D: Obviously not. Um, it has something to do with momentum in the x-direction and in the y-direction, and, since the water that's flowing out has momentum in the x-direction, while the water that's pouring in has momentum in the y-direction, it's actually losing momentum, so it decreases.

I: So, the water that's coming has momentum, the water that's coming in doesn't, so it loses momentum. Did you figure that afterwards, or is that in the solutions, or

D: It's in the solutions.

I: Ok. So that's just, bam, 10 points.

D: Yeah, kind of hurts.

[Problem 3 involves a satellite in an orbit that "just skims the surface" of the planet. It asks for a proof that the period T depends only on the gravitational constant G and the average density of the planet, but it does not depend on the radius. The second part asks for the period given the Earth's density and the value of G.]

I: Um, 3, a satellite in circular orbit, how'd you do this?

D: Um, this is like equations from the last one, m v squared **1**

I: So this is like the last one, like

D: The last midterm, they had a problem like this, except now they want to prove that the period is independent of the radius, but, um, dependent upon density, which is, I think the way they word it, the way they ask this question is kind of bad, if you think about it, because they said that, um, period T is only dependent upon the average density, and it's independent of the radius, but density is dependent on radius, that's, I think, if you really think about it, if you just don't look at it and solve it, of course you're going to get it, but if you really think about it, if I think about it some more, it's going to confuse, because, I think indirectly the density is also related to the radius, so **2**

I: Ok, and so, so what did you do, I mean did that confuse you on this, or did you not think about it.

D: I didn't think about it for, I mean, I know that, if I want to really go into details and be real trivial about it, but again, that's not what they want, so

I: Ok. Ok. And then this part is just plugging into this part, is that right?

D: Yeah.

I: So you just set up m v squared over r equals G M m over r squared, and then you solved v squared equals, so, you've got T equals 2 pi over v, and v equals 2 pi over T, so v squared is 4 pi squared r squared over T squared, so that is that, and then you solve for T.

---

1 *Formulas*: Daniel describes the content as formulas.

2 *Weak Coherence*: Daniel does not "really think about it," because it would confuse him, because it's "not what they want," and, moreover, he only misses 'trivial details.'

D: And I solve for density, back here,

I: Ok. And so that's that. A thin walled hollow cylinder and a hollow spherical ball start rolling down together, which one hits the bottom first. This one did not go well.

D: Um, no. I can't do it in 5 minutes obviously.

I: Right.

D: So, it's in my notes.

I: Did you not have much time for this one?

D: No, I spent 40 minutes on the next one.

I: Oh, you spent 40 minutes on this one and got it, all right. So, 20 points in 40 minutes versus 10 points in 7 minutes.

D: Oh yeah.

I: So what happened, you got I equals m r squared, so can you tell me something about what went wrong, or what were you thinking.

D: Um, well, I got somewhere, and then when I wrote down I equals m r squared, and then I started writing blah blah blah, I just threw it in, so basically I'm just writing this as fast as I can [laugh]. It's in my notebook, the same problem, so I was kind of upset about that. I can't believe the mean was so high though.

I: It is a pretty high mean, actually. And this is just a collision. So let's see, how did you do this? A lot of erasing.

D: Yeah, I did a whole page of work, looked at it, there was a bunch of double integrals there, said wow, I'm really proud of myself, wait a minute, I did it wrong [laugh].

I: It's wrong, oh my.

D: [laugh] Saying, this is really easy, I don't need to take any integrals at all [laugh]. so I felt stupid. I mean I did this in like 8 minutes, and I spent 32 minutes on the integrals.

I: So what were you originally doing?

D: Um, I was trying to integrate everything, and was trying to find inertia like with integration, with dm, and all the other things, and

I: And what made you realize that that was the wrong way to go?

D: Because they would not have given me the center of mass, the I center of mass, for no reason at all. [1]

I: I see.

D: And I just think, wait a minute, I can use the parallel axis theorem to do this.

I: Ok. Ok. So they gave you I [for the] center of mass, and at some point you said wait, they gave me that value, I must have to use it. [pause] Ok, and this is a collision problem.

D: Yeah, they only gave me 18 points for this problem, I don't know where I went wrong,

I: They didn't mark anything wrong, did they.

D: Yeah, and [the professor] was not, didn't say anything about it.

---

[1] *Formulas*: Daniel solves the problem based on what is given, rather than on physical principles.

I: Let's see. Speed of the truck after the collision is, the total kinetic energy, collision elastic, this is not elastic, huh. Did you ask?

D: Yeah, he wasn't too happy about me asking for 2 points.

I: So, ok. All right, um, let's see, I have a list of things that we have to do. you never did, you never talked anything about the homework problems. A few weeks ago I was, people tell me something about how they did these homework problems in this chapter, questions 3 and 8, um, and I don't think we did that, did we?

D: Mm, no.

I: Ok. So this is from way back, I don't know know if you have the homework with you,

D: Yeah, I do.

I: Ok, so if you want to look at that, or whatever, just, um, or just remember, or do it now, or um, problem number 3, and the first part of problem number 8, just how did you so them.

D: Ok, number 8, the first part was pretty easy. Um [pause] this was during spring break, I remember doing this during spring break [flipping pages].

[Talk about his hectic last 2 weeks, only 20 hours of sleep this week, roommate waking him up, etc.]

I: So which one, are doing number 3 first, or number 8?

D: I'm doing number 8, actually. Ok. [unintelligible phrase]. so, velocity will be, ωr, and, this is for velocity at the bottom of the tower, because the radius is just r from the center, and the velocity at the top is going to be ωr plus h, because there's going to be that extra height. The difference magnitude of the velocity is going to be v top minus v, which is ωr plus h minus r, which is going to be ωh. Not much to prove.

I: Ok, now, where did you get this [$v = \omega r$]?

D: It's in the book. [flips pages] It's here.

I: Ok. And I guess that's really all that you needed for this. Ok. And number 3?

D: So, v is, 900 kilometers per hour, radius is, [inaudible] so,

[We pause for a moment while I check the tape to make sure it's moving.]

D: Angular velocity is v over r, so it's 900 km/hr, over 10 km, is equal to 90 per hour, or 0.25 radians over seconds. Just plugging into that. [$v = r\omega$]

I: Ok, so one of the reasons I picked [problem] 3 is just cause I wondered if it seemed weird, um, because this is, an angular velocity, which is things moving in circles, but the plane is moving in a straight line.

D: You're asking me whether it seems weird to me, is that your question?

I: Yeah. Well, how did you know to use this, because the plane isn't going in a straight line, I mean the plane isn't going in, you're standing and watching the plane fly overhead, you're not watching it go in a circle around you, you're just watching it go

D: Well, first of all that's the only reasonable formula that is in the book, but **1**

I: I'm sorry, say that again?

---

1 *Formulas*: Daniel describes problem solving as choosing a formula from the book.

D: This is the only reasonable formula in the book to solve the problem, but, if this is the earth, and this is the plane, that I've been on so many times, [laugh] if it's going to go straight, well, I don't want it to go straight [laugh] I want it to go in a circle, [Daniel draws a circle around the earth] so, even though it seems like it's going straight, I know it's not. [1]

I: Ok. so it's really going in a circle around the earth. Well then

D: It's flying, its motion is the same as the circle, as the curvature of the earth.

I: Why aren't you using the radius from the center of the earth then?

D: Um, that's a good question, I should have thought of that. [pause] I don't know, it's just, um, yeah, that's a good point. I don't know what to tell you. I wonder if the solution set would tell. [looks for it, flips pages] Which homework is this, eight? Um, they use the same equation. [2]

I: Ok. [long pause] Well, anyway, um, let's see. [pause] What would you,what would be different, if you were standing on the surface of the earth, and you're watching a plane go by, what does this have to do with your standing there? When it give angular velocity in relation to someone, how does it, where does the angular velocity of the aircraft, it says relative to you, standing on the earth, if you're standing on the earth and the plane is going by, how would you perceive this as happening, what would you see that would mean that there'd be an angular velocity?

D: What would I see?

I: Yeah. [pause] What does the angular velocity mean in that situation.

D: [pause] I I would say it's just the same thing as here, because, I don't think it's really travelling in a straight line.

I: Well, if it's here, can I use your pen, here's the earth, here's the plane flying that way, um, I guess that the angular velocity if it's here would be this, how fast this angle's changing.

D: Mm-hm, with respect to me it's going to be how fast this angle is changing.

I: Well, I'm wondering if that helps answer the question of why use this radius and not this radius?

D: Yeah, yeah it does.

I: Ok. All right. Let's do other things. I wanted to go through, in also in, ah, somewhere in here, and just read a passage, um, just so that you can tell me what you pay attention to when you're reading this, and what you did and didn't, how much of it makes sense, how much of it doesn't and that sort of thing, just to find out much, what you should get out of the book. So, this is good. So just this page is enough, just from here to here. Um, I don't remember, were you someone who uses the book a lot, or were you not?

D: Yeah, I would say I do.

I: So, just in this page, if you could go through, say what's going on here, but also say, what when you're reading it would you pay attention to mainly, or what would you skip, or that sort of thing.

D: Mm.

I: Maybe you don't skip as you read, maybe you read everything, which,you know, that's

D: Should I, do you want me just to go through this once?

---

[1] *Apparent Concepts*: Daniel makes an apparent but inappropriate application of informal knowledge.

[2] *By Authority*: Daniel justifies his use of the equation by an appeal to authority.

I: Yeah, starting here to here, and just go through, and say well, this isn't too important, or this is real important, or this is the most important, or whatever.

D: [pause] Well, up to the last sentence, I think everything in the passage is important.

I: Ok.

D: It's basically saying that for any system of particles that has no, fixed pattern, if you break [1] it up into little tiny particles, the total kinetic energy of the sum of this whole, of the whole rigid body, which is what they refer to it here as, is the sum of the individual kinetic energy of these little particles.

I: Ok.

D: And the way you find the total kinetic energy, is by using the summation sign, summing up, every single one of that. Um, all the particles move with the same angular velocity along the circular path, yeah, that sort of makes sense, and I think that's important, because, if you just pick a center of mass, and draw, like, if this is going to be the angle it's going to go through, well, if all these particles lie along this line, will move there in the same angular velocity, they have different linear velocity, but it's the same angular velocity, that's what they're saying over there, so the total kinetic energy is just, sum that up again, except that now here you're substituting v with the equation r omega, the square of that, so you're writing that as kinetic energy is 1/2 inertia, omega, because inertia is, um, m, is the summation of m_i r_i. And that's basically what it's saying, the whole thing is that.

[Discussion about reading the book in general. Daniel says he skims first, and then goes through more carefully later.]

I: Um, a problem for you. Take a watch, or whatever, I throw it up in the air. Just after it leaves my hand, but while it's still on the way up, what are the forces on the watch?

D: Um, just as, at the moment it leaves your hand.

I: It's just after it leaves my hand, but while it's still going up. [2]

D: Gravity going down, and initial velocity going up.

I: Ok. Um, so, so there are two forces on it, there is the force of gravity down and the force of velocity up.

D: Mm-hm.

I: Ok, and then can you describe to me what happens to those two as it hits the top, as it, well, hits, as it comes to the top and stops and turns around.

D: Well, um, you can't really describe velocity as a force, because it's different dimensions and everything, but gravity's, um, the initial velocity going up will be decreased in time by gravity, and as it reached the top of the flight, that's when velocity is all taken away by the deceleration of gravity, that's why velocity equals zero, so it has no more, quote unquote energy going up, has been taken by the force of gravity, has to come down, because now the [3] only force acting upon it is the force of gravity.

---

[1] *Concepts*: Daniel describes the essential conceptual content of the passage. Reconciled: The content may be be apparent, which is to say, he may have only a superficial understanding. Later, he does not think of this content in discussing the ring problem.

[2] *No misconception*: Although Daniel first includes the initial velocity as a force, when I ask for further explanation he corrects this.

[3] *By Authority*: The phrases "quote unquote energy going up" and "from what I've learned" give the impression that Daniel's answer is based on authority and does not agree with his intuition.

I: Ok, so, are you saying when it's going up that the velocity up isn't a force, or it is? You said it can't be described as being a force.

D: If, it's, if my word says that, yeah, well, let's just say that, velocity is, from what I've learned, velocity is not the same as force, because velocity, dimensions for velocity is just meters per second, while, um, dimensions for force are Newtons, so they're not the same thing.

I: Ok, so then, when it's, so, so, would it be fair to say that when it leaves my hand, the only force on it is gravity? While it's, after it leaves my hand while it's still going up, the only force on it is gravity.

D: The only force acting on it? Yeah. (Ok.) Yeah, because I would describe the force going up, I will say that that's just kinetic energy that's bringing it up, your watch has kinetic energy, that's why it's moving upwards, while there is gravity acting, pulling it down.

I: Ok, um, good. Done with that. Next one. This is a canyon or something, and here are the two walls of the canyon, and here is a bridge. And this person here weighs 98 pounds, and he's got 3 balls, each of which weighs 1 pound, but the bridge, can hold 100 pounds exactly, maximum, [sketching all this] it's a government made bridge. So, if this person takes these three balls and holds them in his arms and starts walking across the bridge, the bridge is going to collapse. So what he says is, well, I want to get across the bridge, and I want to take these with me, what I'll do is I'll juggle them. Does he make it across the bridge?

D: [pause] I'm trying to figure out [laugh]. I'm trying to figure if one ball is always going to stay up in the air. [pause] Yeah.

I: He does.

D: Mm-hm.

I: And why.

D: Because,the force exerted by, are we, is this in terms of mass, uh, force, 98 pounds, is that, [1]
are you saying that 98 pounds is a force or a mass?

I: Um, his weight is 98 pounds. If he stands on the scale, he weighs 98 pounds.

D: He weighs 98 pounds, and this is 1/2. Yeah, because, the only force that's going to be [2]
acting, exerted on this bridge at all times, if he juggles it right, is only going to be a hundred pounds, because, one ball's always going to be up in the air.

I: Ok, now suppose someone argues, well no, because suppose that you throw the balls really high, say, then when they catch, they catch and go [sound effect] so in catching it, the ball exerts a force in your hand.

D: But the force is just mass times acceleration, and acceleration is mg, mg is always constant. So the force is always constant. It's, exerts a velocity, sure, if you throw it way up in the air, it's going to come down faster, but that doesn't mean that it's going to exert more weight, that's not going to add to the force.

I: Ok. Um, wouldn't you think, just from picture taking a brick or something, one pound is a pretty heavy ball, and you throw it pretty high in the air and you catch it, doesn't it, [gesture]

D: Brings it down.

---

[1] *Formulas, By Authority*: The question indicates a view of physics as involving a set of rules mandated by authority: Daniel wants to know whether to apply the rules associated with masses or those associated with forces.

[2] *Formulas, Misconception*: Daniel applies a rule associated with forces. It is, however, a misapplication, indicating the misconception I was looking for with this problem: he does not expect the ball to exert any more force on the juggler's hand than its weight.

I: Doesn't it push down on you pretty hard when you catch it?

D: Mm-hm. [pause]

I: But that doesn't affect your answer.

D: I [pause] It sort of does, because, um, [pause] well, for one thing, I know if you start jousting on the bridge, and starts exerting a force on it by going up and down, I'm sure the bridge will break, because that's really

I: Ok, so if the guy goes out on the bridge and starts jumping up and down the bridge is going to come down.

D: Yeah. (Ok.) But, juggling, unless it's, you're taking extreme conditions [laugh], if I just think 1
about it, mostly all the energy, all the force is going to be exerted on the arms, and if you do it right, the extra force is going to be dissipated by the motion of your arms, or something like that.

[I argue that "something's got to be holding up the 101 pounds." Daniel remains convinced of his answer.]

I: I want to leave enough time to ask you more general questions, so maybe we won't finish this, but we'll see. This is a ring [sketching], it's mass is 5 kg, it's radius is 2 meters, and it's turning, like that, so that every point on the rim, this point on the rim, a point on the rim is moving with a speed v equals 3 meters per second. Ok? (mm-hm) The question is what is the kinetic energy of this ring.

[Daniel works, flips through his exam and the textbook, checks a table of moments of inertia. At first he looks for the moment of inertia for a solid disk, and I reminds him that the problem involves a ring. Daniel finds the moment of inertia for a ring in the table.]

D: So, kinetic energy is going to be m r^2, plus, times, v over r^2, so the r's are going to cancel, so it's going to be mv^2, it's going to be, [works] um, kg-m/sec, I don't know what the unit for mass is, this is going to be mass.

I: Ok, so it's going to be 9 meters, 9 mass times meters per second. (mm-hm) Ok, so you used, kinetic energy is $I\omega^2$. (mm-hm) What if somebody wanted to solve this using 1/2 mv^2, could they do that?

D: [pause] Yeah, sure.

I: Ok, how would that go?

D: Well, I would just substitute in, solve for v, v is ωr, um [pause] I think this is just a matter 2
of manipulating the formula, can I have a piece of paper? Kinetic energy is supposed to be I ω^2, and v is ωr, um, so ω^2 is v^2 over r^2, v^2 is $\omega^2 r^2$, [pause] it's just going to be I, v^2, over r^2. Is this what you're asking me? This is going to be, oh, you wanted to solve it in terms of m v^2?

I: Right, 1/2 mv^2.

D: Isn't, [pause] are you talking about a general case, or is this in this situation?

I: In this particular case, you started with $I\omega^2$, and I'm just wondering would it be possible, because, the first thing you learned about kinetic energy was to use 1/2 mv^2, and you started with $I\omega^2$, and so you got an answer, and I'm just wondering if it would have been possible to use 1/2 m v^2 in this case, to get an answer.

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[1] *Apparent Concepts*: Daniel finds an apparent, misconceived experiential justification, that "the force is going to be dissipated by the motion of your arms.".

[2] *Formulas*: Daniel describes the question as "just a matter of manipulating the formula."

D: Mm, well, the answer for 1/2 mv^2 has to match the answer for $I\omega^2$. It's, am I using the formula correctly, is it 1/2 $I\omega^2$ or $I\omega^2$? [flips pages] It's 1/2 $I\omega^2$, so, um, in this particular case, the case with the ring, 1/2 mv^2 will also work. (ok) Just because, the moment of inertia of the ring happens to cancel out the r^2 in the bottom of ω. 1

I: Ok. but if it weren't, if the moment of inertia

D: If the moment of inertia was, like 1/2 mr^3, it wouldn't work. Or, something. If the power of r, in the inertia, gas to be r^2 basically. 2

I: Ok, and in another body it might not work out to be r squared, and so you would not be able to use 1/2 mv squared.

D: Mm-hm [Yes].

I: Ok, so if you were going to do that, oh I guess you just did it, so you just

D: Well if I use I omega squared, the final answer

I: Oh, I see, it works, you just showed it actually that it works out to 1/2 mv squared. Ok. So, I'm just, you started with 1/2 I omega squared, why not start with 1/2 m v squared?

D: It's just makes more sense to, because this is a circular motion instead of a linear motion, it makes more sense for me to use omega squared, because omega squared has to deal with angular velocity, instead of angular velocity.

I: All right, done with that, and now more general questions.

I: Um, the question is, and we've talked about this before actually, um, we spent a long time talking about this before, but I wanted to go back to it, is, when you're working in this course, you said in the first time that we met actually, that you liked physics because a lot of the stuff you can see it, you can do things with it, and you can know things just by looking out the window, is the way you put it. Um, and so I'm wondering, how much of that kind of thinking have you found you've been doing in this course, is there a lot of, is that helpful?

D: It can be helpful, and it can be really confusing, just like that juggling problem. (right) Right, um, in chemistry, it would be really hard for me to picture, as I said the atoms moving around, but um, like, um, you, said about the juggling problem, because it's going to exert more force, even though the weight's going to be constant, you know what I'm saying?

I: Right.

D: Um, yeah, because then I can picture, just by experience if, I walk across a dinky bridge, and it will barely hold me, and if you start jousting on it, of course it will break, so, just by remembering that I can answer your question, your specific question. 3

I: So that's a case where it's helpful.

---

1 *Formulas*: Daniel requires only that the answers come out the same. He does not consider the conceptual relationship between the two formulas.

2 *Pieces, Formulas*: Daniel does not relate the current discussion to the passage he had read earlier. There, the concept of moment of inertia was introduced to facilitate the calculation of kinetic energy: $\frac{1}{2} I\omega^2$ was derived from $\frac{1}{2} mv^2$. Daniel reasons based on the literal content of the formulas, not on their conceptual or formal relationship.

3 *Apparent Concepts*: Daniel speaks of using experiential knowledge to "remember" what happens in a particular situation.

D: Mm-hm. And also, you said that, my argument at first was that, well, the force is always going to be constant because the weight's always constant, but, I remember when we had a scale at home, I used to play around, jousting around throwing a tennis ball, and yeah, the thing will go up, actually if you put a lot of speed on it, it will actually have more force, and, just by remembering that, if somebody throws a tennis ball, or hits a tennis ball st me 100 miles per hour, it happens a lot in high schools, [laugh] and if I don't block that stupid thing, it's going to hurt, whereas if someone just throws it at me and hits me, it wouldn't have any effect on me at all, so that changed my mind from saying that force is always constant to force has something to do with it, somehow related to velocity, even though right now I don't know what the relation is, between force and velocity and how it actually effects that mathematical equation, the whole thing. 1

I: Ok. So it's still helpful there.

D: It's still helpful.

I: Ok. But you said sometimes it's helpful and sometimes it's confusing.

D: Yeah, sometimes it's confusing, because I believe in one theory, and then I look out the window, and what I see outside, outside, sort of disproves my theory in the beginning, and then, if it's, vague enough such that I really don't know which one to choose, which one to believe in, it can get really confusing, you know. 2

I: So do you have an example of that?

D: Um, yeah, if you go back to like the juggling problem, you said, um, will he be able to make it, and at first I said, well yeah, because there's only going to be two balls in the hand, and stuff like that, so he should be able to make it, well, then I started thinking about the force that's going to be exerted, said, yeah, that's a good point, that kind of confused me, and then I came up with this thinking that, well, if you do it right [laugh] then you should be able to make it.

I: If you do it right.

D: Meaning that if you juggle it softly enough, you should be able to do it without exerting that much force. But, then I started thinking, well, if you juggle it really lightly, it will still, it does it still affect, does it still increase the force by a little bit, and if it increases the force just by a little bit, then if the bridge strictly can hold a maximum of 100 pounds, then it should break, so it's those kind of thinking that can throw me off sometimes.

I: So, tell me, which threw you off, I 'm sure what was

D: About the juggling problem?

I: Yeah, what was it that threw you off, or might have thrown you off?

D: The fact that I started thinking, I remembered how force may be, may be related to velocity, so in that sense, if I throw the brick high enough, it will acquire enough velocity that might break the bridge, or might joust it, because I know that by throwing something heavy, if I'm balancing on something and if I catch it, it will throw my balance off, because it will joust a little bit.

I: And that threw you off, or could throw you off.

D: That threw me off.

---

1 *Apparent Concepts*: Daniel makes an apparent and inappropriate connection with his experience, which supports a force-causes-motion misconception.

2 *Apparent Concepts*: Daniel speaks of having a variety of ways to think about problems, and not knowing which to "believe in," as opposed to examining the ideas and figuring out where some break down, why one makes more sense, and so on.

I: And that threw you off by saying maybe the b

D: Bridge will break.

I: Ok, when in fact you want to say that the bridge won't break.

D: Mm-hm, because there's only 100 pounds, of force on it, quote, 100 pounds of weight on it. [1] So what threw me off is that I started thinking that if there is a possibility that force is related to velocity, but I don't remember, I don't recall, seeing any formula like that, so that threw me off. (I see.) It may have something to do with momentum, but I'm still confused with, I still don't know the exact difference between momentum and energy.

I: And energy, that's another thing we've been talking about. Um, ok. So there's, sometimes it's helpful and sometimes it's confusing, how do you know which is which, how do you know whether it's being helpful or confusing?

[tape flip]

D: That's pretty obvious, that question is really obvious. It's pretty hard for me to describe when I know it's confusing and when I know it's helpful, because, if it's helpful, I have no doubts about it, I look out the window and I see an example, and the example just proved my theory, and that way it's not confusing.

I: Ok, and if it's confusing you look out the window and see something that's different.

D: Yeah, and I begin to ask what if, and lots of questions.

I: Now when it's confusing, what do you do?

D: What do I do?

I: Well, suppose that you have, I mean, well, here's a case, where you have something about the juggler, and it seems like on the one side you know that the bridge won't break, um, but on the other side you have this kind of thing where, you know, force might be related to velocity, and it's going to come down and break it, so now

D: Well, first of all talking to people helps, I mean, um, I don't necessarily have to ask them whether the bridge is going to break, I'm just going to ask them how force is related to velocity, and if they say that, if they can prove to me that force is related to velocity, that will affect my answer, because, if they that too, then of course the bridge is going to break. If you, if it can only hold 100 pounds maximum, not even a milligram more. But I don't have to ask them to explain to me how the bridge is going to break and why, my question to them is going to be how is force related to velocity.

I: Ok, let me go to another kind of example, and that's, ah, we spent some time talking about this formula, $v = v_0 + at$. (Mm-hm.) And I asked how do you know this, how do you know this is true, and actually I don't remember what you said.

D: I don't either. [laugh]

I: Well how do you, here's an equation, um, tell me something about, what we're talking about right now is, is you have sort of stuff that you just know, from looking out the window

D: How can I prove that, or how can I remember that?

I: Yeah.

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[1] *Apparent Concepts*, *By Authority*: Daniel clearly thinks of his experiential knowledge as potentially relevant. However, he says he discarded it here when he could not 'remember seeing' any connection with the formalism. He did not try to understand the relationship between his intuition and the formalism, to account for the contradiction or to modify his informal knowledge; he simply dismissed it as something that 'threw him off.'

D: That's what you asked me last time, is like, how do I remember, how do I memorize 1
that formula, I mean would I be able to remember it if I haven't taken physics in 10 years, that's sort of what you asked me. (ok) Well, um, for instance if you tell me that acceleration is 10 meters per second, or 5 meters per second, I'll say 10 meters per second, that's a nice round number, and I'm going to run at, accelerate at 10 meters per second, which is probably impossible, but [laugh], so, I'm going to say ok I'm going to start, going to run for one second, and I'm going to accelerate 10 meters per second, so my um velocity after one second is going to be 10 meters per second, because I accelerated to that, and, um that's going to be my final velocity, right? (Ok.) Say my initial velocity si 10, and then I accelerated again, 10 meters per second, then my final velocity will be 20, so just by doing that, I can, I have just used the equation without knowing it, indirectly, just by thinking about it.

I: Ok, so there's, ok, there's a situation where just by thinking about it, you have done this equation pretty much, (mm-hm) ok, is that something that happens a lot in this course?

D: When I was little, yeah, that's how I remembered that when I was little, by actually looking at car speedometers, [laugh] because I grew up with this infatuation of cars, and, so I think ok, if I'm going to accelerate at 50 miles per hour, so zero seconds, 50 miles per second, then in the first second, my speed is going to be 50 mph, right? (right) And I keep on adding and adding and what if I accelerate for 2 seconds, then it's going to be 100, because the first second is going 50 and the second second starts with 50 so it's going to be 100, so just by thinking like that. I mean, know I don't even have to think about it anymore, [unintelligible].

I: Ok, so, but in this course, you're taking physics, and, for example, today, you get a formula $v = r\omega$, and I say well, how do you know that

D: I can't picture that, I have to, either memorize or look it up in the book. That's one 2
example that's not really obvious to me.

I: Ok, and would you say, is it most of the time that it's like this, or is it most of the time that it's like this, in this course?

D: Once it gets to things that are not really apparent, like, if you look up in the sky it's not 3
really apparent to you, um, somebody, you know, there's going to be an angle opening up, that's not really as obvious as driving, or juggling, or something like that. Once it gets to really abstract things, I can't really picture, or use the same method as linear motion.

[Daniel goes on to say that the only time he can do that sort of thinking is with linear kinematics.]

I: Yeah, sure, I understand that. Ok, so the question is, um, when you get, a new formula, v equals r omega, do you, do you try to compare it to looking out the window, do you try to make it fit in to looking out the window kind of stuff, or do you, do you just sort of know that it does or it doesn't, and if it doesn't

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[1] *Concepts*: Daniel gives a description of understanding the formula as closely tied to informal knowledge, and, in fact, describes the common-sense version as using the equation "without knowing it... just by thinking about it." Reconciled: First, that he came up with this explanation is evidently a result of our previous conversation; second, even here, he describes the functionality of this conceptual version as helping him remember the formula, rather than as part of his understanding. In this regard, it is relevant that he remembers my question as how he 'remembers/memorizes,' which is not what I had asked.

[2] *Apparent Concepts*: What Daniel thinks can be understood in this way, by "picturing," is limited. He does not think of constructing or actively seeking out a conceptualization if one is not readily available. In part, this coding is based on my assumption that this particular formula, $v = r\omega$, should be accessible to conceptual understanding, if one thinks to look.

[3] *Apparent Concepts*. Daniel says that his experiential knowledge is only useful with things that are "really apparent."

D: $v = r\omega$ is not, it's something that just, this is the first time I've seen $v = r\omega$, so to use an example like this is, I mean if it's something you have to seen before, I've never seen before, and this is the first time I've seen it, it's pretty hard to find examples, sometimes it's pretty hard to find examples, in relating it to real life situation, not all the time, but in this case, I can't. But, um, I guess it's one of those things where, if I take enough years of physics, then maybe, once I got really familiar with these things, I believe that I can look out the window and perceive, and see that actually happening. So, maybe you just picked a wrong example, in this case [1]

I: Yeah, well this just came up today. If you have another one, please

D: [unintelligible]

I: Um, so, if you get something like this, and it doesn't

D: Click right away?

I: Well, that's, I guess that's part of the question. [pause] Is it, would you say that this is something that makes sense? Would you say it's, here's, let me shift to another example. What a lot of people have, is about circular, a lot of people have, just from everyday kinds of knowing, that when you're going in a circle, you get thrown outward.

D: When you're going in a circle you get thrown out

I: Yeah, you know, people can remember, or have been in a car that's driving in a circle, what you feel is a push, being pushed to the out side, right?

D: Uh-huh, yeah.

I: If you ride in a car that suddenly takes a sharp right turn, you feel pressed up against the door, and so a lot of people have this, looking out the window kind of knowledge that, um, that there's um, if you're going in a circle, you get pushed outward, (mm-hm) and then you take a physics course, and you find out that really you're getting pushed inward.

D: [pause] It's two different forces, though. Centripetal and centrifugal.

I: Ok, so, well go ahead.

D: I'm just making a comment.

I: Well, the question is, for some people, this is a, there are a few different ways of handling it, I mean there's one way to say, well, I was just wrong, you're not thrown outward, that just isn't the way it is, you're pushed inward.

D: My, perception of the whole thing, is that the centripetal force is greater than the centrifugal, maybe I'm mixing backwards, but the force outside, that's pushing me outside is greater than the force that's pushing me inside, that's why it cancels out the the force that's pushing me inside, you know, that's like, just summing up the forces, and, force is pushing me outside happens to be greater in this case. Now, I, wouldn't worry about, after taking physics I know that it is a force that's pushing me inside, but I'm not worried about where it went, because I guess I can just see it as, um, I keep on remembering when you're a kid, and you're playing with the foot things, and they always go straight after you let it go, so, saying, well [2]

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[1] *Weak Concepts*, *Apparent Concepts*: Daniel feels that it may be possible to "actually see that happening" if one has had "enough years of physics." It is not a form of understanding he expects of himself. The coding of *Weak Concepts* is consistent with that of *Apparent Concepts*: Daniel seems to think of the conceptual knowledge someone familiar with physics would have in *Apparent* terms.

[2] *Apparent Concepts*, *Misconception*: Daniel describes an apparent and inappropriate conceptualization.

yeah, maybe it's being pushed towards me, but the force that I'm exerting outwards is so much greater that it cancels out.

I: Ok. So, would you say it's important to you, or it's an extra, if you get a new formula, so I'll use this example still, just because we've been using it, you read something like this in the book, um how important is it to you to have it fit with looking out the window stuff?

D: It would be nice.

I: It would be nice. Is it important to this course?

D: It's, [pause] it's not that important such that I'll have a fit over it if I can't look out the window and see it happening, it would just be a lot more convenient if I can look out and see it happening. **1**

I: It would be more convenient.

D: Yeah, it would make life a lot easier, just because I can remember it.

D: It just, it's more interesting also, because in chemistry, so you mix a bunch of liquids together, but what's the use? I don't see any practical use to it, while if, the distance formula, the velocity formula, the acceleration formula,

I: You can see.

D: Yeah, if I'm going to get home in 6 hours from Berkeley, of course [laugh] I'm going to think about it all the time, like, yeah I'm driving at 75, yeah, I should be home in a few hours, so it's practical, I can actually apply it to real life, and it would be nice if I can do that, while in chemistry, with acid and bases, and I get water, am I gonna drink that water, no. [laugh]

[Etc on chemistry demo, drinking water.]

I: So can you give me, how much of this course, angular momentum stuff, is that stuff that you,that fits this you can see it in real life?

D: No, I have a lot of problems with angular momentum, inertia, center of masses, it's starting to click afterwards, but, it's one of those frustrating things when you're doing the homework and you can't do it, and afterwards, once you've seen it enough, you get more of a feel to it. So even though I haven;'t fully mastered a few topics, in, after, between the first and second midterm, I feel that if I can go, if I can take a course that is based, that is somewhere between basic and advanced, somewhere between basic and 7A, that goes in a slower pace, and I can work one on one with somebody, I can have a much better understanding of this course, and a much better perception of how these concepts work, and maybe then I can really apply a few things that's outside, in the real life situation. Because I see [the professor] up there, and he lectures, and he can come up with examples in real life that I never would think about, but then of course he's a professor, and not just a physics student, [laugh] so I'm sure if I take this course, I mean, I'm not going to do that too. I mean yeah, it would be nice if I can do that, I think physics is a lot more useful for me, a lot more practical for me than chemistry.

I: Ok, um, and just a couple more, cause we're a little bit over already, actually. So, kinetic energy momentum, is an example, that you say you haven't

D: fully mastered, yeah.

I: Fully mastered, but you, on the question on this midterm, the first question was easy.

D: The first question? Yeah.

I: Yeah, which was on kinetic energy.

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[1] *Apparent Concepts*: Daniel does not expect informal knowledge; it only makes things easier sometimes, when it is available, by helping him remember.

D: Well, yeah, um, it's just, because, the difference between kinetic energy and momentum, force and momentum I have problems with, kinetic energy and momentum, it's, the way they ask this question is something that I can figure out the answer to, without much stressing out, because, it's just something about kinetic ene, well, is kinetic energy going to be one half at this point compared to this point? So all I have to do is use the kinetic energy formula, which is 1/2 m v squared, plug in a few values, and check. And with the momentum, I only have to plug in p equals m v and check with different values, so that doesn't give me any problems. While if you say, is kinetic energy the same as momentum, I say wait a minute, one of them is 1/2 m v squared, and one of them is just mv, so the units are different, so I don't think they're the same, but I don't really know what the relationship is.

I: Ok. So in that case, just knowing that they're different formulas, you're not happy with it.

D: Um, knowing that they have different formulas, I can solve this problem, but that doesn't explain to me the difference or, what both of them mean. [1]

I: Ok, and, is that something that you could be able, do you think somebody could or should be able to get that out of this level course, or is that something that you need to take a lot of physics to get to, or

D: I don't think you need a lot of physics to get to it, it's just a matter of sitting down with somebody who knows physics and talking about it. I'm sure if I can talk with some grad student for an hour and talk about these things, and have them explain to me, it wouldn't be much problem at all, but [the professor] just goes so, blitzkrieg pace, that I don't really feel like going, excuse me, what's the difference between momentum and kinetic energy? That would be kind of embarrassing to do in class. [2]

I: All right, so let's stop. Just last thing, really the bottom line of all of this is to find out what aspects of this course are good, and what aspects are not good for you to learn physics, and the reason I ask you to do all these problems is cause I want to find out just how much physics are you learning, and I like to find out how much physics to you feel like you're learning

D: All these things, all the chapters that have been covered so far, some of the problems can be extremely extremely frustrating, like um collision in two dimensions, and what angle the particle separated with, and all those functions, I feel that proving the formula is not really necessary for me, it doesn't matter if I can prove it or not, as long as I know that someone has proven it before, I'm happy, but, I wouldn't mind knowing the formula, it helps, well, you know, I never know when I can use the formula, maybe I will need it someday to figure out something, so I don't mind learning all these formulas at all, and understanding all these concepts, the thing that frustrates me is not being able to do the homework problems because sometimes they are so trivial, and sometimes [the professor] assigns the hard problems, proving this and proving that. Um, I don't really understand the point of, ok, so here's the concept, (right) prove that concept in a homework problem, I believe more in saying here's a concept, here's how it works, having someone explain to me how the concept works, more than, yeah, there's this concept, derive a formula for this concept, to prove it. That's not important to me. [3]

I: Here's a concept, derive the formula for this, to prove it. Ok. So you're, you would say maybe there are too many derivations in the course.

D: In the homework problems.

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[1] *Concepts*: Daniel distinguishes knowing the formulas from knowing what the concepts mean. Reconciled: As with Daniel's earlier remarks about energy and momentum, he seems to be looking for a literal association with something he can perceive.

[2] *By Authority*: Daniel understands learning as having things explained.

[3] *Formulas*: Daniel describes the content as formulas. *Weak Coherence*: He says he does not feel derivations are important for him to know. *By Authority*: Daniel insists that the instructors are responsible for his being able to understand the material.

I: In the homework problems.

D: Because the way I look at it, this is my point of view, is that, ok, so there's a concept, and there's a bunch of physicists out there that's getting paid 40,000 dollars a year, or whatever, and here I am paying 15,000 dollars a year, or however I'm paying, if I have to pay for it, I'm not [laugh] gonna have to, I'm not going to derive this thing for them [laugh], they're going to derive it for me and explain to me how it works. It's, to me is sometimes seems like an overkill. Sure, I understand that maybe doing the homework problem will help me understand the concept, but if it becomes so trivial, such that I can have someone explain it to me, without me having to spend 3 hours worth, getting you know, wasting 3 hours just to get extremely frustrated, it's not worth it.

I: So does this, I wonder how does that fit with, you know, wanting, for example, to really know what the difference is between momentum and energy. Is the derivation something that doesn't help you do that, and so you're

D: It's not that, it's, if you understand the concept, [pause] I think in order to derive the formula, you have to really understand the subject matter first, because how can you prove something, well, it's not, how to say, it's a lot easier to prove something or to come up with the formula for something, if you really understand how the thing works, how it ticks. So if I don't understand how it works, I don't understand this whole thing, why, how do you expect me to prove it, you can't really expect that much for me to prove that thing if I don't understand how it works. So if somebody walks up to me saying, write the formula, or show me how momentum and kinetic energy is different, I would say, well, I don't know.

I: So suppose somebody says, well that's why I gave you the problem,i mean this might be something that, I don't know whether [the professor] would say this or not, but I'll pretend I'm [him] for a moment, he says well that's why I'm giving you the proof, is because if I give that to you then what you are doing is figuring out how it works, that is what a derivation is, is figuring out how it works.

D: Then I'm going to walk up to him and say so I'm paying 400 dollars or whatever for 4 units of this course, so somebody can tell me to prove this thing?

I: Ok, so I'll still play the professor, I don't know whether he'd say these things or not. You want to know more about kinetic energy and momentum than just this is the formula for this one and this is the formula for that one, and so what I'm going to say to you is the way to understand what the difference is, really, is to, is, you can find that in deriving the two, or in doing these derivations, because the derivations, just as you said, involve understanding, and so by doing the derivations you are coming to understand these two formulas.

D: I will probably come back to you and say why should I spend 3 hours and not be able to derive this and get extremely frustrated, while here I am paying 400 dollars that goes indirectly to you somehow, where you can spend 5 minutes and explain this whole thing to me, and make me understand it in less than 10 minutes, the whole thing, how the this whole thing works. I understand, I mean I understand the whole point, but it gets to the point where, I mean, give the students a break, if they can't solve it in 3 hours, and you can explain it to them and make them understand in 10 minutes, I think the 10 minutes is worth his time, 3 hours is not worth my time definitely. [laugh] Because my life goes on whether I understand kinetic energy and momentum or not. But it would be nice, it would be con, I would want to know. And [pause] I guess this is where my economist friends [laugh] [unintelligible] **1**

I: This is what?

D: Economics brains, thinking of business like, here I am paying all this money, and I have to spend time to prove to him that I understand how these things work, it doesn't make sense. Especially when he can explain it to me clearly in 10 minutes, if I can talk to him one on one in

[1] *Weak Coherence, Formulas, By Authority*: Daniel sticks to and elaborates on his assertion that an instructor should "make [him] understand" the formulas.

10 minutes, I'm sure I can be able to understand the whole concept, so it doesn't make sense to me.

I: Ok, but he says, yeah but I really can't explain it to you, because you have all this looking out the window knowledge, that you really need to figure out the connections between, I mean I can tell you things, but if I tell you things, that's just, I mean, I told you the formulas.

D: Ok, then I would probably come back to him saying, well, um, maybe I need some directions, and maybe sitting with a graduate student or anybody one on one, and having them guide me through this whole thing, in order for me to understand this thing fully, is why I'm taking this course, instead of having you tell me to do it on my own.

I: Ok, so let's go back, general comments about the course, a little too many derivations in the homework, (yeah) um, maybe not enough time one one one with graduate students.

D: Discussion sections, it's, um it's catch twenty, well, I mean, it sort of affects each other, cause during the discussion section the graduate students spend so much of the time trying to explain how the homework works. Well if you assign homework problems that will, that's more of concept related instead of trivial related, like, more related to the basic concepts and building up from them, the graduate students will be able to talk more about how these things work, and slowly bring in hints and ideas of how to solve the problem, while, this is how, the sections I've sort of satisfied with works, is that, here he is explaining some physics at this level, and homeworks and other problems are at this level, and in the middle there is nothing. So I'm saying, why don't you give me problem sets about this level, have the TA explain at this level, and the transition is a lot easier.

I: Oh, so now there's a case where you don't want the TA to explain everything.

D: I didn't say that, I just said that

I: Haven't you actually said before that you didn't like one of the TAs cause he just gives you the whole homework?

D: Oh, yeah, it's convenient, but

I: So where does the economics mind work there?

D: How does the economics mind work there? (yeah) Because I'm not getting as much as I can, because that way I'm just, it's like somebody's just giving me money, and I'm not working for it, no, it's not like that, it's like, they're just stuffing food in my mouth and I'm actually not working, it's actually deteriorating me as a person, because my brain is not being stimulated, because all I have to do is integrals, I mean, if all I am doing is that, I'm going to take a math class. **1**

I: Ok, um, maybe it's a question of degree, if the professor wants you to do derivations, that's too much stimulation, or something.

D: Mm-hm. It's more, my point of view is, if you want to assign that problem, with H7A problems, or concentrate on that thing alone, because [the professor] is trying to explain what the book's saying, and he's pretty much general about how the whole things works, and then he gives problems to us way above what he says in lecture, and he doesn't give any hints about how to approach a problem at all, and I think that makes no sense, that sort of deteriorates the whole quality of the class.

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[1] *Independent*: As earlier, Daniel believes he needs to 'work for it' to learn. Unreconciled.